SHIFT FROM A SLAB

Shift From A Slab

In the scenario depicted, we have a glass slab of thickness t and refractive index n placed in air, as illustrated. Notably, the rays originating from the object undergo refraction twice at the two edges of the slab before they reach the observer.

Apparent position of image after first refraction:

We know that, $$\frac{d'}{d} = \frac{n_2}{n_1}$$

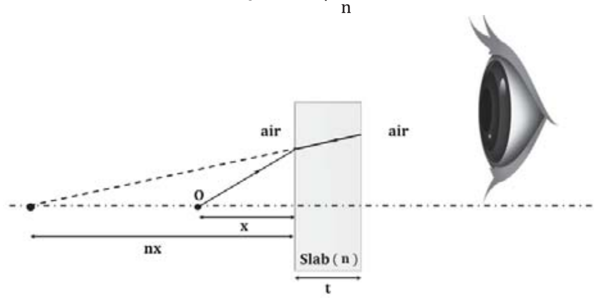
$$\frac{n}{l} = \frac{d'}{x} \Rightarrow d' = nx$$

Apparent position of image after second refraction:

We know that, $$\frac{d'}{d} = \frac{n_2}{n_1}$$

$$\frac{1}{n} = \frac{d'}{nx+t}$$

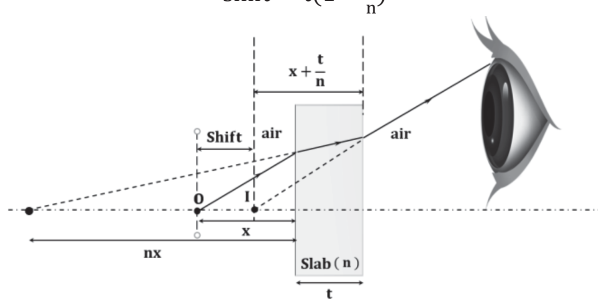
$$d' = x + \frac{t}{n}$$



Hence, the distance between the object and the position of the image is altered due to the presence of the slab.

$$\text{Shift} = x + t - (x + \frac{t}{n})$$

$$\text{shift} = t(1 - \frac{1}{n})$$



Since the shift is not influenced by the position of the object (x), the same displacement will be observed regardless of where the object is located.

Shift from a slab when surrounding medium is not air

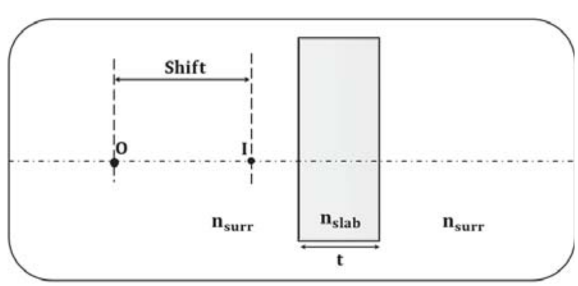
$$\text{Shift} = t(1 - \frac{n_{surr}}{n_{slab}})$$

The rays must be paraxial.

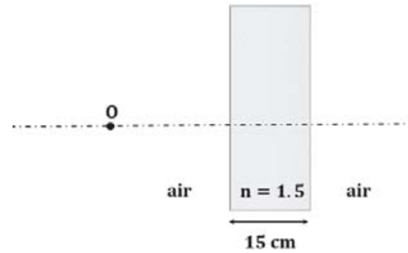
The medium on both sides of the slab should be identical.

The displacement is unrelated to the distance of the object from the slab (x).

The displacement is determined from the object.



Ex. Find the shift from a slab in given scenario



Sol. Given,

$$n_{surr} = n_{air} = 1$$
$$n_{slab} = 1.5$$

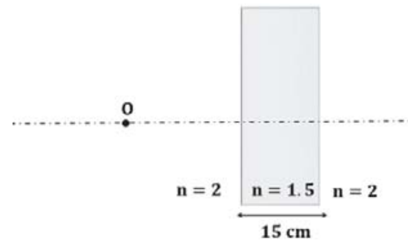
Thickness of the slab, $t = 15$ cm

We have, $$shift = t(1 - \frac{n_{surr}}{n_{slab}})$$

$$S = 15(1 - \frac{2}{3}) = +5 \text{ sm}$$

Shift in the direction of incident rays is positive.

Ex. Find the shift from a slab in given scenario



Sol. Given,

$$n_{surr} = 2$$
$$n_{slab} = 1.5$$

Thickness of the slab, $t = 15$ cm

We have, $$shift = t(1 - \frac{n_{surr}}{n_{slab}})$$

$$S = 15(1 - \frac{2 \cdot 2}{3}) = -5 \text{ cm}$$

What does negative shift mean?

Shift in the direction opposite to incident rays is negative.

Shift from a slab

$$Shift = t(1 - \frac{n_{surr}}{n_{slab}})$$

The rays must be nearly parallel.

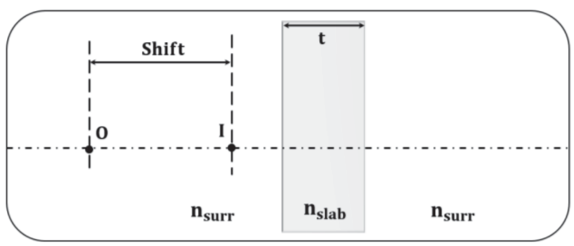
The medium on both sides of the slab should be identical.

The displacement is unaffected by the distance of the object from the slab (x).

The displacement occurs starting from the object.

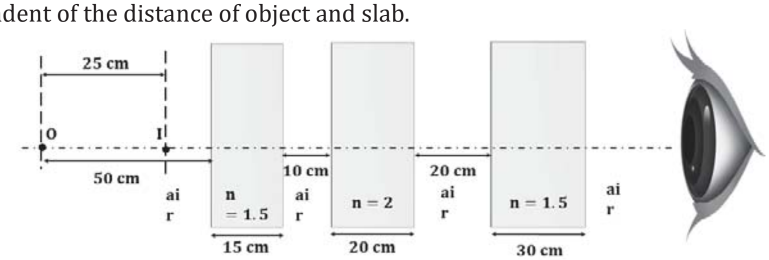
If the displacement is positive, consider it in the direction of the working rays.

If the displacement is negative, interpret it in the direction opposite to that of the working rays.



Ex. Find the position of object as seen by observer

Sol. Net shift in slabs can be found by adding the shift in individual slabs. We know that shift is independent of the distance of object and slab.



We have, $$shift = t(1 - \frac{n_{surr}}{n_{slab}})$$

Shift in individual slabs: $$shift = t(1 - \frac{n_{surr}}{n_{slab}})$$

$$S_1 = 15\left(1 - \frac{2}{3}\right) = +5 \text{ cm}$$
$$S_2 = 20\left(1 - \frac{1}{2}\right) = +10 \text{ cm}$$
$$S_3 = 30\left(1 - \frac{2}{3}\right) = +10 \text{ cm}$$

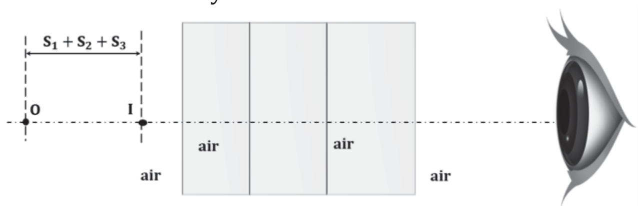
Net shift: $$S_{net} = S_1 + S_2 + S_3 = 5 + 10 + 10 = 25 \text{ cm}$$

Shift in a Composite slab

Net shift is summation of individual slabs shifts ($S_1 + S_2 + S_3$), it is found in the medium where object and observer are present.

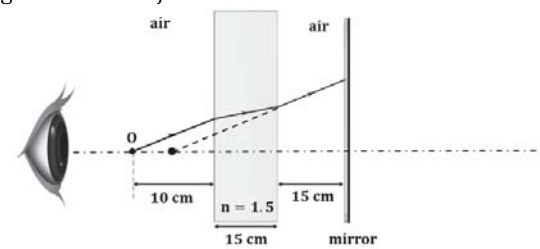
The object and the observer need to be in the identical medium.

The displacement is not affected by the distance between the slabs.



Ex. Find the distance of the image from the mirror as seen by the observer

Sol. Here, the rays will get refracted from slab and form the image I_1 which will not be visible to observer. This image becomes object for the mirror.



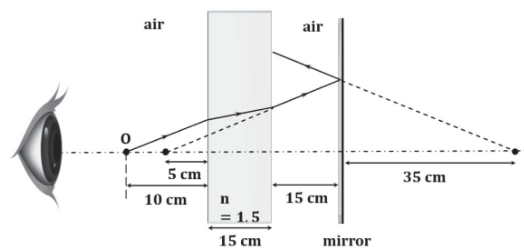
$$\text{Position of } I_1\text{: shift } = t\left(1 - \frac{n_{surr}}{n_{slab}}\right) \Rightarrow S_1 = 15\left(1 - \frac{2}{3}\right) = +5 \text{ cm}$$

Position of I_2:

Image I_2 is formed after reflection of rays from mirror.

As it is formed due to plane mirror.

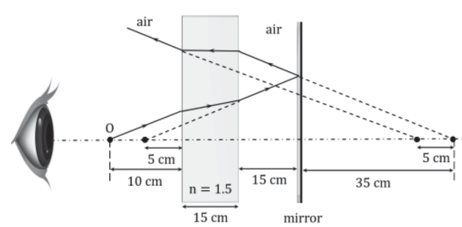
Object distance = Image distance= 5 + 15 + 15 = 35 cm



Position of I_3:

Image I_3 is formed after refraction of rays from slab.

$$\text{Position of } I_1\text{: shift } = t\left(1 - \frac{n_{surr}}{n_{slab}}\right) j \Rightarrow S_1 = 15\left(1 - \frac{2}{3}\right) = +5 \text{ cm}$$



As the shift is positive, image moves in direction of incident rays. Thus, the final image will be at 30 cm from the mirror.

30 cm (Right of Mirror)

Autocollimation

In Autocollimation, the rays trace back along the path of the incident rays, causing the image to form directly on the object itself.

Shift from a slab: Object and Observer are in different mediums

Since the medium of the object and observer differs, we cannot utilize the previous formula.

Let d’ be the distance of first image from slab.

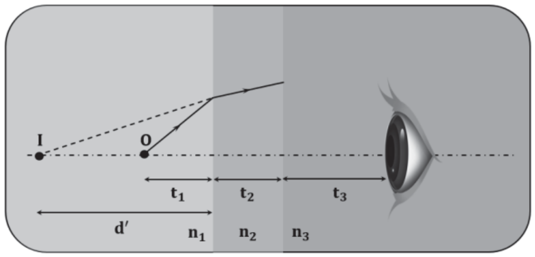
Thus, after first refraction,

$$\frac{n_2}{n_1} = \frac{d'}{t_1}$$
$$d' = \frac{n_2}{n_1} \cdot t_1$$
$$d' = \frac{n_2 t_1}{n_1}$$

$n_2 =$ Refractive index of slab

$n_1 =$ Refractive index of medium of object

$t_1 =$ Distance between object and slab



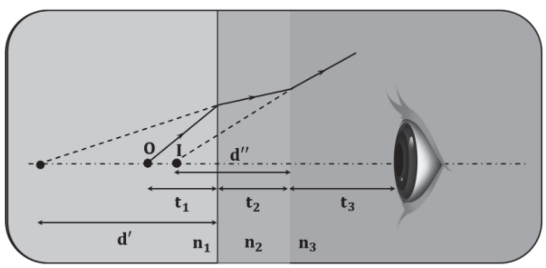
Shift from a slab; Object and Observer are in different mediums

Let d’ denote the distance of the first image from the slab.

Therefore, following the second refraction,

$$\frac{n_3}{n_2} = \frac{d''}{\frac{n_2}{n_1}}t_1 + t_2.$$
$$d'' = \frac{n_3 t_1}{n_1} + \frac{n_3 t_2}{n_2}.$$

$n_3 =$ Refractive index of medium of observer $t_3 =$ Distance between observer and slab



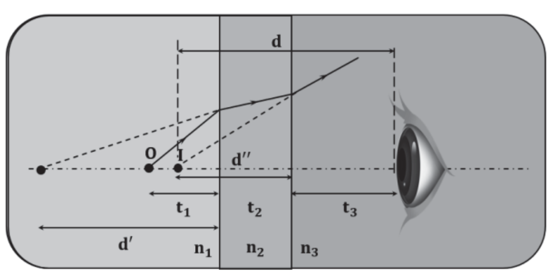
Shift from a slab: Object and Observer are in different mediums

Let d be the distance of final image from observer.

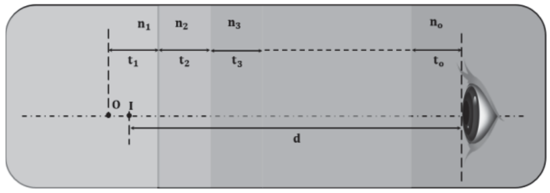
Thus, total shift:

$$d = d'' + t_3$$
$$d = \frac{n_3 t_1}{n_1} + \frac{n_3 t_2}{n_2} + \frac{n_3 t_3}{n_3}$$
$$= n_3[\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}]$$
$$= n_0[\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_0}]$$
$$d = n_3(\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3})$$

Here,$n_3 = n_o =$ Refractive index of medium of observer



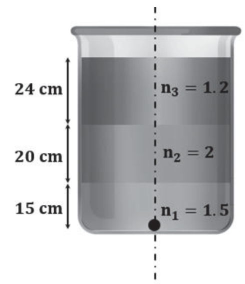
Shift from a slab; Object and Observer are in different mediums (n mediums)



The net shift can be calculated using this formula if n different mediums are present between object and observer as shown in the figure.

$$d = n_o\left(\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_2} - - - + \frac{t_o}{n_o}\right)$$

Ex. Find the location of the image formed by the object placed at bottom of container.



Sol. Given,.

$n_0 = 1$	$n_1 = 1.5$	$n_2 = 2$	$n_3 = 1.2$
$t_0 = 0$ cm	$t_1 = 15$ cm	$t_2 = 20$ cm	$t_3 = 24$ cm

We have,

$$d = n_o\left(\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_2} - - + \frac{t_o}{n_o}\right)$$

$$= 1\left[\frac{15}{1.5} + \frac{20}{2} + \frac{24}{1.2} + \frac{0}{1}\right]$$

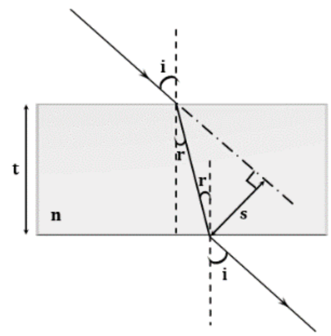
$$= 10 + 10 + 20 = 40 \text{ cm}$$

Lateral shift from a slab

If the incident rays deviate from being paraxial, the emergent rays from the slab will align parallel to the incident rays.

Deviation (δ) is zero for a slab

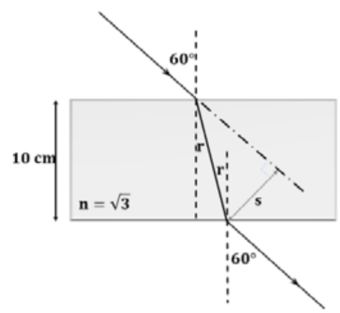
The lateral shift represents the space separating these two parallel lines. This shift is insignificant when dealing with paraxial rays.



$$\frac{\triangle ACD}{\text{Sin}(i-r)} = \frac{S}{AC}$$

$$s = t\sec(r)\sin(i - r)$$

Ex. Find lateral shift when a ray strikes a glass slab of refractive index 3 at an angle of 60° with the normal.



Sol. From Snell's law ($n_1 \sin i = n_2 \sin r$)

$$1 \cdot \text{Sin}60^\circ = \sqrt{3} \cdot \text{Sinr}$$

$$1 \cdot \text{Sin}60^\circ = \sqrt{3} \cdot \text{Sinr}$$

$$\frac{\sqrt{3}}{2} = \sqrt{3}\text{Sinr} \Rightarrow r = 30^\circ$$

We have, the lateral shift:

$$s = t\sec(r)\sin(i - r)$$

$$= 10 \cdot \text{Sec}30^\circ \cdot \text{Sin}(60 - 30^\circ)$$

$$= 10 \cdot \frac{2}{\sqrt{3}} \cdot \frac{1}{y} = \frac{10}{\sqrt{3}} \text{cm}$$