COMBINATIONS OF MIRROR

Velocity in Spherical Mirror

Let,

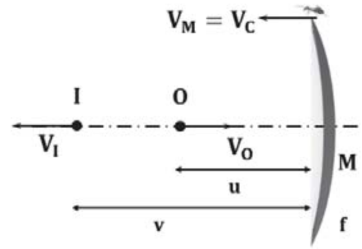
$V_I =$ Velocity of image $V_O =$ Velocity of object $V_M =$ Velocity of mirror

We have, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Differentiating with respect to time,

$$\frac{-1}{u^2}\frac{du}{dt} + \frac{-1}{v^2}\frac{dv}{dt} = 0 \text{ (Since, f = Constant)}$$

$$\frac{dv}{dt} = \frac{-v^2}{u^2}\frac{du}{dt}$$



Here,

$\frac{dv}{dt}$ = velocity of image w.r.t pole or mirror $= V_I - V_M$

$\frac{du}{dt}$ = velocity of object w.r.t pole or mirror $= V_O - V_M$

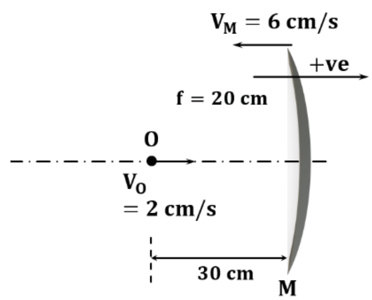
Hence, we derive the subsequent relationship between velocities at a moment for a spherical mirror.

$$V_I - V_M = -\frac{v^2}{u^2}(V_O - V_M)$$

Velocities should be substituted based on the sign convention.

This equation holds true for the components of velocities aligned with the principal axis.

Ex. An object and concave mirror are moving towards each other as shown. If the velocity of object and mirror are 2 cm/s and 6 cm/s respectively, find out the velocity of image V_I.



Sol. Given $u = -30$ cm; $f = -20$ cm

Distance of image, $v = \frac{uf}{u-f}$

$$v = \frac{(-30)\times(-20)}{(-30)-(-20)} = -60 \text{ cm}$$

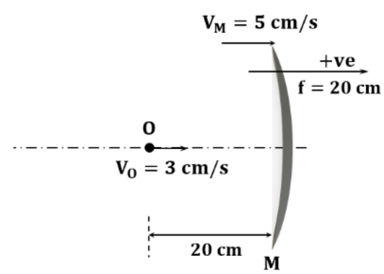
We have, $V_I - V_M = -\frac{v^2}{u^2}(V_O - V_M)$

$$V_I - (-6) = -\frac{(-60)^2}{(-30)^2}(2 - (-6))$$

$$V_I = -38 \text{ cm/s}$$

Here, minus sign of image velocity indicates that it is moving opposite to the object.

Ex. An object and convex mirror are moving in same direction as shown. If the velocity of object and mirror are 3 cm/s and 5 cm/s respectively, find out the velocity of image V_I.



Given, $u = -20$ cm ; $f = 20$ cm

Distance of image, $v = \frac{uf}{u-f}$

$$= \frac{(-20)\times(20)}{(-20)-(20)} = 10 \text{ cm}$$

We have, $V_I - V_M = -\frac{v^2}{u^2}(V_O - V_M)$

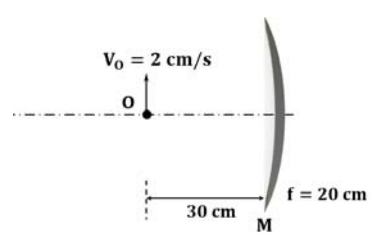
$$V_I - (5) = -\frac{(10)^2}{(-20)^2}(3 - (5))$$

$$V_I = 5.5 \text{ cm/s}$$

Here, plus sign of image velocity indicates that it is moving in same direction of the object.

Ex. An object is moving with 2 cm/s in the direction as shown. Find out the velocity of image V_I.

Sol Given, $u = -30$ cm; $f = -20$ cm



Distance of image, $v = \frac{uf}{u-f}$

$$= \frac{(-30)\times(-20)}{(-30)-(20)} = -60 \text{ cm}$$

Rate of change in height of object = Velocity of object,

$$\frac{dh_o}{dt} = V_O = 2 \text{ cm/s}$$

Rate of change in height of image = Velocity of image,

$$V_i = \frac{dh_i}{dt}$$

Concept

Velocity parallel to principal axis:

Differentiate $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$$V_{IM}=-\frac{v^2}{u^2}(V_{OM})$$

Velocity perpendicular to principal axis:

Differentiate $\frac{h_i}{h_o}=-\frac{v}{u}$

$$V_i=-\frac{v}{u}V_o$$

We have, $\frac{h_i}{h_o}=-\frac{v}{u}$

Differentiating w.r.t. time, $\frac{dh_i}{dt}=-\frac{v}{u}\frac{dh_0}{dt}$

$$V_i=-\frac{(-60)}{(-30)}\times 2$$

$$V_i=-4$$

Ex. An object is moving with 5 cm/s in the direction as shown. Find out the speed of image V_I.

Sol. As the object is moving at 37° to principal axis, u, v and h_o are changing.

u_x = Component of velocity of object along principal axis

u_y = Component of velocity of object perpendicular to principal axis

Given, $u=-30$ cm; $f=-20$ cm

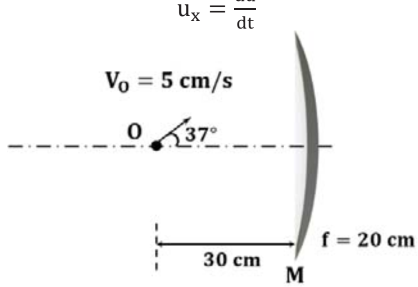
Distance of image, $v=\frac{uf}{u-f}$

$$=\frac{(-30)\times(-20)}{(-30)-(20)}=-60 \text{ cm}$$

$$u_x=V_O\cos 37^\circ=5\cos 37^\circ=4 \text{ cm/s}$$

$$u_y=V_O\sin 37^\circ=5\sin 37^\circ=3 \text{ cm/s}$$

$$u_x=\frac{du}{dt}$$



Component of velocity of image along principal axis.

$$\frac{dv}{dt}=\frac{-v^2}{u^2}\frac{du}{dt}=\frac{-(-60)^2}{(-30)^2}\times 4=-16 \text{ cm/s}$$

We have, $\frac{h_i}{h_o}=-\frac{v}{u}$

$$uh_i=-vh_o$$

Differentiating w.r.t. time,

$$u\frac{dh_i}{dt}+h_i\frac{du}{dt}=-[v\frac{dh_o}{dt}+h_o\frac{dv}{dt}]$$

At the instant when object and image both are on principal axis,

$$h_i=h_o=0$$

$$u\frac{dh_i}{dt}=-[v\frac{dh_o}{dt}]$$

$$(-30)\frac{dh_i}{dt}=-[(-60)\times 3]$$

$$\frac{dh_i}{dt}=-6 \text{ cm/s}$$

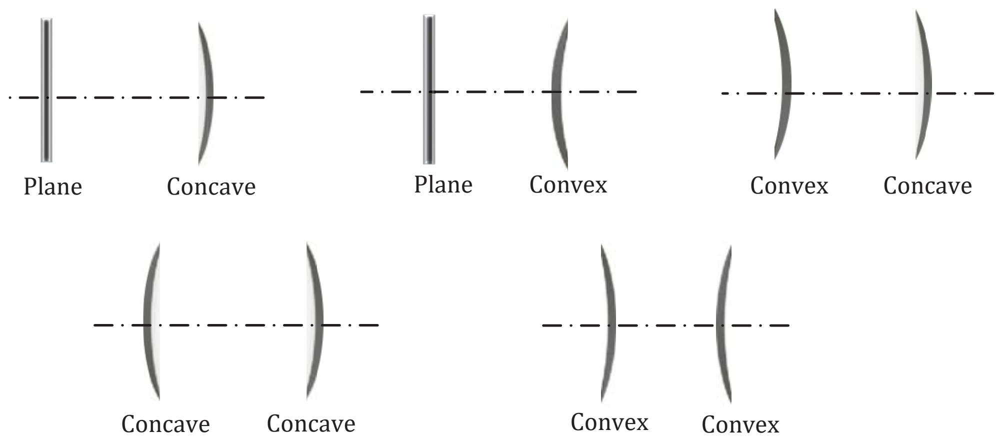
Thus, the velocity o images, $V_i=vx\hat{\imath}+\frac{dh_i}{dt}\hat{\jmath}$

$$V_i=-16\hat{\imath}-6\hat{\jmath}$$

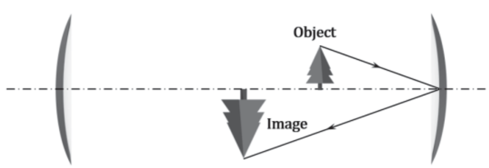
$$|V_i|=\sqrt{(-16)^2+(-6)^2}=\sqrt{292} \text{ cm/s}$$

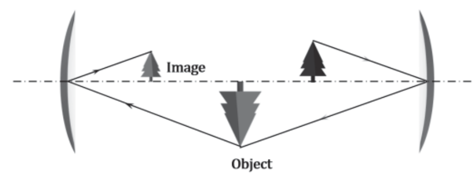
Combination of Mirror

The figure illustrates various combinations. When encountering such combinations in questions, consider the image formed by the first mirror as the object for the next mirror.

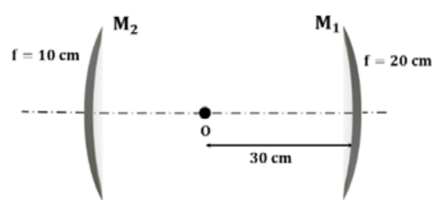


First reflection **Second reflection**





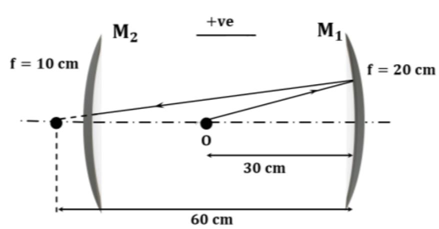
Ex. Two concave mirrors are placed 50 cm apart facing each other and a point object O is placed between them as shown. Find the location of image formed after two reflections - first at M_1 and second at M_2.



Sol. First Reflection

Given, $u = -30 \text{ cm}; f = -20 \text{ cm}$

Distance of image, $v = \frac{uf}{u-f} = \frac{(-30)\times(-20)}{(-30)-(20)} = -60 \text{ cm}$



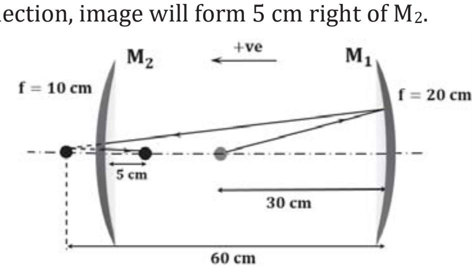
As the distance between mirrors is 50 cm, image after first reflection will form 10 cm to the left of M_2.

Second Reflection

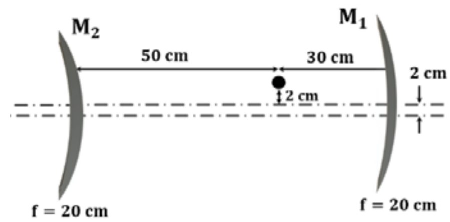
Given, $u = 10 \text{ cm}; f = -10 \text{ cm}$

Distance of image, $v = \frac{uf}{u-f} = \frac{(-10)\times(10)}{(-30)-10} = -5 \text{ cm}$

Thus, after second reflection, image will form 5 cm right of M_2.



Ex. A concave mirror M_1 and convex mirror M_2 of focal length 20 cm each are placed facing each other 80 cm apart. The principle axis of the M_1 lies 2 cm above the principle axis of M_2. A point object is placed between the mirrors as shown. Find the position and nature of the image produced by successive reflections, first at the concave mirror and then at the convex mirror.



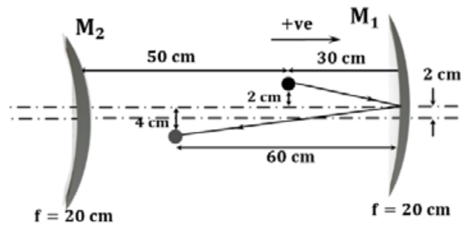
Sol. Given, $u = -30\text{ cm}; f = -20\text{ cm}$

Distance of image, $v = \frac{uf}{u-f} = \frac{(-30)\times(-20)}{(-30)-(20)} = -60\text{ cm}$

First Reflection: Height of the image w. r. t. M_1:

We have, $\frac{h_i}{h_o} = -\frac{v}{u}$

$$h_i = -2 \times \frac{-60}{-20} = -4\text{ cm}$$



As the distance between mirrors is 80 *cm*, image after first reflection will form 20 cm to the right and 2 cm below the principal axis of M_2.

$$v_1 = -60\text{ cm}, h_1 = -4\text{ cm}$$

Second Reflection:

Given, $u = -20\text{ cm}; f = 20\text{ cm}$

Distance of image, $v = \frac{uf}{u-f}$

$$= \frac{(-20)\times(20)}{(-20)-(20)} = 10\text{ cm}$$

Height of the image w.r.t

We have, $\frac{h_i}{h_o} = -\frac{v}{u}$

$$h_i = -(-2) \times \frac{10}{-20} = -1\text{ cm}$$