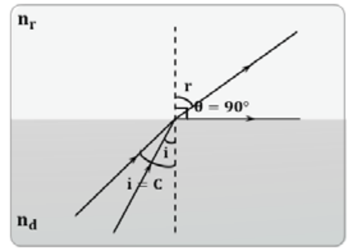
## TOTAL INTERNAL REFLECTION

### Critical Angle



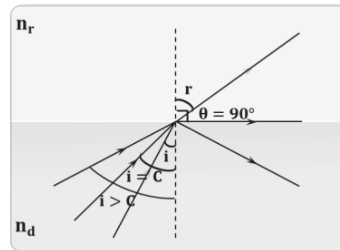

As light travels from a denser medium (with refractive index $n_d$) to a rarer medium (with refractive index $n_r$), the angle of refraction r is greater than the angle of incidence i.

When a ray transitions from a denser medium to a rarer one, the angle of incidence at which the angle of refraction is 90° is termed the critical angle C.

From Snell's law of refraction:

$$n_d \sin C = n_r \sin 90°$$
$$n_d \sin C = n_r \sin 90°$$
$$\sin C = \frac{n_r}{n_d}$$
$$C = \sin^{-1}\left(\frac{n_r}{n_d}\right)$$

### Total Internal Reflection (TIR)




When a ray travels from a denser to a rarer medium with an angle of incidence exceeding the critical angle, the interface between the two mediums behaves as a reflecting surface. This occurrence is referred to as "Total internal reflection."

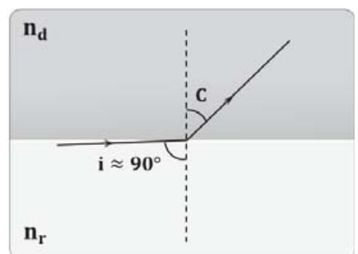
### Condition For TIR

The ray must travel from a denser medium to a rarer one.

$$i > C$$

As the ray transitions from a denser to a rarer medium, if the angle of incidence is below the critical angle, a minimal portion of the ray reflects back into the incident medium, with refraction being the primary effect. However, when the angle of incidence surpasses the critical angle, the entire incident ray reflects off the interface and returns to the incident medium. This phenomenon is termed Total Internal Reflection.

### Grazing Incidence



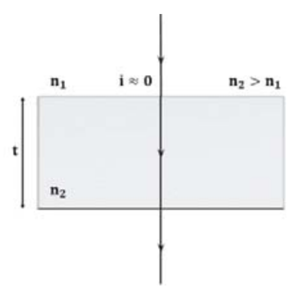

When a ray passes from a rarer medium to a denser one and approaches the critical angle, denoted as C, the phenomenon is termed as "grazing incidence," where the angle of incidence, denoted as i, is approximately 90°, causing the ray to refract at C.

In this case

$$n_r \sin 90° = n_d \sin r$$
$$\sin r = \frac{n_r}{n_d}$$
$$r = \sin^{-1}\left(\frac{n_r}{n_d}\right) = C$$

### Min. And Max. Lateral Shift

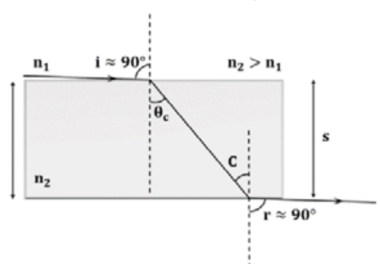
### Minimum Lateral Shift




When light rays approach the interface of two mediums almost perpendicular to the surface, they undergo no deviation, resulting in the absence of lateral displacement of the ray.

$$d_{min} = 0$$

### Maximum Lateral Shift




When light rays approach the interface of two mediums at nearly parallel angles, known as grazing incidence, the resulting lateral shift of the ray occurs.

$$s = t \sec r \sin(i - r)$$

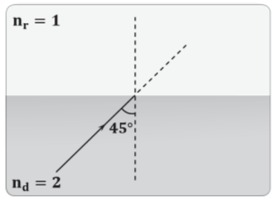
Here,

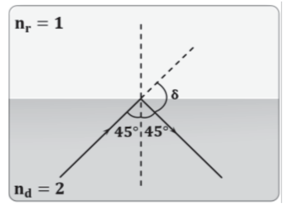
$$i = 90°, r = \theta_c$$
$$s = t \sec \theta_c \sin(90° - \theta_c)$$
$$s = t \sec \theta_c \cos \theta_c$$
$$s = t$$
$$d_{max} = t = \text{ Thickness of slab}$$

**Ex.** Find deviation (δ) when a ray traveling from a denser medium to a rarer medium is incident at an angle of 45° at the interface as shown in figure.

**Sol.** Since the light ray is going from denser to rarer medium, first of all let us check the critical angle. In this case, the critical angle is given by.

$$C = \sin^{-1}\left(\frac{n_r}{n_d}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^c$$



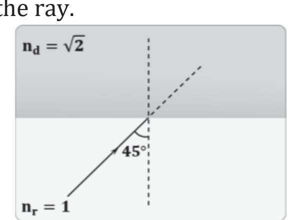

Since the angle of incidence is greater than the critical angle, total internal reflection will take place. Hence, light will come back to the incident medium, as shown in the figure.

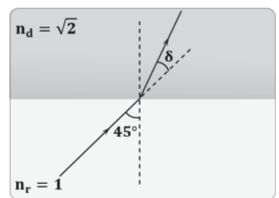
The angle of deviation in clockwise direction is given by,

$$\delta = (180^\circ - 2i) = (180^\circ - [2 \times 45^\circ]) = 90^\circ$$

**Ex.** Find deviation (δ) for the ray shown in the figure, take 1 and $\sqrt{2}$ as refractive indices of the rarer and denser media, respectively.

**Sol.** Since the light ray is going from rarer to denser medium, total internal reflection will not be taken place by the ray.




Now,

$$n_r \sin 45^\circ = n_d \sin r$$
$$\sin 45^\circ = \sqrt{2} \sin r$$
$$\frac{1}{\sqrt{2}} = \sqrt{2} \sin r$$
$$\sin r = \frac{1}{2}$$
$$r = 30^\circ$$

The angle of deviation in anti-clockwise direction is given by,

$$\delta = (i - r) = (45^\circ - 30^\circ) = 15^\circ$$

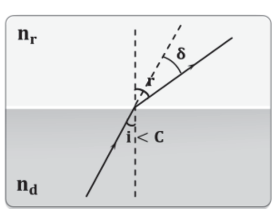
**Graph between δ and *i***

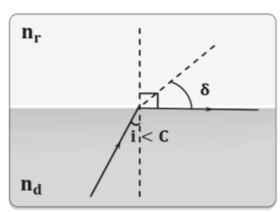
**Ray Moving From Denser To Rarer Medium**

$$i \leq C$$

Given that i < C, total internal reflection will not occur. Therefore, normal refraction will take place. Consequently,

$$n_d \sin i = n_r \sin r$$
$$\sin r = \frac{n_d}{n_r} \sin I$$
$$r = \sin^{-1}\left(\frac{n_d}{n_r} \sin(i)\right)$$



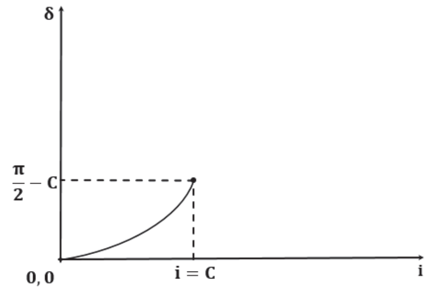




The angle of deviation in clockwise direction is given by,

$$\delta = r - I$$
$$\delta = \sin^{-1}\left(\frac{n_d}{n_r} \sin(i)\right) - I$$

| i | δ |
|---|---|
| 0 | 0 |
| C | $\frac{\pi}{2} - C$ |

Once the angle of incidence equals the critical angle, the angle of refraction corresponds to, $\frac{\pi}{2}$ Thus, the angle of deviation in clockwise sense becomes, $\frac{\pi}{2} - C$.

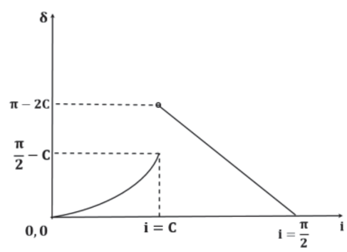


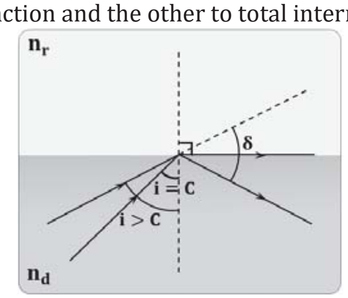

When $i$ is greater than c, total internal reflection occurs, leading to the angle of deviation in a clockwise direction being $\delta = \pi - 2i$

The equation depicts a line with a negative slope, forming a straight line.

When the angle of incidence slightly exceeds the critical angle, the angle of deviation attains $(\pi - 2C)$.

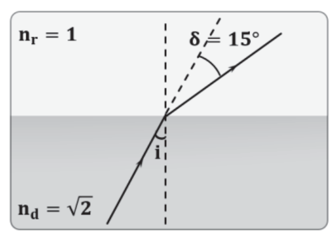
If $S < (\frac{\pi}{2} - C)$ In such a scenario, two distinct values of the angle of incidence ($i_1$ and $i_2$) arise from two separate phenomena: one attributed to refraction and the other to total internal reflection.




$\delta = \pi - 2i$

**Ex.** Find angle of incidence (i) for the given ray such that angle of deviation (δ) is 15°. Take refractive indices of denser and rarer mediums to be $\sqrt{2}$ and 1, respectively.




**Sol.** In this case, the critical angle:

$$C = \sin^{-1}\left(\frac{n_r}{n_d}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Given, $\delta = 15^\circ$

Therefore, in this case, $\delta(= 15^\circ) < \left[\frac{\pi}{2} - C\right] = 45^\circ$

So, two values of angle of incidence are possible.

Case 1: Refraction:

Angle of refraction, $r = i + 15^\circ$

By applying Snell's law, we get, $n_d \sin i = n_r \sin(i + 15^\circ)$

$$\sqrt{2}\sin i = \sin(i + 15^\circ)$$

Method to find sin 15°: $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$$\sin 15^\circ = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$\sin 15^\circ = \left[\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right] - \left[\frac{1}{\sqrt{2}} \times \frac{1}{2}\right]$$

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Method to find cos 15°: $\cos 15^\circ = \sin(90^\circ - 15^\circ)$

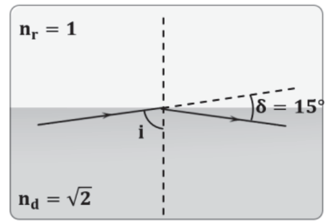
$$\cos 15^\circ = \sin 75^\circ$$

$$\cos 15^\circ = \sin(45^\circ + 30^\circ)$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sqrt{2}\sin i = \sin i\cos 15^\circ + \cos i\sin 15^\circ$$
$$\sqrt{2} = \cos 15^\circ + \cot i\sin 15^\circ$$
$$\sqrt{2} = \frac{\sqrt{3}+1}{2\sqrt{2}} + \cot i\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$
$$4 = (\sqrt{3}+1) + \cot i(\sqrt{3}-1)$$
$$3-\sqrt{3} = \cot i(\sqrt{3}-1)$$
$$\sqrt{3}(\sqrt{3}-1) = \cot i(\sqrt{3}-1)$$
$$\cot i = \sqrt{3}$$
$$i = 30^\circ$$

Thus, for the case of refraction, the angle of incidence should be 30°.



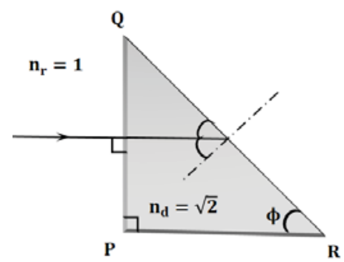

Case 2: Total internal reflection
In this case, the angle of deviation is given by

$$\delta = \pi - 2i$$
$$15^\circ = 180^\circ - 2i$$
$$i = \frac{180^\circ - 15^\circ}{2} = \frac{165^\circ}{2}$$
$$i = 82.5^\circ$$

Thus, for the case of TIR, the angle of incidence should be 82.5°.

$$i = 30^\circ, 82.5^\circ$$

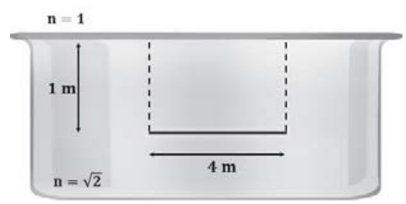
**Ex.** Find out the range of $\phi$ such that the ray always show TIR at QR.



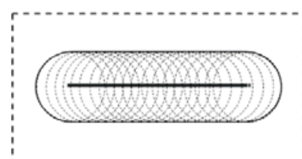

**Sol.** The ray always show TIR at QR if the angle of incidence at QR i.e., $(90^\circ - \phi) >$ Critical angle (C).
In this case, the critical angle is given by,

$$C = \sin^{-1}\left(\frac{n_r}{n_d}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$
$$(90^\circ - \phi) > C$$
$$\phi < (90^\circ - 45^\circ)$$
$$\phi < 45^\circ$$

**Ex.** If a tube light (line source ) of length 4 m , placed at depth 1 $m$ from the surface of the water as shown, find the shape and area of the bright patch formed on the surface of the water.




**Sol.** Consider each point of tube (line) as a point source of light. This point source can be seen by an observer on water surface if the total internal reflection of ray does not happen. Thus, it will be visible in a circular patch, radius of which is defined by critical angle.



Critical angle: $C = \sin^{-1}\left(\frac{n_r}{n_d}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

If R is the radius of the circular patch, then,

$$\tan 45^\circ = \frac{R}{1} \Rightarrow R = 1\text{m}$$

Hence, the area on fluid surface through which light will be seen by the observer is given by,

$$\text{Area} = \text{Area of rectangle } + 2 \times \text{ Area of semicircle}$$
$$= \text{Length of tube } \times (1+1) + \left(2 \times \frac{\pi R^2}{2}\right)$$
$$= (4 \times 2) + (\pi \times 1^2) = (8+\pi)\text{m}^2$$