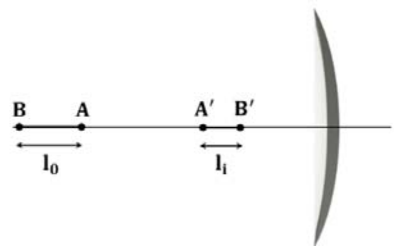
IMAGE FORMATION BY CONCAVE MIRROR

Longitudinal Magnification

- When the object is positioned along the principal axis, the longitudinal magnification is determined by the ratio of the image length to the object length.
- Magnification will be positive when the orientation of the object and image is identical.
- Magnification will be negative when the orientation of the object and image is opposite to each other.

$$m = -\frac{l_i}{l_0}$$

Ex. An object AB of length 30 cm is placed in front of a concave mirror as shown. Focal length of mirror is 20 cm. Find the magnification for AB.



Sol. Position of image: $v = \frac{uf}{u-f}$

Point A: $u = -60; f = -20$

$$v = \frac{(-60)(-20)}{-60+20}$$

$$= \frac{+1200}{-40} = -30 \text{ cm}$$

Point B: $u = -90; f = -20$

$$v = \frac{(-90)(-20)}{-90+20}$$

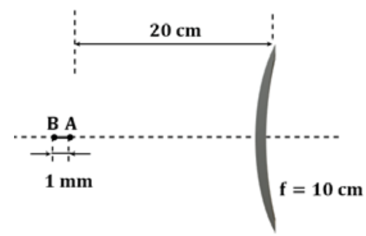
$$= \frac{1800}{-74} = \frac{-180}{7}$$

Length of image: $l_1 = 30 - \frac{180}{7} = \frac{30}{7}$

Magnification: $m = \frac{l_0}{l_0} = \frac{30/7}{30} = \frac{1}{7}$

$$m = \frac{-1}{7}$$

Ex. An object AB of length 1 mm is placed in front of a convex mirror as shown. Focal length of mirror is 10 cm. Find the length of image l_i.



Sol. Here, $l_o \ll u$, thus we can use short trick.

$$v = \frac{uf}{u-f}$$

$$m = \frac{l_i}{l_0} = \frac{dv}{du} = -\frac{v^2}{u^2}$$

$$u = -20f = +10$$

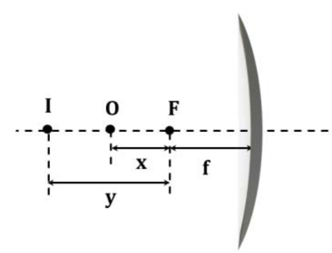
$$v = \frac{(-20)(10)}{-20-10} = \frac{+20d}{734}$$

$$\frac{dv}{du} = \frac{l_i}{1mm} = -(\frac{v^2}{u^2})$$

$$\frac{l_i}{1mm} = -(\frac{2013}{20})$$

$$l_i = -\frac{1}{9} \text{ mm}$$

Newton's Formula



$$u = -(x+f)f = -f.$$

$$v = -(y+f)$$

$$\frac{1}{(y+f)} + \frac{1}{(x+f)} = \frac{1}{f}$$

$$\frac{1}{y+f} = \frac{1}{f} - \frac{1}{(x+f)}$$

$$\frac{1}{y+f} = \frac{x+f-f}{f(x+f)}$$

$$f(x+f) = x(y+f)$$

$$fx + f^2 = xy + fx$$

$$xy = f^2$$

Valid for both concave & convex mirror for all cases.

$$f^2 = xy$$

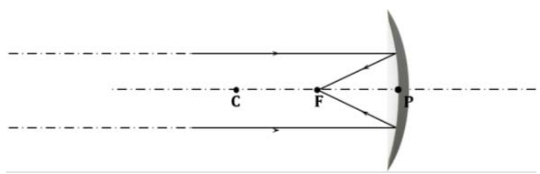
Utilized when both the distances of the image and the object are specified from the focal point. Suitable for validation purposes. Unable to determine whether the formed image is situated to the right or left of the focal point.

Virtual Image

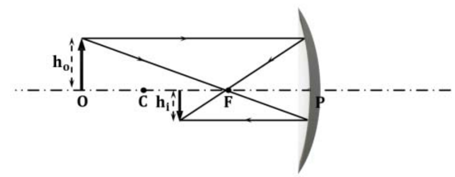
Indeed, it is possible to capture a photograph of a virtual image; however, only a real image can be projected onto a screen for photography.

Image Formation By Concave Mirror

Case 1:

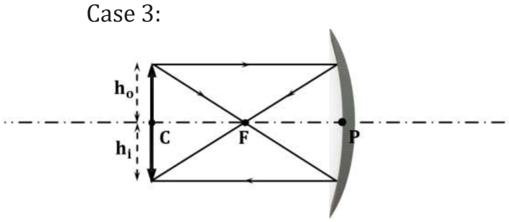


case 2:

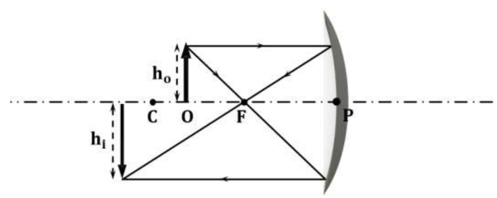


Object	Image	Nature	
$0 = -\infty$	$I = -F$	Point Image	Case 1
$-\infty < 0 < -C$	$-C < I < -F$	Real, inverted, Diminished	Case 2

Case 3:

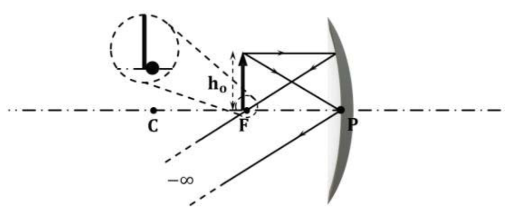


Case 4:



Object	Image	Nature	
$0 = -\infty$	$I = -F$	Point Image	Case 1
$-\infty < 0 < -C$	$-C < I < -F$	Real, inverted, Diminished	Case 2
$O = -C$	$I = -C$	Real, Inverted, Same Image	Case 3
$-C < O < -F$	$-\infty < I < -C$	Real, Inverted, Magnified	Case 4

Case 5:



Object	Image	Nature	
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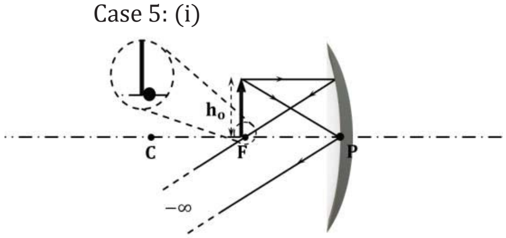
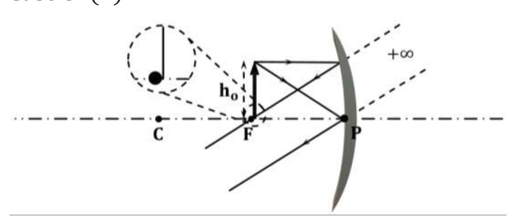
$O = -C$	$I = -C$	Real, Inverted, Same Image	Case 3
$-C < O < -F$	$-\infty < I < -C$	Real, Inverted, Magnified	Case 4
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5

Case 5: (i)



Case 5: (ii)



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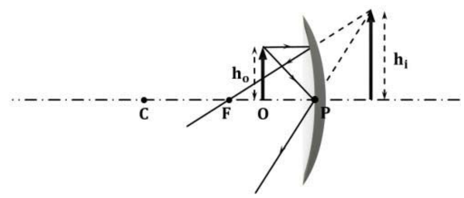
$-\infty < 0 < -C$	$-C < I < -F$	Real, inverted, Diminished	Case 2
$O = -C$	$I = -C$	Real, Inverted, Same Image	Case 3
$-C < O < -F$	$-\infty < I < -C$	Real, Inverted, Magnified	Case 4
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5 (i)
$O = -F$	$I = +\infty$	Visual, Exed, Magnified	Case 5 (ii)

Case 6:



Object	Image	Nature	
$0 = -\infty$	$I = -F$	Point Image	Case 1
$-\infty < 0 < -C$	$-C < I < -F$	Real, inverted, Diminished	Case 2
$O = -C$	$I = -C$	Real, Inverted, Same Image	Case 3
$-C < O < -F$	$-\infty < I < -C$	Real, Inverted, Magnified	Case 4
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5 (i)
$O = -F$	$I = +\infty$	Visual, Erect, Magnified	Case 5 (ii)
$-F < O < P$	$P < I < +\infty$	Visual, Erect, Magnified	Case 6

Case 7:

When converging rays intersect the mirror, the object is rendered virtual. If this virtual object is positioned a distance x behind the mirror's pole but not at an infinite distance, then $u = +x$. Consequently,

$$\frac{I}{v} + \frac{I}{x} = \frac{I}{f}$$

$$\frac{1}{v} = -\frac{1}{x} - \frac{1}{f}.$$

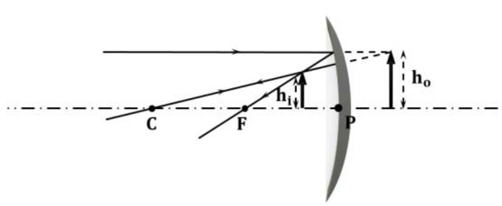
$$x = \infty$$

$$\downarrow$$

$$x = 0$$

$$v = -f$$

$$v = 0$$



Given that the image distance v is negative, it indicates that the image will materialize in front of the mirror, thus ensuring that the image is real.

Moreover, considering that u and v bear opposite signs, as inferred from the general formula of transverse magnification, it can be deduced that both the image height and the object height share identical signs, being positive. Consequently, the image is upright.

When the object is at infinity, the image forms at the focus. Therefore, $u > v$ which implies $h_i < h_O$. Therefore, the image will be diminished in size.

Monty's ray diagram for Concave Mirror

Object	Image	Nature	
$0 = -\infty$	$I = -F$	Point Image	Case 1
$-\infty < 0 < -C$	$-C < I < -F$	Real, inverted, Diminished	Case 2
$O = -C$	$I = -C$	Real, Inverted, Same Image	Case 3
$-C < 0 < -F$	$-\infty < I < -C$	Real, Inverted, Magnified	Case 4
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5
$O = -F$	$I = -\infty$	Real, Inverted, Magnified	Case 5 (i)
$O = -F$	$I = +\infty$	Visual, Erect, Magnified	Case 5 (ii)
$-F < O < P$	$P < I < +\infty$	Visual, Erect, Magnified	Case 6
$P < 0 < +\infty$	$-F < I < P$	Real, Erect, Diminished	Case 7

Rays 1, 2, and 3 depicted in the illustration correspond to cases 1 through 6 outlined in the table. Ray 4 illustrated in the figure corresponds to case 7 as outlined in the table.

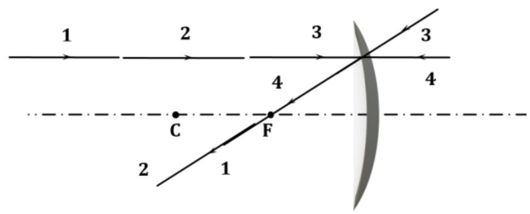


Image Formation by Concave Mirror: Graphical Representation

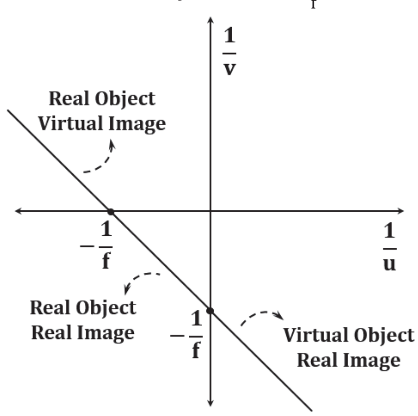
We know that, $\frac{1}{v} + \frac{1}{4} = (\frac{1}{f})$

As f remains constant and negative for a concave mirror let $\frac{1}{f} = c$ now, by choosing $\frac{1}{u} = x$ and $\frac{1}{v} = y$ we get. $y + x = -C$

[Here we've written c with sign of the focus]

This illustrates a linear relationship characterized by a negative slope, as demonstrated in the figure.

$$y = 0 \quad x = -c = \frac{-1}{f}$$
$$x = 0 \quad y = -c = -\frac{1}{f}.$$

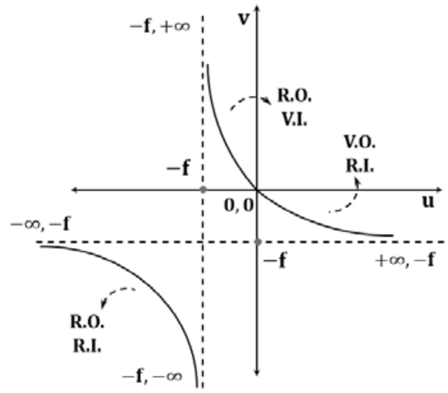


In the 2nd quadrant $\frac{1}{u} = x = -ve$ and $\frac{1}{v} = y = +ve$ thus, $u = -ve$ and $v = +ve$. Hence, this portion of the straight line refers "Real object" and "Virtual image".

In the 3rd quadrant, $\frac{1}{y} = x = -ve$ and $\frac{1}{v} = y = -ve$, thus, $u = -ve$ and $v = -ve$. Hence, this portion of the straight line refers "Real object" and "Real image".

In the 4 th quadrant, $\frac{1}{x} = x = +ve$ and $\frac{1}{v} = y = -ve$, thus, $u = +ve$ and $v = -ve$. Hence, this portion of the straight line refers "Virtual object" and "Real image".

Since f is constant, if we plot "v" vs "u" graph, then the graph will be a hyperbola.



Ex. Which of the following statements is/are true about concave mirror? Concave mirror always forms:

(A) Real image of real objects.
(B) Smaller image of virtual objects.
(C) Real image of virtual objects.
(D) Virtual image of real objects.

Sol. When the object is placed between the pole and focus of the concave mirror, the image becomes virtual. So, option (A) is false. When the object is virtual, the image gets diminished. So, option (B) is true. When the object is virtual, the real image is formed in between the pole and focus. (C) is true. When the object is real, the real image can also form. Thus, concave mirror not always form virtual image. Thus, option (D) is false.

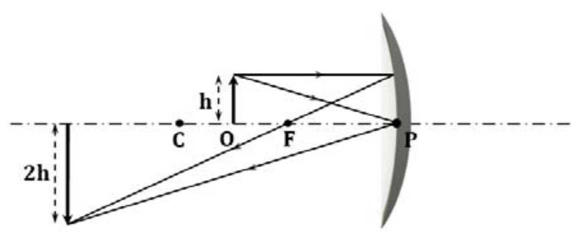
Ex. An object is placed in front of a concave mirror whose focal length is 10 cm. If image formed is double the height of object, then find out the distance of the object from the mirror.

Sol.

1. We know that when the real object is placed between the centre of curvature and the focus of the concave mirror, the characteristics of the image are: Real, inverted and magnified.

2. Also when the real object is placed between the focus and the pole of the concave mirror, the characteristics of the image are: Virtual, erect and magnified.

Case 1: It is given that the focal length of the mirror is, $f = -10$ cm Since the height of the image is required to be double of the height of the object, we can say that image distance is also double of the object distance.



Let $u = -x$. Thus, $v = -2x$ (since both the image is real). Now, by applying the mirror formula, we get,

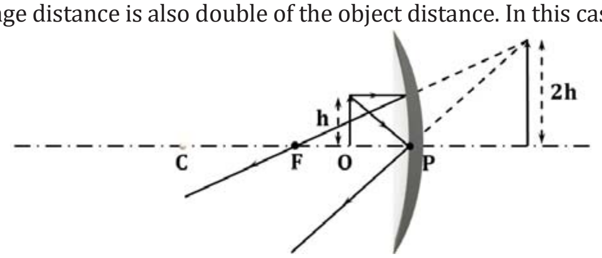
$$\frac{1}{2x} + \frac{1}{x} = \frac{1}{10}$$

$$\frac{1+2}{2x} = \frac{1}{10}$$

$$x = 15 \text{ cm}$$

Therefore, the object distance is, $u = -15$ cm

Case 2: Since the height of the image is required to be double of the height of the object, we can say that image distance is also double of the object distance. In this case, the image is virtual.



Therefore, if $u = -x$, then, $v = +2x$. Now, by applying the mirror formula, we get,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{2x} - \frac{1}{x} = \frac{-1}{10}$$

$$\frac{1}{x} - \frac{1}{2x} = \frac{1}{10} x = +5$$

$$\frac{1}{2x} = \frac{1}{10} x = +5$$

Therefore, the object distance is, $u = -5$ cm