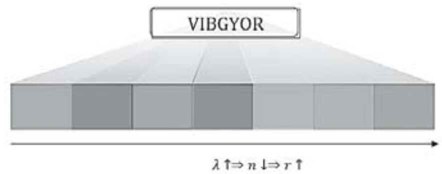
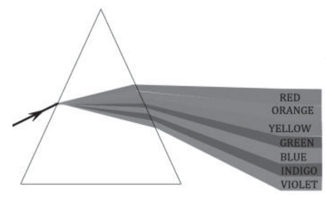
## DISPERSION

### Dispersion




White light is composed of seven colors (VIBGYOR). Experimentally, it can be seen that when a white light pass through a transparent medium (say, R.I. is $n$), it gets separated in its constituents colors. This phenomenon is known as "Dispersion".

$$n = a + \frac{b}{\lambda^2} + \cdots \ldots\ldots \text{(Cauchy's formula)}.$$

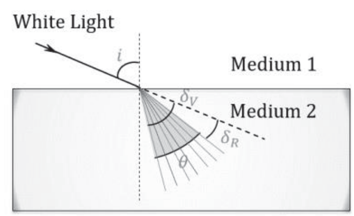



This formula shows that the refractive index of a medium depends on the wavelength of a light ray. As wavelength decreases, the refractive increases.

If the white light ray is moving from vacuum to a medium of R.I. $n$, for a certain angle of incidence $i$, the angle of refraction is, $r = \sin^{-1}\left(\frac{\sin i}{n}\right)$. Thus, as wavelength increases, $n$ decreases which in turn will increase $r$. Since the wavelength of red is maximum among others, its angle of refraction is also maximum.

We can pass white light through a transparent prism to observe the dispersion phenomenon, as shown in the figure

### Angular Dispersion

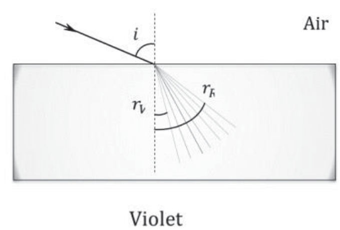



It is the difference in the angles of deviation of two extreme colors (violet and red) of the spectrum of white light

Maximum deviation is suffered by violet whereas the minimum deviation is suffered by red. The yellow colour defines the mean deviation or the deviation of the parent white light. If $\delta_V$ and $\delta_R$ are the deviation suffered by violet and red, then mathematically, the angular dispersion is defined as

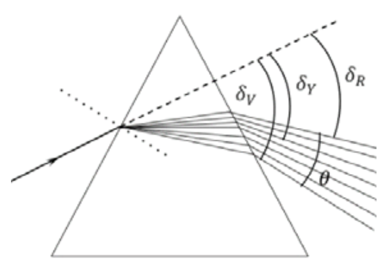
$$\theta = \delta_V - \delta_R$$

**Ex.** White light travels from Air to a different Medium and during this process gets scattered into different colors. Which color out of the scattered colors will be having the least angle of refraction?




**Sol.** For a certain angle of incidence $i$, the angle of refraction is $= \sin^{-1}\left(\frac{\sin i}{n}\right)$. Thus, as wavelength increases, $n$ decreases (from Cauchy's formula) which in turn will increase $r$. Since the wavelength of violet is minimum among others, its angle of refraction is also minimum.

### Dispersive Power




When white light is incident on a prism, the dispersion phenomenon can be observed. The maximum deviation is suffered by violet whereas the minimum deviation is suffered by red. The angular dispersion is defined as, $\theta = \delta_V - \delta_R$

The yellow colour defines the mean deviation or the deviation of the parent white light.

Dispersive power is a property of medium which shows the strength of the medium by which it can split the white light into its constituent lights.

When we want to find the dispersive power of a medium, the following steps has to be obeyed:

1. Form thin prism (its angle of deviation, $\delta = (n-1)A$ ) using the given medium.
2. Calculate $\delta_V$ and i.e., $\delta_V = (n_V - 1)A$ and
3. Calculate magnitude of angular dispersion, $\theta = \delta_V - \delta_R = (n_V - n_R)A$
4. Calculate mean deviation, $\delta_Y = (n_Y - 1)A$
5. Now, the dispersive power of the medium, $\omega = \frac{\text{Angular Dispersion}}{\text{Mean Deviation}} = \frac{n_V - n_R}{n_Y - 1}$

It is defined as the ratio of angular dispersion ($\theta$) and the mean deviation ($\delta_Y$)

$$\omega = \frac{\text{Angular Dispersion}}{\text{Mean Deviation}} = \frac{n_V - n_R}{n_Y - 1}$$

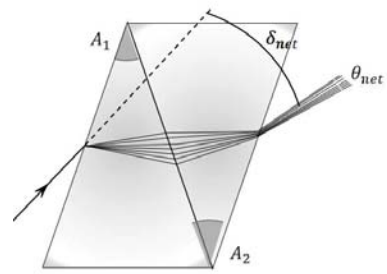
It is the property of medium only, It is a dimensionless quantity

When $n_Y$ is not given then it can be approximated as:

$$n_Y \approx \frac{n_V + n_R}{2}$$

**Dispersion and deviation**

Consider the arrangement of thin prisms as shown. Suppose the angle of deviation of yellow ray is $\delta_1$ (C.W.) and $\delta_2$ (A.C.W.) when it crosses the thin prisms having angle $A_1$ and $A_2$, respectively. Therefore, the net deviation is,




$$\delta_{net} = \delta_1 - \delta_2$$

Similarly, if the angular dispersion of yellow ray is $\theta_1$ (C.W.) and $\theta_2$ (A.C.W.) when it crosses the thin prisms having angle $A_1$ and $A_2$, respectively, then net angular dispersion will be,

$$\theta_{net} = \theta_1 - \theta_2$$

Therefore, successive deviations ($\delta_1$and $\delta_2$) and dispersions ($\theta_1$ and $\theta_2$) suffered by yellow ray are given by,

$$\delta_1 = (n_{y_1} - 1)A_1$$

$$\delta_2 = (ny_2 - 1)A_2$$

$$\theta_1 = (n_{v_1} - n_{R_1})A_1$$

$$\theta_2 = (n_{v_2} - n_{R_2})A_2$$

$$\delta_{net} = (\delta_1 - \delta_2) = \left(n_{Y_1} - 1\right)A_1 - \left(n_{Y_2} - 1\right)A_2$$

$$\theta_{net} = (\theta_1 - \theta_2) = (n_{V_1} - n_{R_1})A_1 - (n_{V_2} - n_{R_2})A_2$$

The dispersive power of the prism having angle $A_1$ is,

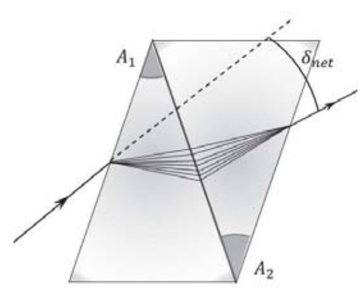
$$\omega_1 = \frac{\theta_1}{\delta_1} = \frac{n_{V_1} - n_{R_1}}{n_{Y_1} - 1}$$

The dispersive power of the prism having angle $A_2$ is,

$$\omega_2 = \frac{\theta_2}{\delta_2} = \frac{n_{V_2} - n_{R_2}}{n_{Y_2} - 1}$$

**Dispersion without deviation**

We know that if the dispersion and the deviation happens consequently in the given arrangement of thin prisms, then the net deviation and net angular dispersion will be,




$\delta_{net} = \left(n_{Y_1} - 1\right)A_1 - \left(n_{Y_2} - 1\right)A_2$

$\theta_{net} = (n_{V_1} - n_{R_1})A_1 - (n_{V_2} - n_{R_2})A_2$

The dispersive power of two prisms is given by,

$$\omega_1 = \frac{n_{V_1} - n_{R_1}}{n_{Y_1} - 1}, \quad \omega_2 = \frac{n_{V_2} - n_{R_2}}{n_{Y_2} - 1}$$

Suppose we want that the deviation of light takes place without dispersion i.e.,

$$\delta_{net} \neq 0 \text{ and } \theta_{net} = 0.$$

For $\theta_{net} = 0$, we have, $(n_{v_1} - n_{R_1})A_1 = (n_{v_2} - n_{R_2})A_2$

$$\omega_1\left(n_{y_1} - 1\right)A_1 = (n_{y_2} - 1)\omega_2 A_2$$

$$(n_{y_2} - 1)A_2 = (n_{y_1} - 1)A_1 \frac{\omega_1}{\omega_2}$$

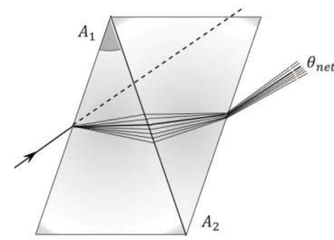
By substituting the value of $(n_{Y2} - 1)_2$ in the expression of $\delta_{net}$, we get,

$\delta_{net} = (n_{y_1} - 1)A_1 - (n_{y_2} - 1)A_2$

$\delta_1 - \delta_1 \frac{\omega_1}{\omega_2} = \delta_1 \left[1 - \frac{w_1}{\omega_2}\right]$ [ Since $(x_{y1} - 1)A_1 = \delta_1$]

$\delta_{net} = (n_{Y_1} - 1)A_1 - (n_{Y_2} - 1)A_2$

$\theta_{net} = (n_{V_1} - n_{R_1})A_1 - (n_{V_2} - n_{R_2})A_2$




The dispersive power of two prisms is given by,

$\omega_1 = \frac{n_{V_1} - n_{R_1}}{n_{Y_1} - 1}, \quad \omega_2 = \frac{n_{V_2} - n_{R_2}}{n_{Y_2} - 1}$

Suppose we want that the deviation of light takes place without dispersion i.e., $\theta_{net} \neq 0$ and $\delta_{net} = 0$. This is possible if emergent yellow colored ray becomes parallel to the incident ray.

For $\delta_{net} = 0$, we have,

$$(n_{y_1} - 1)A_1 = (n_{y_2} - 1)A_2$$

$$\frac{(n_{V_1} - n_{R_1})A_1}{\omega_1} = \frac{(n_{V_2} - n_{R_2})A_2}{\omega_2}$$

$$(n_{V_2} - n_{R_2})A_2 = \frac{\omega_2}{\omega_1}(n_{V_1} - n_{R_1})A_1$$

By substituting the value of $(n_{V2} - n_{R2})_2$ in the expression of $\theta_{net}$, we get,

$$\theta_{net} = (n_{V_1} - n_{R_1})A_1 - (n_{V_2} - n_{R_2})A_2$$

$$= (n_{V_1} - n_{R_1})A_1 - (n_{V_1} - n_{R_1})A_1 \frac{w_2}{w_1}$$

$$= \theta_1 \left[1 - \frac{w_2}{w_1}\right] \quad [\text{ since } \theta_1 = (n_{V_1} - n_{R_1})A_1]$$

$$\delta_{net} = (n_{Y_1} - 1)A_1 - (n_{Y_2} - 1)A_2$$

$$\theta_{net} = (n_{V_1} - n_{R_1})A_1 - (n_{V_2} - n_{R_2})A_2$$

The dispersive power of the prism having angle $A_1$ is,

$$\omega_1 = \frac{n_{V_1} - n_{R_1}}{n_{Y_1} - 1}$$

**Deviation without Dispersion**

$$\theta_{net} = 0$$

$$\delta_{net} = \delta_1 \left[1 - \frac{\omega_1}{\omega_2}\right]$$

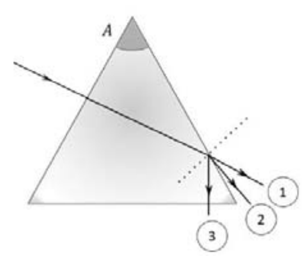
**Dispersion and Deviation**

$$\delta_{net} = 0$$

$$\theta_{net} = \theta_1 [1 - \frac{\omega_2}{\omega_1}]$$

**Ex.** A combination of violet, green and red light enters normally into a prism and travels to the other face of prism, there it splits into three rays as shown. Identify the color of the rays 1, 2 and 3.

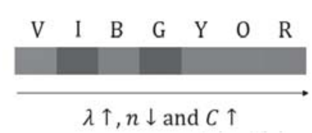
**Sol.** First of all, since the incident ray enters normally into the prism, the refracted ray also become normal to that refracting surface. Therefore, $r_1 = 0$. Hence, $r_2 = A$.




We know that as wavelength increases, the refractive index of the medium decreases (from Cauchy's formula). Now, if the light is moving from denser medium (say, R.I. $n$) to rarer medium (say, vacuum), then the critical angle for that medium will be,

$$C = \sin^{-1}\left(\frac{1}{n}\right)$$

Therefore, as refractive index decreases, the critical angle increases. Hence, the descending order of the critical angle for the given combination of light will be,




$$c_R > c_g > c_V$$

Assume that the angle of incidence at point P is, $A = 45°$

Since the ray 3 undergoes TIR whereas other two rays (i.e., 1 and 2) emerge out of the prism, the critical angle for 3 ray $C_3 < A$, and the critical angle for $1^{st}$ and $2^{nd}$ ray $C_1, C_2 > A$

Therefore from the figure, we can say, $C_1 > C_2 > C_3$.

We also know that:

$$c_R > c_g > c_V.$$

Hence

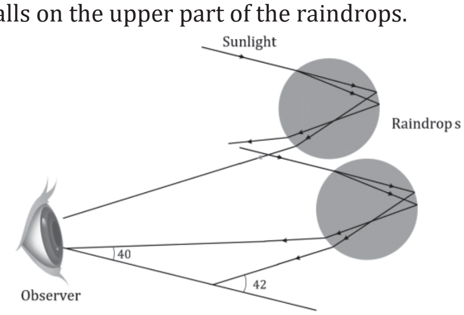
3rd ray: violet 2nd ray: green 1st ray: Red

**Application of TIR (Primary and Secondary Rainbow)**

When the sunlight falls on the rain drops, the dispersion phenomenon happens and we can see the rainbow.

**Rainbow: Primary**

In this case, sunlight falls on the upper part of the raindrops.




**Rainbow: Secondary**

In this case, sunlight falls on the lower part of the raindrops