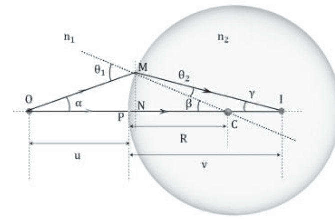
SPHERICAL REFRACTION



Spherical Refraction

P: Pole of the transparent sphere
C: Centre of curvature
R: Radius of curvature
PC: The line joining the point P and C is known principal axis

Formula of Spherical Refraction

O: Object I: Image
Assumption
The angle θ_1 and θ_2 are small ⇒ The rays are paraxial ⇒,and γ are small ⇒ The incident and refracted rays are close to the principal axis ⇒ The point M is very much close to the principal axis ⇒ The distance PN is negligible
Derivation of the formula:

$$\triangle\ \mathrm{OMC}\alpha + \beta = \theta_1$$
$$\triangle\ \mathrm{CMI}\ \ \theta_2 + \gamma = \beta.$$
$$n_1 \cdot \sin\theta_1 = n_2 \sin\theta_2$$
$$n_1\theta_1 \approx n_2\theta_2 \Rightarrow n_1(\alpha+\beta) = n_2(\beta-\gamma)$$
$$n_2\gamma + n_1\alpha = (n_2 - n_1)\beta$$
$$n_2 \tan\gamma + n_1 \tan\alpha = (n_2 - n_1)\tan\beta$$
$$n_2 \cdot \frac{MN}{NI} + n_1 \cdot \frac{MN}{NO} = (n_2 - n_1)\cdot\frac{MN}{MC}$$
$$\frac{n_2}{v} + \frac{n_1}{u} = \frac{(n_2-n_1)}{R}$$

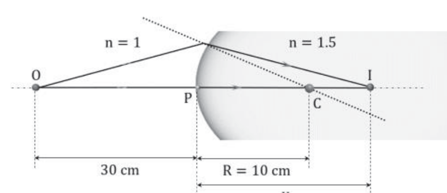
Generalized formula: $$\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2-n_1)}{R}$$

Pole is origin and the direction of incident ray is taken as + ve direction.
u, v, R are kept with their sign.
Only valid when rays are paraxial.
Rays originate from medium n_1 and goes into n_2.

Ex. An object is placed in a medium of refractive index n = 1 in front of a spherical glass slab of refractive index n = 1.5 as shown. Find the nature and the position of the image formed due to the object O.

Sol. Given,
$n_1 = 1$ $n_2 = 1.5$ $u = -30$ $R = +10$
By applying the formula of spherical refraction, we get,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2-n_1}{R}$$
$$\frac{1.5}{v} + \frac{1}{30} = \frac{1.5-1}{10}$$
$$\frac{1.5}{v} = \frac{1}{20} - \frac{1}{30}$$
$$\frac{3}{2v} = \frac{1}{60}$$
$$v = +90\ \text{cm}$$



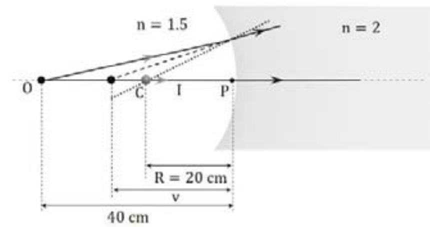
Since this is the case of refraction, real image will form when the rays actually meet behind the surface.
v = +ve = Real image
v = −ve = Virtual image
Therefore, in this case the image will be real and it will form at 90 cm to the right of the pole. This is the case of "Real image of real object".
90 cm (Real)

Ex. An object is placed in a medium of refractive index $n = 1.5$ in front of a spherical glass slab of refractive index $n = 2$ as shown. Find the nature and the position of the image formed due to the object O.

Sol. Given,

$N_1 = 1.5 \qquad n_2 = 2$

$u = -40 \qquad R = -20$



By applying the formula of spherical refraction, we get,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{2}{v} - \frac{1\cdot5}{-40} = \frac{2-1\cdot5}{-20}$$

$$\frac{2}{v} + \frac{3}{80} = \frac{-1}{40}$$

$$\frac{2}{v} = \frac{-3}{80} - \frac{1}{40}$$

$$\frac{2}{v} = \frac{-3-2}{80}$$

$$\frac{2}{v} = \frac{-5}{8016}$$

$$v = -32 \text{ cm}$$

This is the case of "Virtual image of real object"$-$ 32 cm (Virtual)

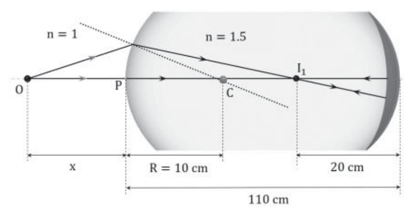
Ex. An object is placed in a medium of refractive index $n = 1$ in front of a spherical glass slab of refractive index $n = 1.5$ in which a concave mirror of focal length $f = 10\ cm$ is fixed on the other end as shown. Find x such that the final image is formed on the object O itself.

Sol. The final image will form on the object ( ) itself i.e., auto-collimation will happen if:

1. The image formed by the spherical glass slab is located at the centre of curvature (C_m) of concave mirror.
2. The image formed by the spherical glass slab is located at the pole of concave mirror.

Case-1

The image formed by the spherical glass slab is located at the centre of curvature (C_m) of concave mirror.



Since the focal length of the mirror is $f = 10\ cm$, the distance of Cm from the point P will be

$$(110 - 2f) = (110 - 20) = 90 \text{ cm}$$

$$1^{st} \text{ Ref } v = +90 \quad u = -x$$

$$\frac{1.5}{90} - \frac{1}{-x} = \frac{1.5-1}{+10}$$

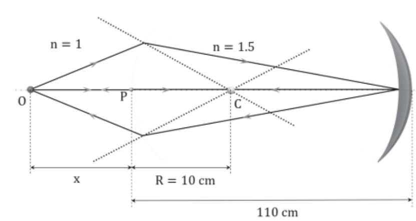
$$\frac{1}{60} + \frac{1}{x} = \frac{1}{20}$$

$$\frac{1}{x} = \frac{1}{20} - \frac{1}{60}$$

$$x = 30\text{cm}$$

Case-2

In this case, the image formed by the spherical glass slab is located at the pole of concave mirror. Therefore, image distance of the image due to the refraction at glass slab is,= +110



By applying the formula of spherical refraction, we get

$$\frac{1.5}{110} - \frac{1}{-x} = \frac{1\cdot5-1}{10}$$

$$\frac{3}{220} + \frac{1}{x} = \frac{1}{20}$$

$$\frac{1}{x} = \frac{1}{20} - \frac{3}{220}$$

$$\frac{11-3}{220}$$

$$x = \frac{220}{8} = 27.5\text{cm}$$