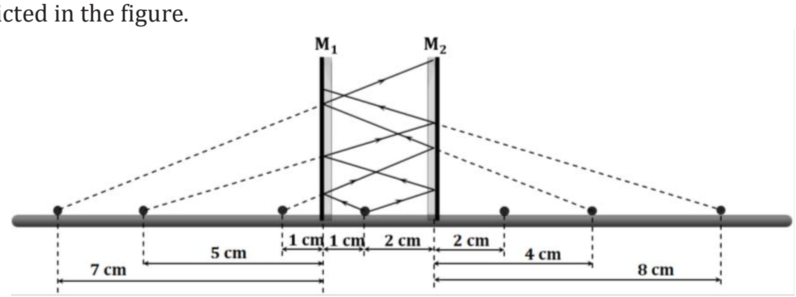
NUMBER OF IMAGE FORMED BETWEEN TWO PLANE MIRRORS

Number Of Images Between Two Parallel Plane Mirrors

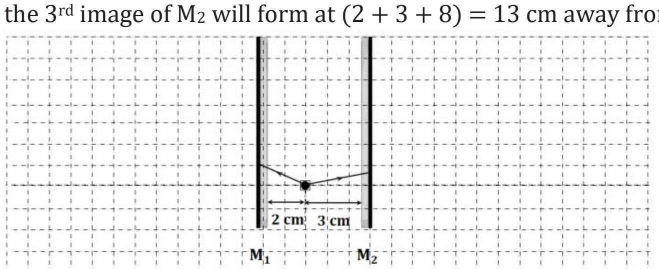
- Consider an object O positioned 1 cm from M_1 and 2 cm from M_2, as illustrated in the diagram.
- The ray originating from the object O, upon striking mirror M_1, produces an image I_1 located 1 cm away on the opposite side of the object with respect to M_1.
- Now, image I_1 serves as the new object for mirror M_2. Consequently, the distance from M_2 to the object is $(1 + 1 + 2) = 4$ cm, resulting in the formation of image I_2 at a distance of 4 cm from M_2, as depicted in the figure.



- Likewise, image I_2' serves as the new object for mirror M_1. Consequently, the distance from M_1 to the object is $(1 + 2 + 4) = 7$ cm, resulting in the formation of image I_3 at a distance of 7 cm from M_1, as depicted in the figure.
- Currently, rays originating from object O, upon striking mirror M_2, generate an image I_1'located 2 cm away on the opposite side of the object with respect to M_2, as depicted in the figure.
- The image I_1' now serves as the new object for mirror M_1. Consequently, the distance from M_1 to the object is $(1 + 2 + 2) = 5$ cm, resulting in the formation of image I_2 at a distance of 5 cm from M_1, as depicted in the figure.
- Likewise, image I_2 serves as the new object for mirror M_2. Consequently, the distance from M_2 to the object is $(5 + 1 + 3) = 8$ cm, resulting in the formation of image I_3'at a distance of 8 cm from M_2, as depicted in the figure. In this manner, the process of image formation persists indefinitely. Consequently, the number of images will be infinite.

Ex. A point object is placed between two parallel mirrors M_1 and M_2 at 2 cm from M_1 and 3 cm from M_2 as shown. Find out the distance of first three images from the mirrors formed on both sides after reflection from each mirror.

Sol. The object is 2 cm away from M_1 and 3 cm away from M_2. Therefore,
The 1st image of M_1 forms at 2 cm away from it. This 1st image of M_1 is responsible for formation of 2nd image of M_2 and hence, the 2nd image of M_2 will form at $(2 + 2 + 3) = 7$ cm away from M_2. This 2nd image of M_2 is responsible for formation of 3rd image of M_1 and hence, the 3rd image of M_1 will form at $(2 + 3 + 7) = 12$ cm away from M_1.
Since the object is 3cm away from M_2, the 1st image of M_2 forms at 3cm away from it. This 1st image of M_2 is responsible for formation of 2nd image of M_1 and hence, the 2nd image of M_1 will form at $(2 + 3 + 3) = 8$ cm away from M_1. This 2nd image of M_1 is responsible for formation of 3rd image of M_2 and hence, the 3rd image of M_2 will form at $(2 + 3 + 8) = 13$ cm away from M_2.



Short Trick:

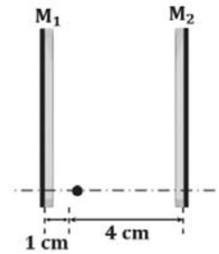
To determine the distances of the images formed when an object is positioned between two parallel mirrors, we should adhere to the following steps:

1. Determine the distance (d) separating the two parallel mirrors.
2. Calculate the distance of the initial image resulting from the reflections off both mirrors.
3. Add the distance d to the distance of the initial images produced by both mirrors to determine the distance of the subsequent images.

4. Continue adding the distance d to the distance of the images produced by both mirrors in each step to find the distances of the subsequent images.
In the previous example, the distance between two mirrors is, $d = (2 + 3) = 5$ cm.
The distance of the 1st images due to M_1 and M_2 are 2 cm and 3 cm, respectively.

Number of the image	Distance of the image formed due M_1	Distance of the image formed due to M_2
1st Image	2 cm	3 cm
2nd Image	8 cm	7 cm
3rd Image	12 cm	13 cm

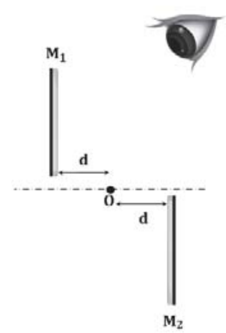
The arrows indicate that we should incorporate the distance d in a manner consistent with the steps outlined in the "Short trick."



Ex. A point object is placed between two parallel mirrors M_1 and M_2 at 1 cm from M_1 and 4 cm from M_2. Find out the distance of first three images from the mirrors formed on both sides after reflection from each mirror.

Sol. The distance between two mirrors is, $d = (1 + 4) = 5$ cm.
The distance of the 1 st images due to M_1 and M_2 are 1 cm and 4 cm, respectively.

Number of the image	Distance of the image formed due M_1	Distance of the image formed due to M_2
1st Image	1 cm	4 cm
2nd Image	9 cm	6 cm
3rd Image	11 cm	14 cm



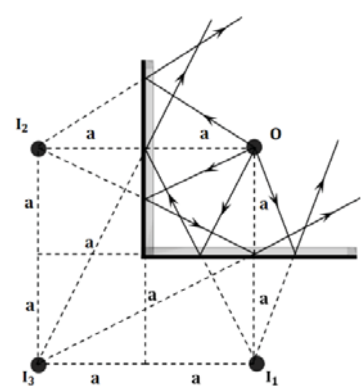
Ex. A point object is placed between two parallel mirrors M_1 and M_2 at d distance from M_1 and M_2, respectively, as shown. Maximum number of images, the observer can see are:

(A) 1 (B) 2
(C) 3 (D) ∞

Sol. In this case, although two images will be formed but the image I_1' is not in the field of view of the observer. Therefore, the observer can see the image I_1 only. Therefore, the observer can see only one image.
Thus, option (A) is the correct answer.

Number Of Images Between Two Inclined Plane Mirror

Number Of Images In Inclined Mirrors

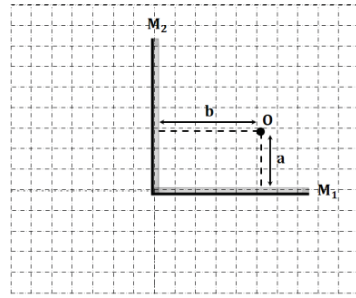


Initially, consider the greenish-yellow rays originating from the object O. The rays situated to the right side of the normal drawn from object O on mirror M_1 undergo a single reflection (due to M_1 only) and produce the image I_1. Meanwhile, the rays positioned to the left side of the normal undergo two reflections (due to both M_1 and M_2). However, these rays, situated to the left side of the normal, appear to emanate from I_1 and strike mirror M_1. Consequently, I_1 serves as the object for M_2 and generates the image I_3.
Next, consider the ocean blue rays originating from the object O. The rays situated above the normal drawn from object O on mirror M_2 undergo a single reflection (due to M_2 only) and create the image I_1. Conversely, the rays located below the normal undergo two reflections (due to both M_1 and M_2). However, these rays, positioned below the normal, appear to emanate from I_2 and strike mirror M_1. Consequently, I_2 serves as the object for M_1 and generates an image. However, due to symmetry, this image coincides exactly with I_3.
Therefore, we can conclude that when two plane mirrors are arranged perpendicular to each other and the object is positioned at the angle bisector (meaning it is equidistant from both mirrors), a total of three images are produced.

Ex. A point object O is placed at a and b from two perpendicular mirrors M_1 and M_2, respectively, as shown. Number of images, the object can make in both mirrors are:

(A) 3 (B) 4 (C) 6 (D) ∞

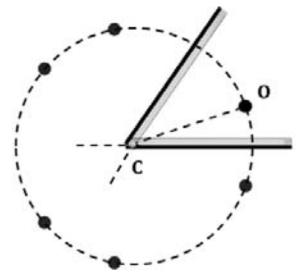
Sol. The object is a distance away from the mirror M_1 and b distance away from mirror M_2. Therefore, it forms an image I_1 at distance a from the mirror M_1 and forms another image I_2 at distance b from the mirror M_2.



If we extend the mirror M_1 and M_2, then I_2 is at distance a above the mirror M_1 and I_1 is at distance b to the right side of M_2.
Hence, the image of I_1 will form at distance b to the left side of M_2 and the image of I_2 will form at distance a below M_1. Both images coincide with each other and form one single image I_3.
Therefore, total three images are formed and hence, option (A) is the correct answer.

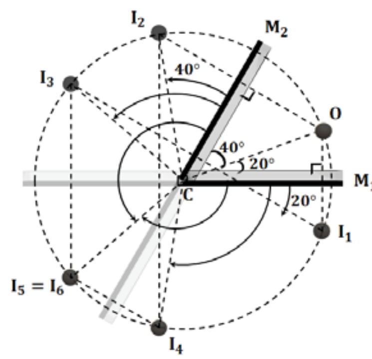
Number Of Images In Inclined Mirrors: Circle Concept

When an object is situated between two inclined plane mirrors, all the images of that object will be located on a circle with its center at the intersection point of the mirrors, denoted as C. The radius of this circle is equivalent to the distance from the object to the intersection point of the mirrors.



Ex. Find the number of images formed of the object O placed between the two mirrors M_1 and M_2 inclined at 60° with each other as shown.

Sol. We know that all the images of object O will lie on the circle having its center at the point of intersection of mirrors, C and Radius of the circle is equal to the distance OC.
The object is inclined at 20° from M_1 and 40° from M_2.Therefore,
The 1st image of M_1 forms at 20° from it. This 1st image of M_1 is responsible for formation of 2nd image of M_2 and hence, the 2nd image of M_2 will form at (20° + 40° + 20°) = 80° from M_2. This 2nd image of M_2 is responsible for formation of 3rd image of M_1 and hence, the 3rd image of M_1 will form at (40° + 20° + 80°) = 140° away from M_1.

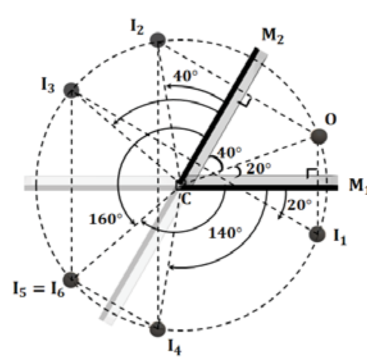


Mirror M_1	Mirror M_2
20°	40°
100°	80°
140°	160°

Since the object is inclined at 40 ° from M_2, the 1st image (I_2) of M_2 forms at 40° from it. This 1st image of M_2 is responsible for formation of 2nd image (I_4) of M_1 and hence, the 2nd image of M_1 will form at (40°+20°+40°) = 100° from M_1. This 2nd image of M_1 is responsible for formation of 3rd image (I_6) of M_2 and hence, the 3rd image of M_2 will form at (40°+20°+100°) = 160° away from M_2. Since the 3rd images (I_5 and I_6) of both the mirrors coincide, the total number of images formed is 5.

Coinciding Images

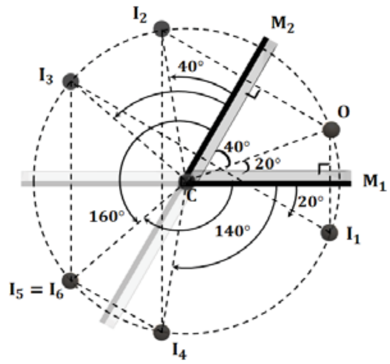
How can we determine whether the last two images coincide or not?
For images to coincide with each other, the sum of the angles formed by the final images on the mirrors plus the angle between the two mirrors must equal 360°.
When the last two images are coinciding with each other, it should be treated as a single image



Short Trick:

Angle between the mirrors $\theta = 60°$

Once the positions of the first images for both mirrors are determined, subsequent images can be found by incrementally adding the angle between the mirrors to the positions of the initial images, as illustrated in the table.



Mirror M_1	Mirror M_2
20° +60°	40° +60°
100° +60°	80° +60°
140°	160°

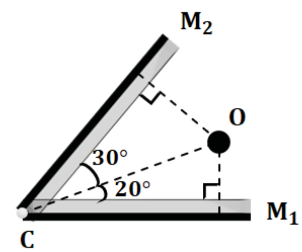
$140° + 60° \geq 180°$ & $160° + 60° \geq 180°$

So, we will stop counting images at this step. $n = 5$

Ex. Find the number of images formed of the object O placed between the two mirrors M_1 and M_2 inclined at 50° with each other as shown.

Sol. Here, the angle between the mirrors is 50°. The object is inclined at 20° with mirror M_1 and 30° with mirror M_2.

Thus, the 1st image of M_1 forms at 20° from it and the 1st image of M_2 forms at 30° from it. By using short trick of adding the angle between mirrors to first images, we get the locations of further images as shown in table.



Here, Sum of the angles made by the last images on the mirrors + angle between the two mirrors =

$$120° + 170° + 50 \neq 360°$$

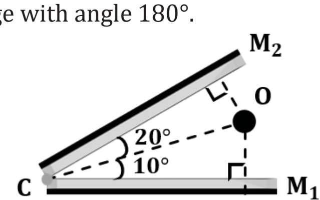
Thus, the last two images are not coinciding.

Mirror M_1	Mirror M_2
20°	40°
100°	80°
140°	160°

Here, Sum of the angles made by the last images on the mirrors + angle between the two mirrors = When the position of object becomes 180° to the plane of mirror, it cannot form the image as the object is parallel to this plane. Hence, we should not count the image with angle 180°.

Thus, the number of images formed is 7.

Ex. Find the number of images formed of the object O placed between the two mirrors M_1 and M_2 inclined at 30° with each other as shown.



Sol. Here, the angle between the mirrors is 30°. The object is inclined at 10° with mirror M_1 and 20° with mirror M_2.

Thus, the 1 st image of M_1 forms at 10° from it and the 1st image of M_2 forms at 20° from it. By using short trick of adding the angle between mirrors to first images, we get the locations of further images as shown in table.

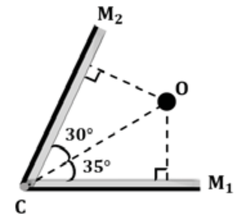
Mirror M_1	Mirror M_2
20°	10°
40°	50°
80°	70°
100°	110°
140°	130°
160°	170°

Here, Sum of the angles made by the last images on the mirrors + angle between the two mirrors =

$$170° + 160° + 30° = 360°$$

Thus, as the last two images are coinciding total images formed are

$$12 - 1 = 11; n = 11$$



Ex. Find the number of images formed of the object O placed between the two mirrors M_1 and M_2 inclined at 65° with each other as shown.

Sol. Here, the angle between the mirrors is 65°. The object is inclined at 35° with mirror M_1 and 30° with mirror M_2.

Thus, the 1st image of M_1 forms at 35° from it and the 1st image of M_2 forms at 30° from it. By using short trick of adding the angle between mirrors to first images, we get the locations of further images as shown in table.

Mirror M_1	Mirror M_2
30°	35°
100°	95°
160°	165°

Here, Sum of the angles made by the last images on the mirrors + angle between the two mirrors =

$$160° + 165° + 65° \neq 360°$$

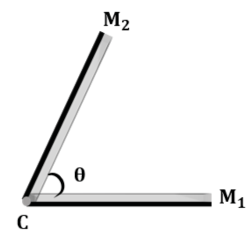
Thus, as the last two images are not coinciding, total images formed are 6.

$$n = 6$$

Number Of Images Between Inclined Mirrors

If the ratio $\frac{360°}{\theta}$ If θ represents the angle between two mirrors and is an integer, the following formulas can be utilized to determine the number of images.

$\frac{360°}{\theta}$	Position of Object	Number of Images
Even Integer	Symmetric and Unsymmetric Both	$\frac{360°}{\theta} - 1$
Odd Integer	Symmetric	$\frac{360°}{\theta} - 1$
Odd Integer	Unsymmetric	$\frac{360°}{\theta}$



Vector Form of Reflected Ray

Consider

$\hat{a}$ = unit vector along incident ray

$\hat{b}$ = unit vector along reflected ray

$\hat{n}$ = unit vector along normal

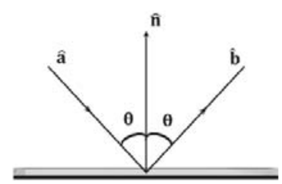
From parallelogram as shown in the figure,

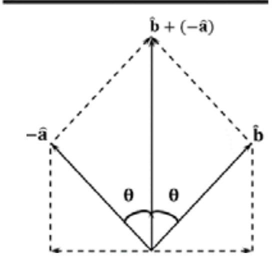
$$\hat{b} + (-\hat{a}) = \vec{n} = 2\cos\theta\hat{n}. \quad \dots (1)$$

Also, from dot product, $\cos\theta = \hat{a}\cdot\hat{n}$

By substituting in equation (1) we get,

$$\hat{b} = \hat{a} - 2(\hat{a}.\hat{n})(\hat{n})$$





Ex. Find the unit vector along the reflected ray if the vector along the incident ray is $\hat{a} = 3\hat{\imath} - 4\hat{\jmath}$ and the normal is as shown.

Sol. Here, normal is along y axis.

Thus, unit vector along normal is, $\hat{n} = \hat{\jmath}$

Unit vector along incident ray, $\hat{a} = \frac{+3\hat{\imath}-4\hat{\jmath}}{5}$

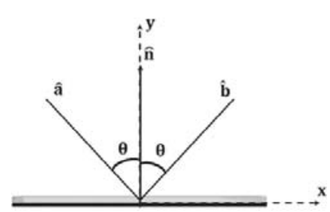
We know that, $\hat{b} = \hat{a} - 2(\hat{a}.\hat{n})\hat{n}$

By substituting the values of vectors,

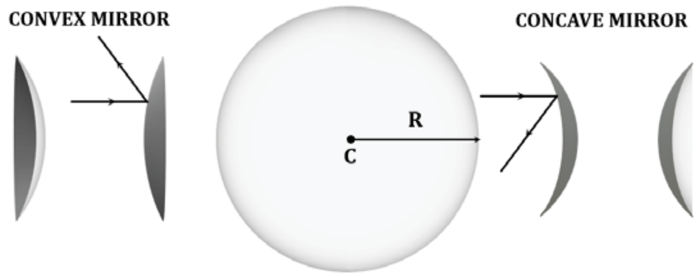
$$\hat{b} = \frac{3\hat{\imath}-4\hat{\jmath}}{5} - 2\left(\left(\frac{3\hat{\imath}-4\hat{\jmath}}{5}\right)\cdot\hat{\imath}\right)\hat{\jmath}$$

$$= \frac{3\hat{\imath}-4\hat{\jmath}}{5} + \frac{8}{5}\hat{\jmath} = \frac{3\hat{\imath}+4\hat{\jmath}}{5}$$

$$\hat{b} = \frac{3\hat{\imath}+4\hat{\jmath}}{5}$$



Introduction to spherical mirror



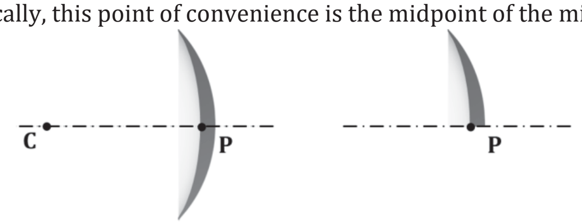
Convex Mirror
A mirror that reflects light outward is known as a convex mirror.

Concave Mirror
A mirror that reflects light on its inner surface is referred to as a concave mirror.

Centre Of Curvature
The sphere from which it is sliced is termed the center of curvature.

Radius Of Curvature
The radius of this sphere is known as the radius of curvature.

Pole A pole is a point on the entirety of the sphere from which measuring the object and image is convenient; typically, this point of convenience is the midpoint of the mirror.



Principal Axis
The line connecting the center of curvature and the pole is referred to as the principal axis.

Radius Of Aperture (r)

r = Radius of aperture; R = Radius of curvature
The aperture radius indicates the size of the mirror, while the curvature radius describes the curvature of the mirror.

Ex. Compare the size and curviness of the mirror A & B
1. Mirror-A, Radius of aperture = 10 mm, Radius of curvature = 5 cm
2. Mirror-B, Radius of aperture = 5 mm, Radius of curvature = 10 cm

Sol. r_A= 10 mm R_A = 5 cm; r_B = 5 mm R_B= 10 cm
The aperture radius indicates the mirror's size, whereas the curvature radius defines the curvature of the mirror.
Size – A > B Curviness – A > B