MIRROR FORMULA AND SIGN CONVENTION

Mirror Formula

α, β, γ are also small; NP is negligible.

$$\frac{\triangle OMC}{\alpha+\theta=\beta} \quad \dots (1)$$

$$\frac{\Delta CMI}{\beta+\theta=\gamma} \quad \dots (2)$$

From Equation (1) and (2) $\alpha - \beta = \beta - \gamma.\gamma + \alpha = 2\beta$

θ is small

$$\alpha - \beta = \beta - \gamma.\gamma + \alpha = 2\beta$$ $\sin\theta \approx \theta$

$$\tan\gamma + \tan\alpha = 2\tan\beta$$ $\cos\theta \approx 1$

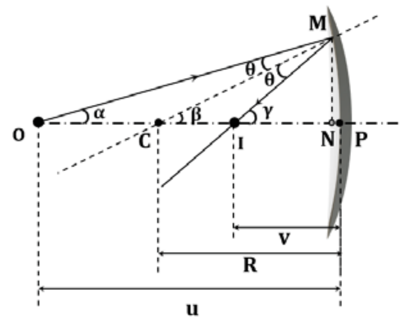
$$(\Delta MNI)(\Delta OMN)(\Delta CMN)$$ $\tan\theta \approx \theta.$

$$\frac{MN}{NI} + \frac{MN}{NO} = \frac{2\cdot MN}{NC}$$ $\gamma \approx \tan\gamma$

$$P_1 = vPO = 4PC = R$$ $\beta \approx \tan\beta$

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$ $\alpha \approx \tan\alpha$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$



Generalized Mirror Formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Valid for both mirrors.

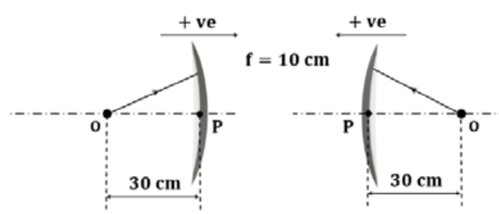
For all cases when we put u, v, & f with sign

Sign Convention

Assume pole as origin.

Wherever the rays of work come, there is positive.

Let's analyze the setup involving a concave mirror with an object O positioned 30 cm in front of the mirror to determine the image distance from the mirror. However, Ram and Shyam have depicted the ray diagram as illustrated in the figure. It is given that the focal length of the mirror is 10 cm.



Regarding Ram, the positive direction is from left to right because the incident ray travels in that direction. Conversely, for Shyam, the positive direction is from right to left since the incident ray moves in that direction. Hence, we derive the following:

Student Name	u	F
Ram	–30	–10
Shyam	–30	–10

Thus, employing the mirror formula yields:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

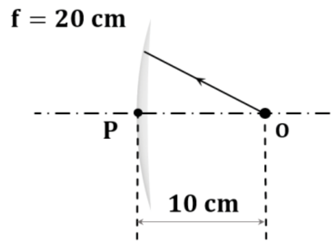
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$v = \frac{uf}{u-f}$$

$$= \frac{-30(-10)}{-30+10}$$

$$= \frac{300}{-20} = -15 \text{ cm}$$

Ex. Find out the position of image when object is placed as shown in the figure.



Sol. According to the sign convention, we can write the following:

The distance of the object, u = −10 cm

The focal length of the mirror, f = −20 cm

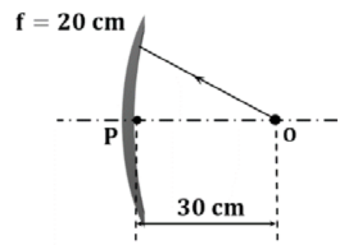
Hence, by applying mirror formula, we get,

$$v = \frac{uf}{u-f} = \frac{(-10)(-20)}{-10+20} = \frac{200}{10} = 20 \text{ cm}$$

Therefore, the image forms at a distance 20 cm behind the mirror. In this case, the reflected light doesn't meet at a point to form the image rather it seems like the light comes from the image. This type of image is known as "Virtual image"

$$v = 20\text{cm}$$

Ex. Find out the position of image when object is placed as shown in the figure.



Sol. According to the sign convention, we can write the following

The distance of the object, u = −30 cm

The focal length of the mirror, f = −20 cm

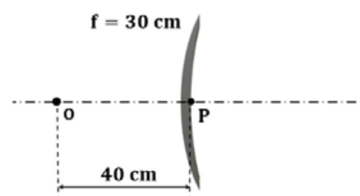
Hence, by applying mirror formula, we get,

$$v = \frac{(-30)(-20)}{-30+20} = \frac{60\phi}{-1\phi} = -60\text{cm}$$

Therefore, the image forms at a distance 60 cm in front of the mirror. In this case, the reflected light meets at a point to form the image and this type of image is known as "Real image"

$$v = -\ 60 \text{ cm}$$

Ex. Find out the position of image when object is placed as shown in the figure.



Sol. According to the sign convention, we can write the following:

The distance of the object, u = −40 cm

The focal length of the convex mirror, f = +30 cm

Hence, by applying mirror formula, we get,

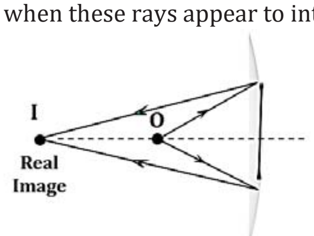
$$v = \frac{(-40)(+30)}{-40-30} = \frac{+1200}{+70} = \frac{120}{7} \text{ cm}$$

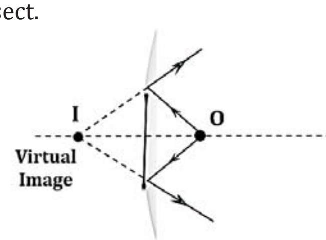
Therefore, the image forms at a distance $\frac{120}{7}$cm behind the mirror and the image will be a "Virtual image".

$$v = \frac{120}{7} \text{ cm}$$

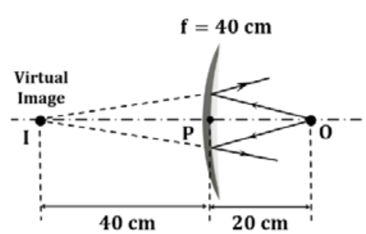
Real And Virtual Image

A real image of an object forms when the reflected rays physically intersect, whereas a virtual image forms when these rays appear to intersect.





Ex. Find out the position and nature of image.



Sol. According to the sign convention, we can write the following:

The distance of the object, u = −20 cm

The focal length of the convex mirror, f = −40 cm

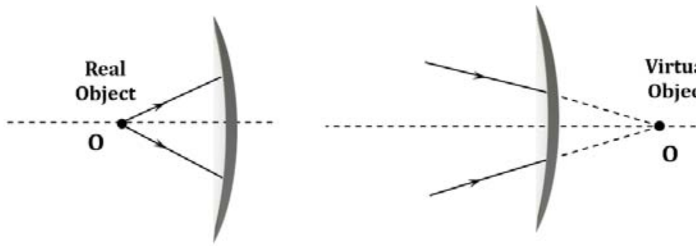
Hence, by applying mirror formula, we get,

$$v = \frac{(-20)(-40)}{-20+40} = \frac{800}{289} = 40 \text{ cm}$$

Therefore, the image forms at a distance 40 cm behind the mirror and hence, the image will be a "Virtual image".

$$v = +40 \text{ cm, Virtual}$$

When incident rays seem to converge at a point, the resulting object is termed a virtual object.
When incident rays seem to spread out from a point, the object is referred to as the real object.



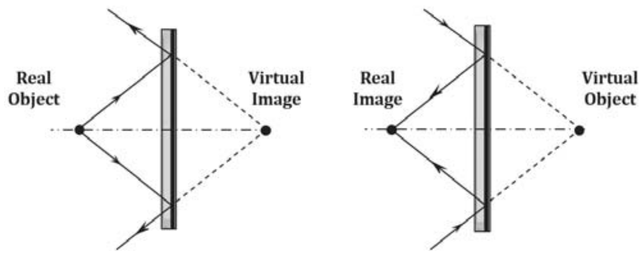
If an object is positioned 30 *cm* in front of a concave mirror with a focal length of 20 *cm*, the resulting image distance will be.

$$v = \frac{(-30)(-20)}{-30+20} = -60 \text{ cm}$$

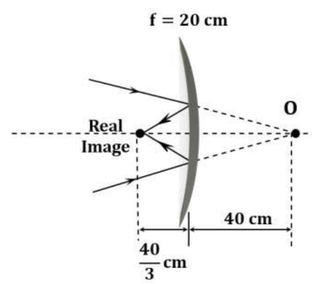
Hence, the image will be produced at a distance of 60 *cm* in front of the mirror, constituting a real image.

Now, let's contemplate a scenario where a mirror M_2 of any kind is positioned in front of the convex mirror, as depicted in the diagram. Whether M_2 is convex, concave, or a plane mirror, the incident rays upon it will converge. The real image produced by the concave mirror M_1 will serve as the virtual object for mirror M_2.

Plane mirrors can create both real and virtual images, contingent upon the characteristics of the object.



Ex. Determine the position and characteristics of the image generated by a concave mirror with a focal length of 20 cm when a virtual object is positioned 40 cm away from the mirror.



Sol. Since the object is virtual, it will be behind the concave mirror and according to the sign convention, we can write the following:

The distance of the object, u = + 40 cm.

The focal length of the concave mirror, f = −20 cm

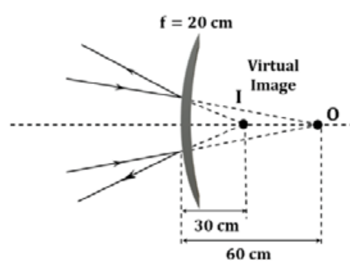
Hence, by applying mirror formula, we get,

$$v = \frac{(40)(-20)}{40+20} = \frac{-800}{+60} = -\frac{40}{3}$$

Therefore, the image forms at a distance $\frac{40}{3}$ cm in front of the mirror and hence, the image will be a "Real image".

$$v = -\frac{40}{3} \text{ cm, Real}$$

Ex. Find the position and the nature of the image formed by a convex mirror of focal length 20 *cm* when a virtual object is placed at distance of 60 *cm* from the mirror.



Sol. Since the object is virtual, it will be behind the convex mirror and according to the sign convention, we can write the following:

The distance of the object, u = +60 cm

The focal length of the convex mirror, f = + 20 cm

Hence, by applying mirror formula, we get

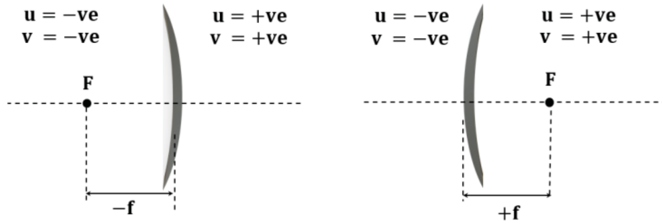
$$v = \frac{60\times20}{60-20}$$

$$= \frac{1200}{49} = 30 \text{ cm}$$

Therefore, the image forms at a distance 30 cm behind of the mirror and hence, the image will be a "Virtual image".

$$v = +30\text{cm, Virtual}$$

Important Observations

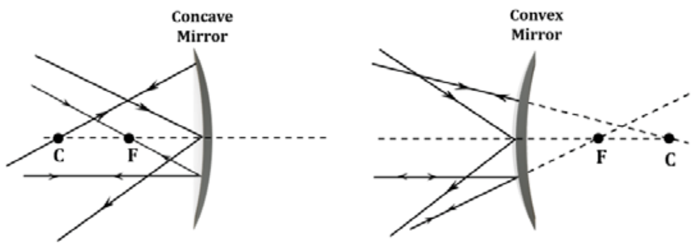


u, for a real object is always negative.
v, for a real image in always negative.

u, for a virtual object is always positive.
v, for a virtual image is always positive.

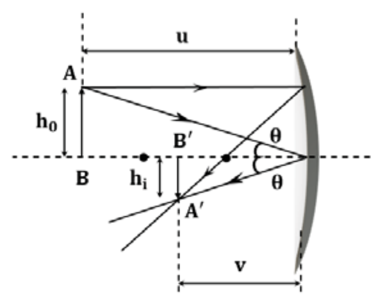
Standard Rays

1. When incident rays align parallel to the principal axis, the reflected ray will either intersect or appear to intersect at the focus of the mirror.
2. If the incident ray intersects or seems to intersect at the focus of a spherical mirror, then the reflected ray will run parallel to the principal axis of the mirror.
3. Should the incident ray traverse through the center of curvature of a spherical mirror, the reflected ray will also travel through the center of curvature. Thus, if the object is situated at the center of curvature, the resultant image will also reside at the center of curvature.
4. When the incident ray strikes the pole at an angle θ relative to the principal axis, the reflected ray will mirror the same angle with the principal axis.



Transverse Magnification

When an object AB with a height h_0 is positioned perpendicular to the principal axis of the mirror, the resulting image A'B' will also align perpendicular to the principal axis. If the image's height is h_i, then the transverse magnification is defined as,



$$m = \frac{h_i}{h_0}$$

From $\triangle$ ABP, we get, $\tan\theta = \frac{h_0}{u}$.

From $\Delta A'B'P$, we get, $\tan\theta = \frac{h_i}{v}$.

Therefore, by combining these two relation, magnitude wise we get the following:

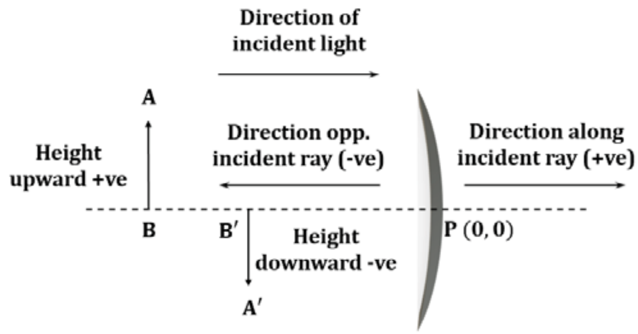
$$\frac{h_i}{h_0} = \frac{v}{u}$$

Under these circumstances, both the object and image reside in front of the mirror, resulting in negative quantities in accordance with the sign convention. Assuming the upper side of the principal axis to be positive and the lower side negative, the generalized formula for transverse magnification is as follows:

$$m = \frac{h_i}{h_0} = -\frac{v}{u}$$

[All quantities in this formula must be included with their respective signs when solving problems.]
If $|m| > 1$, then $|h_i| > |h_0|$. This suggests that the image size is greater than that of the object. When $|h_i| > |h_0|$, it also implies that, $|v| > |u|$.
If $|m| < 1$, then $|h_i| < |h_0|$. This implies that the size of the image is lesser than that of the object. When $|h_i| < |h_0|$, it also implies that, $|v| < |u|$.

$$m = \frac{h_i}{h_0} = -\frac{v}{u}$$



Nature of Image

Enlarged Image $|h_i| > |h_0|$ and $|v| > |u|$

Diminished Image $|h_i| < |h_0|$ and $|v| < |u|$

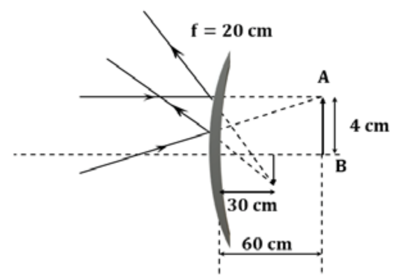
Inverted Image

The object and image are situated on opposite sides of the principal axis.

Erect Image

Object and image are positioned on the same side of the principal axis.

Ex. Consider the following ray diagram, find the position and the nature of the image formed by a convex mirror.



Sol. In this problem, the object AB is placed behind the convex mirror and above the principal axis. Hence, according to the sign convention, we can write the following:

The distance of the object, $u = +60$ cm

The focal length of the concave mirror, $f = +30$ cm

The height of the object is, $h_o = +\ 4$ cm

Hence, by applying the mirror formula, we get,

$$v = \frac{60.20}{60-20} = \frac{1200}{40} = 30 \text{ cm}$$

Now, by applying the formula of transverse magnification, we get,

$$\frac{h_i}{h_0} = -\frac{v}{u}$$

$$\frac{h_i}{+4} = -(\frac{30}{6y})$$

$h_i = -2\text{cm} -$ inverled smaller

$$v = +30 \text{ cm}; h_i = -2 \text{ cm}$$