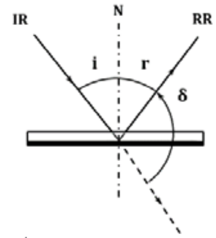
ROTATION OF MIRROR

Angle Of Deviation

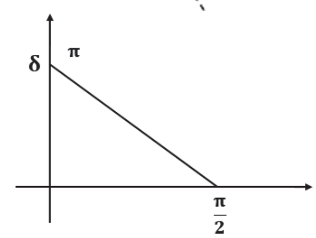


- The angle of deviation refers to the angle formed between the direction of the reflected ray and the direction of the incident ray when there is no interface present.
- The angle of deviation can be measured either clockwise or counterclockwise, depending on the context.
- The symbol δ represents the angle of deviation.
- In the case of a plane mirror, the angle of deviation can be determined using the provided formula.

$$\delta = \pi - 2i$$

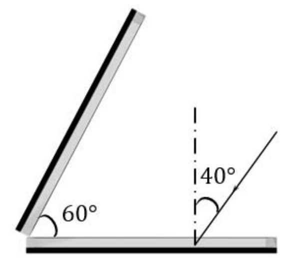
δ –Angle of Deviation i – Angle of incidence

- The graph will exhibit a straight line with a downward slope, as illustrated in the nearby diagram.



Ex. Calculate the total deviation resulting from two reflections in the described situation.

Sol. Since angle of incidence on the mirror M_1 is 40°, the angle of reflection will be 40°. Therefore, the reflected ray will make an angle 50° with the horizontal, as shown in the figure.



Hence, the angle of deviation, $\delta_1 = 180^\circ - 2 \times 40^\circ = 100^\circ(c \cdot w)$

From the knowledge of basic geometry, it can be said that the reflected ray from the mirror M_1 makes an angle of 70° with mirror as shown in the figure and hence, the angle of incidence on the mirror M_2 is 20°.

Hence, the angle of deviation due to M_2 is given by,

$$\delta_2 = 180^\circ - 2 \times 20^\circ = 140^\circ(c \cdot w)$$

Therefore, net angle of deviation after two reflection,

$$\Delta_{net} = \delta_1 + \delta_2$$

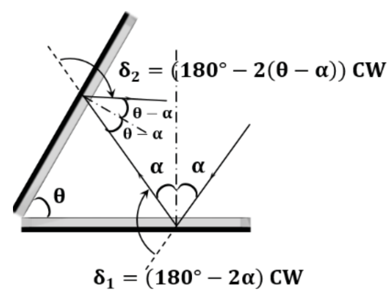
$$\delta_{net} = 240^\circ(c \cdot w)$$

$$120^\circ(A \cdot c \cdot w)$$

$$\delta_{net}|_{2R} = 240^\circ CW \text{ or } 120^\circ ACW$$

Angle Of Deviation: General Case

Let the angle of incidence be α and the angle between two mirrors beθ.



The angle of deviation due to M_1 is given by,

$$\delta_1 = 180^\circ - 2\alpha(c \cdot w)$$

Since the angle between two mirrors is θ and the reflected ray from M_1 makes an angle (90°−α) with the horizontal, it can be said that the reflected ray from the mirror M_1 will make an angle of $[180^\circ - \theta - (90^\circ - \alpha)] = [90^\circ - (\theta - \alpha)]$ with mirror M_2 and hence, the angle of incidence on the mirror M_2 is $(\theta - \alpha)$ as shown in the figure.

Hence, the angle of deviation due to M_2 is given by,

$$\delta_l = 180^\circ - 2(\theta - \alpha) = (c \cdot w)$$

Therefore, net angle of deviation after two reflections

$$\delta_{net} = \delta_1 + \delta_2$$

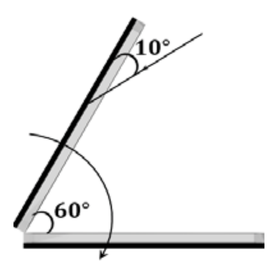
$$= 360^\circ - 2\theta(C \cdot w). = 2\theta(A \cdot c \cdot w)$$

$$\delta_{net}|_{2R} = 2\theta ACW$$

Angle Of Deviation: Short Trick

The angle of deviation remains unaffected by the angle of incidence.

Net angle of deviation δ_{net} is always 2θ following two reflections within two inclined plane mirrors.



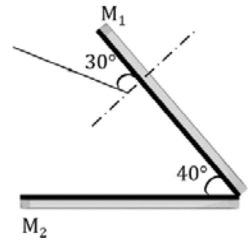
Steps to identify the direction of angle of deviation:

Consider the mirror on which the ray falls first as the first mirror.

Then make another mirror from the first mirror.

The direction in which the first mirror will rotate is the direction of that angle of deviation. $\delta_{net}|_{2R} = 120^\circ$

Ex. Determine the net deviation after two reflections.



Sol. M_1 is the first mirror since the light falls upon it initially.
The angle of inclination between two mirrors is, $\theta = 40°$. Therefore, the angle of deviation after two reflections will be, $\delta_{net} = 2\theta = 80°$.
Since to make the mirror from the mirror it should rotate anti - clockwise, the direction of angle of deviation will be ACW.

$$\delta_{net}|_{2R} = 80°\text{ACW or } 280°\text{CW}$$

Rotation Of Mirror

Imagine rotating the mirror by an angle θ in a clockwise direction.

Different notations that are used are as follows:

A_1 represents the incident ray, N_1 denotes the normal of the plane mirror prior to rotation, N_2 represents the normal of the plane mirror following rotation, R_1 signifies the reflected ray before the mirror's rotation, and R_2 indicates the reflected ray subsequent to the mirror's rotation.

From the figure, we get the following:

The angle between A_1 and N_2, $\angle A_2ON_2 = i + \theta$ this implies that the angle between R_2 and N_2 is also,$\angle R_2ON_2 = (i + \theta)$

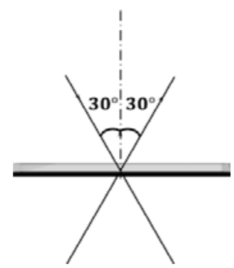
The angle between R_1 and N_2 is, $\angle R_1ON_2 = i - \theta$ Therefore, the angle between R_1 and R_2, i.e. the angle of rotation of the reflected ray,

$$\angle R_1OR_2 = \angle R_2ON_2 - \angle R_1ON_2$$
$$\angle R_1OR_2 = (i + \theta) - (i - \theta)$$

$\angle R_1OR_2 = 2\theta$ (in the same sense as rotation of mirror)

When the mirror is rotated by an angle θ, the reflected ray also rotates by 2θ in the same direction.

Ex. By what angle mirror should be rotated such that reflected ray becomes horizontal?

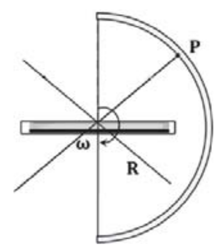


Sol. The reflected ray will become horizontal only if it is rotated by 60° in clockwise direction or, 120° in anti-clockwise direction.
Now, we know that if we rotate the mirror by angle θ then the reflected ray rotate by 2θ in the same sense. Therefore

1. To rotate the reflected ray by $2\theta = 60°$ in clockwise direction, the mirror should be rotated by $\theta = 30°$ in clockwise direction
2. To rotate the reflected ray by $2\theta = 120°$ in anti-clockwise direction, the mirror should be rotated by $\theta = 60°$ in anti-clockwise direction $\theta = 30\text{CW or } 60\text{ACW}$

Ex. If the mirror is rotating by ω rad/s, find the speed v with which spot P moves on the circular screen.

Sol. We know that if we rotate the mirror by angle θ then the reflected ray rotate by 2θ in the same sense.



The angular velocity of the mirror, $\omega = \frac{d\theta}{dt}$

Therefore, the angular velocity of the spot on the screen is given by,

$$\omega' = \frac{d2\theta}{dt} = 2\omega$$

Hence, the speed of the spot P on the circular screen will be given by,

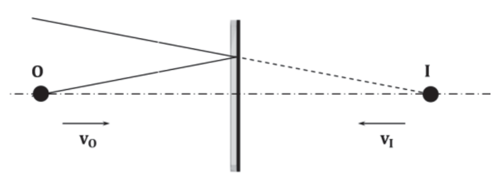
$$v = \omega' R = 2\omega R$$
$$v = 2\omega R$$

Velocity Of Image

When the mirror is fixed

If the object moves toward the mirror with a velocity v_o and the mirror remains stationary, then the image will move toward the mirror with a velocity v_I, as depicted in the illustration. Therefore, the relationship can be expressed as follows: $v_o = -v_I$

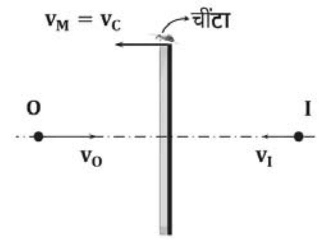
The negative sign indicates that the velocities of the object and the image are in opposite directions relative to the mirror.



When the mirror is moving

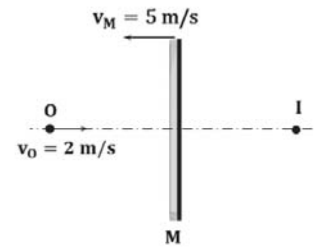
Suppose the mirror moves with a velocity v_M toward the object, as depicted in the figure. Consequently, the velocity of the ant residing on the mirror will also be, $v_C = v_M$.

From the ant's perspective, the velocities of the object and the image will be in opposite directions. Hence, the relationship between the velocities of the object and the image relative to the ant can be expressed as:



$$v_{oc} = -v_{IC}$$
$$v_{0M} = -v_{IM}.$$
$$V_0 - V_M = -(V_I - V_M)$$
$$V_0 - V_M = -V_I + V_M$$
$$v_I = 2v_M - v_O$$

Ex. An object and mirror are moving towards each other as shown. If the velocity of object and mirror are 2 m/s and 5 m/s respectively, find out the velocity of image v_I .



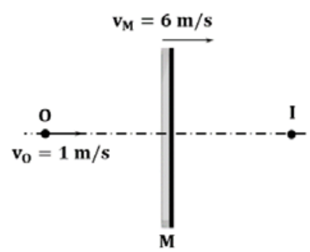
Sol. We know that $v_I = 2v_M - v_O$

Where v_I is the velocity of the object and v_O is the velocity of the mirror.

Assuming +ve x-axis as shown in the figure, we get,

$$v_O = +2 \text{ m/s}$$
$$v_I = -5 \text{ m/s}$$
$$V_I = 2(-5) - 2 = -12 \text{m/sec}$$
$$v_I = -12 \text{ m/s}$$

Ex. An object and mirror are moving in the same direction as shown. If the velocity of object and mirror are 1 m/s and 6 m/s, respectively, find out the velocity of image v_I .



Sol. We know that $v_I = 2vM - v_O$

Assuming +ve x-axis as shown in the figure, we get,

$$v_O = +1 \text{ m/s}$$
$$v_I = +6 \text{ m/s}$$
$$V_I = 2 \times 6 - 1 = 11 \text{m/sec}$$
$$V_I = 11 \text{m/sec}$$