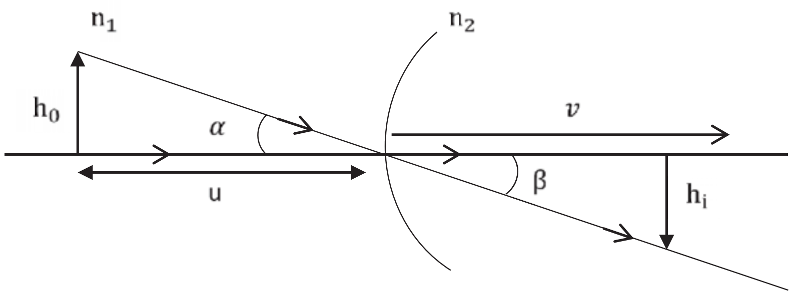
LENSES

Magnification in spherical refraction

Consider an object placed in front of a spherical surface, as shown in the figure



$$\text{Magnification} = \frac{\text{Height of image}}{\text{Height of object}}$$

Snell's law: $n_1 \sin\alpha = n_2 \text{Sin}\,\beta$

For paraxial rays, α and β are very small. Thus, $\sin\alpha = \tan\alpha$ and $\sin\beta = \tan\beta$

$$n_1 \tan\alpha \approx n_2 \tan\beta$$

$$\eta_1 \frac{h_0}{u} = \eta_2 \cdot \frac{h_i}{v} \qquad [\because \tan\alpha = \frac{h_u}{u} \text{ and } \tan\beta = \frac{h_1}{v}]$$

$$m = \frac{h_1}{h_0} = \frac{n_1}{n_2} \cdot \frac{v}{u}.$$

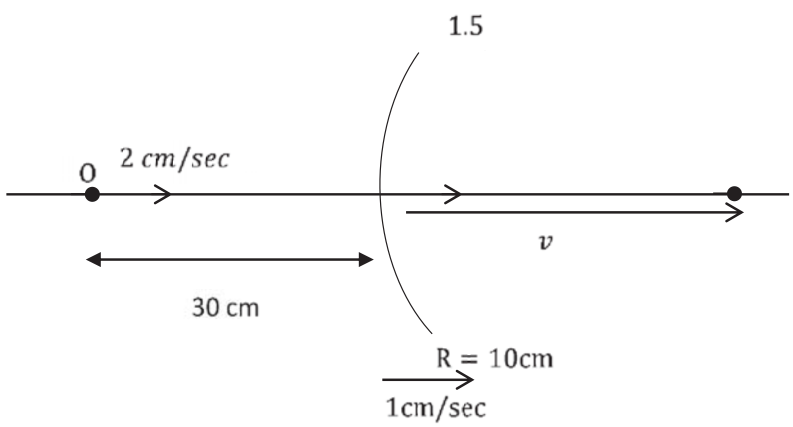
While solving the problems, h , h_o , u and v should be put with sign.

Velocity in spherical refraction

Generalized formula for spherical refraction,

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Differentiating with respect to time,



$$\frac{-n_2}{v^2} \cdot \frac{dv}{dt} + \frac{n_1}{u^2} \cdot \frac{du}{dt} = 0$$

$$\frac{dv}{dt} = \frac{n_1}{n_2} \cdot \frac{v^2}{u^2} \cdot \frac{du}{dt}.$$

$$V_{IS} = \frac{n_1}{n_2} \cdot \frac{v^2}{u^2} \cdot V_{OS}.$$

$\frac{du}{dt} =$ Velocity of object relative to spherical surface $= v_{IS}$

$\frac{dv}{dt} =$ Velocity of image relative to spherical surface $= v_{OS}$

Image position:

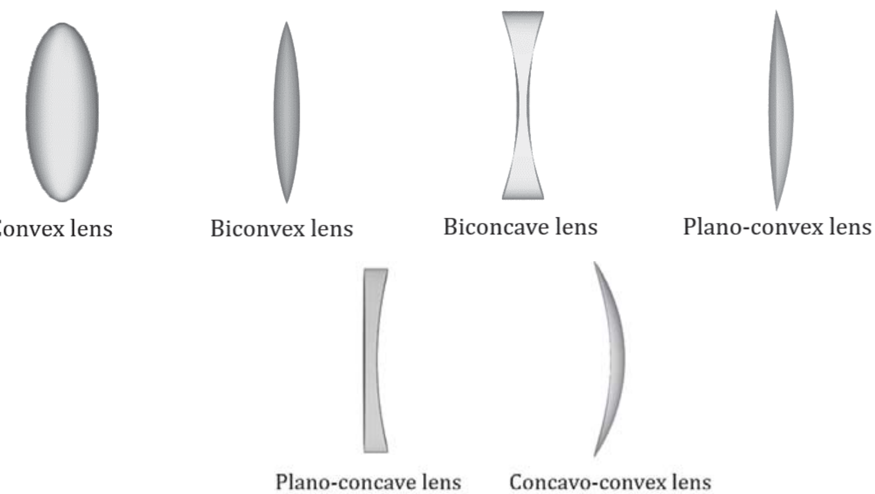
$$\frac{1\cdot5}{v}+\frac{1}{30}=\frac{1\cdot5-1}{10}$$
$$\frac{3}{2v}=\frac{1}{20}-\frac{1}{30}$$
$$V=90\text{cm}$$

Image Velocity:

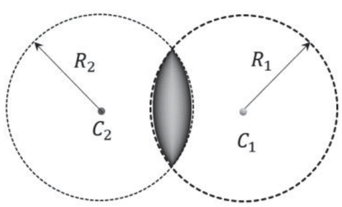
$$V_{IS}=\frac{n_1}{n_2}\cdot\frac{V^2}{u^2}\cdot V_{0S}$$
$$V_I-V_S=\frac{n_1}{n_2}\cdot\frac{V^2}{u^2}\cdot(V_0-V_S)$$
$$V_I-1=\frac{1}{3I_2}\left[\frac{90}{30}\right]^2\cdot[2-1].$$
$$v_1=\frac{2}{3}\cdot3^4+1=7\text{ cm/sec}$$

Types of lenses, concave and convex lens

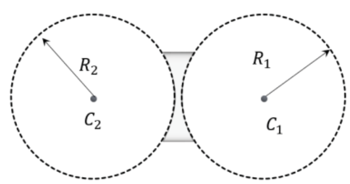
- A lens is a piece of transparent glass which concentrates or disperses light rays when passes through them by refraction.
- Light gets refracted twice when passing through lens.
- At least one surface of a lens is spherical.



Convex Lens



Concave Lens



Centre of Curvature

The centre of sphere from which a lens is formed

Radius of Curvature

The radius of sphere from which a lens is formed

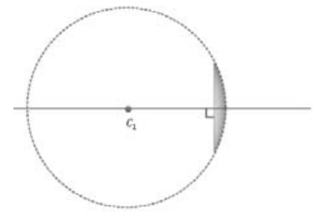
Optical axis

The line joining centre of curvature

Its not necessary to have same radius of curvature for both surfaces of a lens.

Optical Axis

Optical axis of plano-convex or plano-concave lens is found by drawing a perpendicular line to plane surface of the concerned lens, passing through centre of curvature of spherical surface.



Thin Lens

The thin lens is a lens where its thickness is significantly small as compared to other dimensions like distance of object/image

- Lens is bounded by two spherical surfaces.
- These bounding surfaces can be convex, concave or plane.
- A lens is called thin when,
- Radius of aperture specifies the size of lens.

Convex Lens

Optical Axis

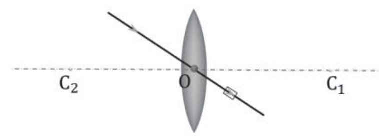
Line joining both the centres of curvature.

Optical Centre

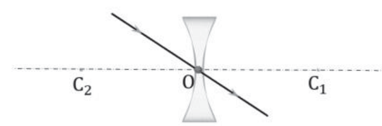
Central point of the lens through which a ray of light passes without any deviation

Aperture

Actual diameter of the circular outline of a spherical lens.
Central portion of thin lens behaves as a slab.
Shift through the slab is zero for thin lens.



Convex Lens

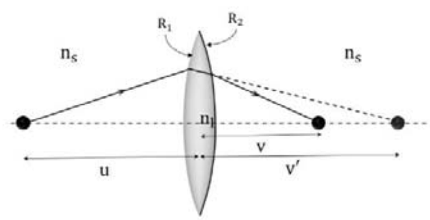


Concave Lens

Lens maker's formula

Assumptions:

Rays should be paraxial Medium on both sides should be same Lens should be thin Assume Optic center as origin v, u, R_1 and R_2 should be taken with sign



First spherical refraction: $\frac{n_l}{v'} - \frac{n_s}{u} = \frac{n_l - n_s}{R_1} - (1)$

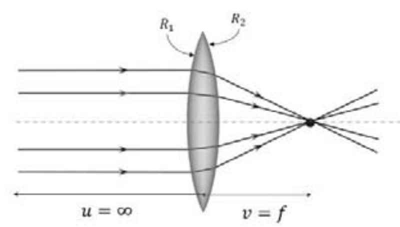
Second spherical refraction: $\frac{n_s}{v} - \frac{n_l}{v'} = \frac{n_s - n_l}{R_2} - (2)$

Adding equations (1) and (2),

$$\frac{n_s}{v} - \frac{n_s}{u} = n_l - n_s\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \left[\frac{n_l}{n_s} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_l}{n_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



We have,

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_l}{n_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) - (1)$$

Focus:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} - (2)$$

From equation (1) and (2),

$$\frac{1}{f} = \left(\frac{n_l}{n_5} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$