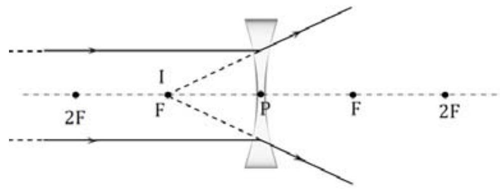
VELOCITY OF IMAGE, POWER OF LENS AND LENS CUTTING

Image formation by diverging lens

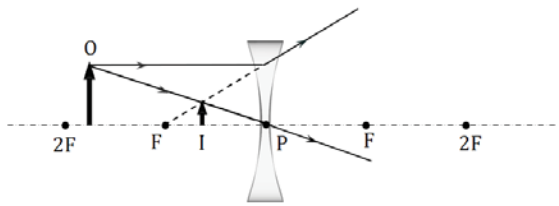
Real object

Case: 1



Object	Image	Nature
$O = -\infty$	$I = -$f	Virtual, Point Sized

Case: 2



In this case, the object is real and the object's distance is,

$$u = -x$$

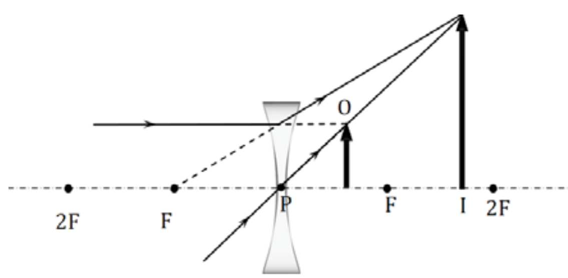
Therefore,
$$\frac{1}{v} + \frac{1}{x} = -\frac{1}{f}$$
$$\frac{1}{v} = -\frac{1}{x} - \frac{1}{f}.$$

v is always - ve. Hence, the image will be virtual.

Object	Image	Nature
$-\infty < O < P$	$-F < I < P$	P Virtual, Erect, Diminished

Virtual object

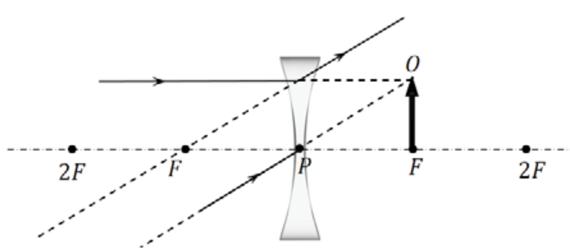
Case: 3



Object	Image	Nature
$P < O <$ F	$P < I < +\infty$	Real, Erect, Enlarged

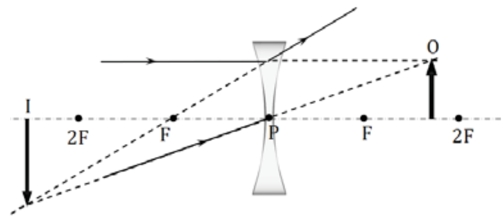
For the case of virtual object, the image gets shifted from $+\infty$ to $-\infty$ when the object crosses 1^{st} focus of any lens (converging or diverging) from left to right

Case: 4



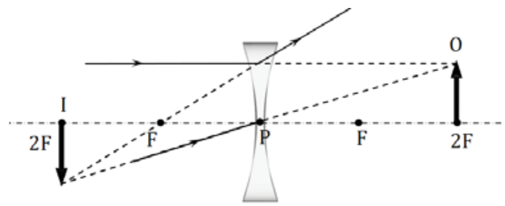
Object	Image	Nature
$O =$ F	$I = -\infty$ or $+\infty$	No comments

Case: 5



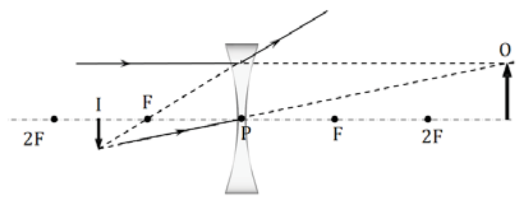
Object	Image	Nature
$F < O < 2F$	$-\infty < I < -2F$	Virtual, Inverted, Enlarged

Case: 6



Object	Image	Nature
$O = 2F$	$I = -2F$	Virtual, Inverted, Same

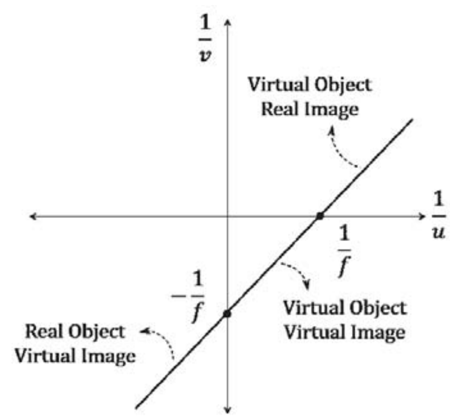
Case: 7



Object	Image	Nature
$2F < O < \infty$	$-2F < I < -F$	Virtual, Inverted, Diminished

The rays 1, 2 ,and 3 in the figure represent the case 1 to case 6 described in the table.
The ray 4 in the figure represents the case 7 described in the table.

Graphical Representation



We know that $\frac{1}{v} - \frac{1}{u} = \frac{-1}{f}$

Since f is a constant, let choose $\frac{1}{f} = c$. Now, by

Choosing $\frac{1}{u} = x$ and $\frac{1}{v} = y$, we get, $y - x = -c$

This represents a straight line with positive slope (i.e., +1), as shown in the figure.

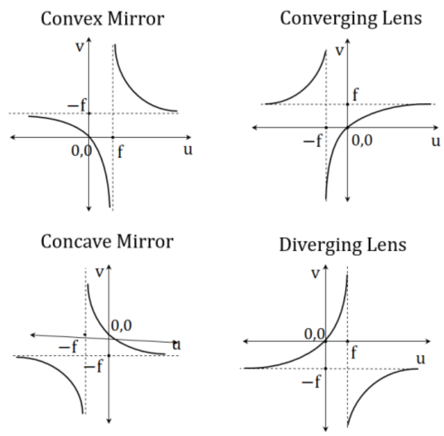
For, $x = 0 \quad y = \frac{-1}{f}$

$y = 0 \quad x = \frac{1}{f}$

In the 1st quadrant $\frac{1}{u} = x = +ve$ and $\frac{1}{v} = y = +ve$, thus, $u = +ve$ and $v = +ve$. Hence, this portion of the straight line refers "Virtual object" and "Real image".

In the 3rd quadrant $\frac{1}{u} = x = -ve$ and $\frac{1}{v} = y = -ve$, thus, $u = -ve$ and $v = -ve$. Hence, this portion of the straight line refers "Real object" and "Virtual image".

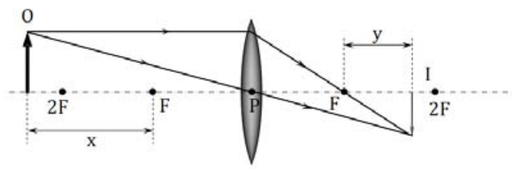
In the 4th quadrant $\frac{1}{u} = x = +ve$ and $\frac{1}{v} = y = -ve$, thus, $u = +ve$ and $v = -ve$. Hence, this portion of the straight line refers "Virtual object" and "Virtual image".



Newton's Formula (lens)

$$xy = f^2$$

x is distance of object 1[st] focus and y is distance of image from 2[nd] focus



Longitudinal Magnification

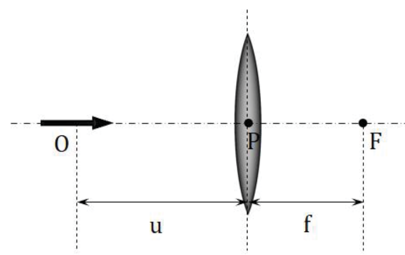
We have: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Differentiating both sides of the equation, we get,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$-\frac{1}{v^2} \cdot dv + \frac{1}{u^2} \cdot du = 0$$

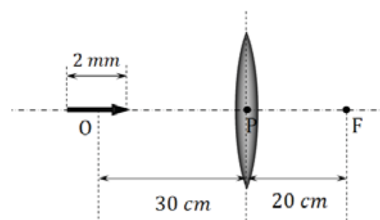
$$\frac{dv}{du} = \frac{v^2}{u^2}$$



dv is length of image and du is length of object

Applicable for very small length of the object (i.e., when $du << u$)

Ex. An object of length 2 mm is placed at distance of 30 cm from a convex lens of focal length 20 cm as shown. Find the longitudinal length of the image formed.



Sol. Length of the object: $du = 2$ mn

Distance of the object from the lens: $u = 30$ cm

Focal length of the lens: $u = 30$ cm

Therefore, the image distance will be,

$$v = \frac{ut}{u+t} = \frac{(-30)(20)}{-30+20} = \frac{600}{10} = 60u$$

Here, $du << u$. So, by applying the formula of longitudinal magnification, we can get the length of the image as given below:

$$\frac{dv}{du} = \frac{v^2}{u^2} \Rightarrow \frac{dv}{2mn} = \left(\frac{60}{30}\right)^2 = 4$$

$$dv = 8 \text{ mm}$$

Velocity of Images

Velocity parallel to principal axis

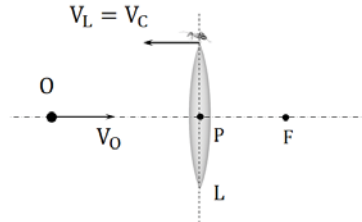
V_I = Velocity of image V_O = Velocity of object V_L = Velocity of lens

We have, $\frac{1}{u} - \frac{1}{v} = \frac{1}{f}$

Differentiating with respect to time,

$$\frac{-1}{u^2}\frac{du}{dt} - \frac{-1}{v^2}\frac{dv}{dt} = 0 \text{ (Since, f = Constant)}$$

$$\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$$



Here,

$\frac{dv}{dt}$ = Velocity of image w.r.t. pole of the lens $= V_{I,L} = V_I - V_L$

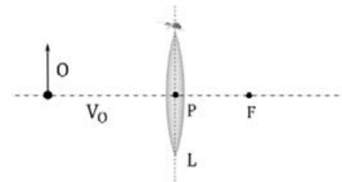
$\frac{du}{dt}$ = Velocity of object w.r.t. pole of the lens $= V_{O,L} = V_O - V_M$

Velocity perpendicular to principal axis:

We know: $\frac{h_i}{h_o} = \frac{v}{u}$

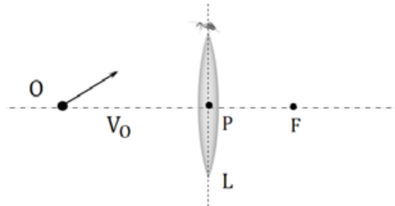
Differentiating w.r.t time t, we get, $\frac{dh_i}{dt} = \frac{v}{u}\frac{dh_o}{dt}$

$$V_i = \frac{v}{u}V_o$$



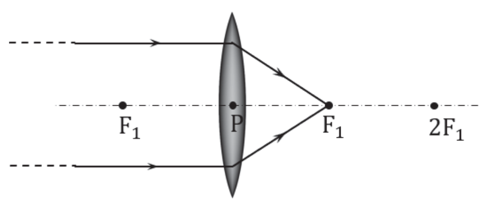
Velocity at any angle with principal axis: Velocity at any angle with principal axis:

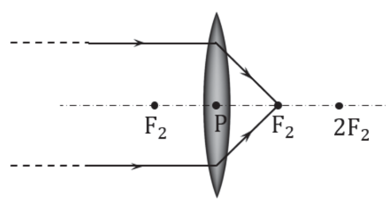
- Divide the velocity of the object in two components: one parallel to principal axis and the other perpendicular to the principal axis.
- Apply previously obtained two formulae and obtain the component velocity.
- Do vector addition to find the final answer



Power of Lens

The reciprocal of focal length (in meter) is defined as the optical power.





More the bending of light rays, higher the power of the lens.

The power of lens is defined as:

$$P_{Lens} = \frac{1}{f(\text{ in m})}$$

Put the value of focal length f with sign while solving the problems.

S.I. Unit of power of lens is *dioptre* (D) but focal length in this case should be in *meter* (m).

$$f = 20 \text{ cm}$$

$$p = \frac{1}{0.2} = 5D$$

Diverging lens has negative optical power and converging lens has positive optical power. Same convention is valid in case of mirrors.

The power of mirror is defined as:

$$P_{Mirror} = -\frac{1}{f(\text{ in m})}$$

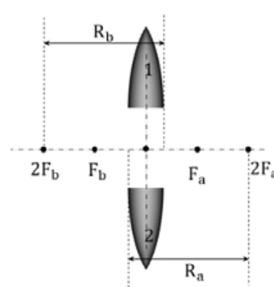
Cutting of Lens

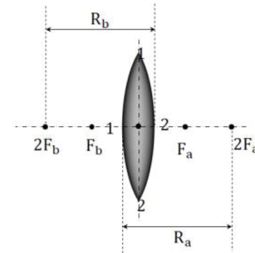
Focal length after parallel and perpendicular cutting

Cutting of a lens parallel to the principal axis.

After cutting the lens, the radius of curvature R_a and R_b doesn't change. Thus, the focal length of both part 1 and part 2 remains same.

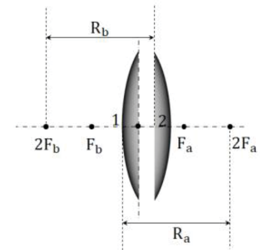
$$f_1 = f_2 = \frac{2(\mu - 1)}{R}$$





Cutting of a lens perpendicular to the principal axis.

Before cutting:
$$\frac{1}{f_i} = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$
$$\frac{1}{f_1} = \frac{(\mu-1)2}{R}$$
$$f_1 = \frac{R}{2(\mu-1)} = f$$
$$f = \frac{2(\mu-1)}{R}$$



After cutting:

Each part becomes plano -convex lens.
$$f_1 = f_2 = 2f = \frac{(\mu-1)}{R}$$

Ex. A convex lens of focal length 20 *cm* is cut parallel to the principal axis into two equal halves as shown and the parts are moved 2 *mm* away from either side of the principal axis. Find the separation between the images of the object *O* formed by these parts.

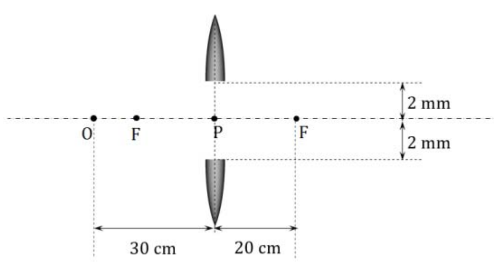
Sol. Since the lens is cut parallel to the principal axis into two equal halves and the parts are moved 2 *mm* away from either side of the principal axis, the principal axis of each part will also get separated by 2 *m*, as shown in the figure.

Now,

Distance of the object from the lens: $u = 30$ cm

Focal length of the lens: $u = 30$ cm

Therefore, the image distance will be,
$$v = \frac{ut}{u+t} = \frac{(-30)(20)}{-30+20} = \frac{600}{10} = 60u$$



For 1st part:

Image is at I_1
$$u = -30\ c, v = +60\ cm \text{ and } ho = -2 \text{ mm}$$

Therefore,
$$\frac{h_i}{h_0} = \frac{v}{u}$$
$$h_i = \frac{v}{u} h_o$$
$$h_i = \frac{60}{(-30)}(-2)$$
$$h_i = +4 \text{ mm}$$

For 2nd part:

Image is at I_2

$u = -30\ c, v = +60\ cm$ and $ho = +2$ mm

Therefore,
$$h_i = \frac{60}{(-30)}(+2)$$
$$h_i = -4 \text{ mm}$$

Therefore, the separation between the images of the object *O* formed by these parts is,
$$I_1 I_2 = (4 + 2 + 2 + 4) = 12 \text{ mm}$$