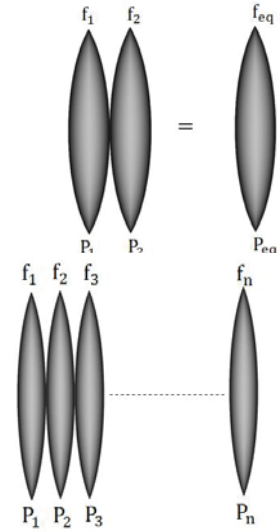
COMBINATION OF LENSES

Combinations of lenses



Equivalent focal length of lenses is $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

Equivalent power of lenses is $P_{eq} = P_1 + P_2$

Equivalent focal length of n-lenses is

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots + \frac{1}{f_n}$$

Equivalent power of n-lenses is

$$P_{eq} = P_1 + P_2 + \cdots + P_n$$

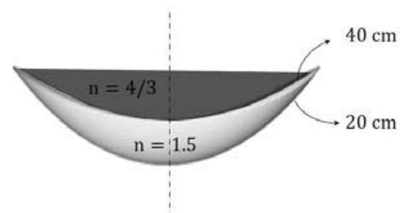
- Rays should be paraxial
- Lenses should be thin
- Lenses should be in contact with each other
- Object and observer should be in same medium
- $f_1, f_2, \ldots\ldots\ldots f_n$ should be taken with sign
- f_{eq} value comes with the sign
- f_{eq} is positive $\Longrightarrow$ converging lens
- f_{eq} is negative $\Longrightarrow$ diverging lens
- $f_1, f_2, f_3 \ldots \ldots \ldots f_n$ are the individual focal lengths with respect to the object and the observer medium

Focal length of the middle lens

$$\frac{1}{f_2} = \left(\frac{2}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The R.I. of the surroundings for the middle lens will be the R.I. of object's medium not the R.I. of the corresponding lenses.

Ex. Water is placed in a concavo-convex lens having spherical surfaces of radii of curvature 20 cm and 40 cm as shown. Find the equivalent focal length of the system.



Sol. The water on the concavo-convex lens forms a plano-convex lens having R.I. 4/3. Hence, this is a combination of two lenses:

1. Plano-convex lens 2. Concavo-convex lens

Let the individual focal length of the plano-convex lens is f_1 and that of concavo-convex lens is f_2.

$$\frac{1}{f_1} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{40} - \frac{1}{\infty}\right) = \frac{1}{120}$$

$$\frac{1}{f_2} = (1 \cdot 5 - 1)\left(\frac{1}{20} - \frac{1}{40}\right) = \frac{1}{80}$$

Therefore, the equivalent focal length of the system will be,

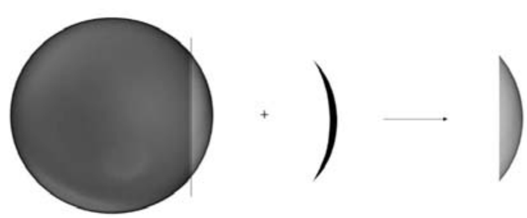
$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{120} + \frac{1}{80}$$

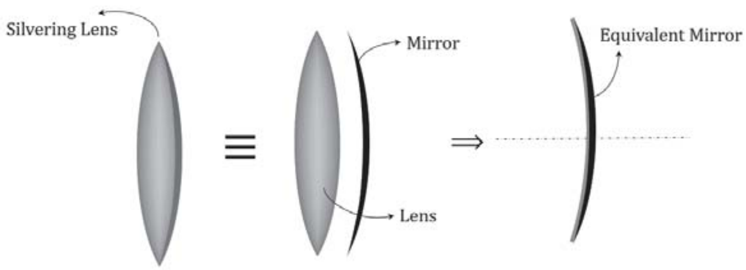
$$\frac{2+3}{240}$$

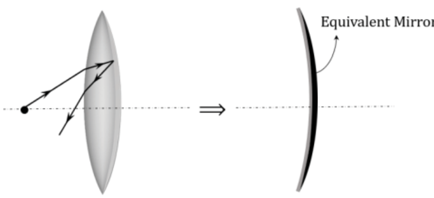
$$f_{eq} = \frac{240}{5} = 48 \text{ cm}$$

Silvering of Lenses



Cut the solid sphere along the dotted line | Do silvering on the curved surface of the cut out piece | A plano-convex lens with silvering on the curved surface





Upon incidence on the silvered lens, the incident ray reflects back to incident medium. Therefore, the equivalent system will be a mirror.

Since the incident ray crosses the lens two times and strikes the mirror only once, the equivalent power of the system will be, $P_{eq} = 2P_l + P_M$

Since the equivalent system acts as a mirror, the relation between equivalent power and equivalent focal length of the system will be,

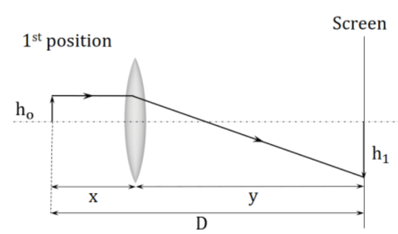
If the focal length of the lens and the mirror are f_l and f_M, respectively, then the equivalent focal length will be, $\frac{1}{f_{eq}} = \frac{1}{f_M} - \frac{2}{f_l}$ Since, $P_{eq} = 2P_l + P_M$

While finding the focal length of the lens , the R.I. of the surrounding medium that we should consider will be n_1 only because the ray doesn't go to the medium n_2.

- If focal length of equivalent mirror is negative implies concave mirror
- If focal length of equivalent mirror is positive implies convex mirror
- Medium need not be same on both the sides of the mirror
- Rays should be paraxial
- Lenses should be thin
- Lenses and mirror should be in contact with each other
- $f_1, f_2, \ldots\ldots f_n$ and f_m should be taken with sign
- Surrounding medium of lenses should be n_1

u–v method

First position



To find the focal length of the lens

Distance between real object and the screen: D

Height of the object: h_0

For 1 position of the lens, the image of the real object forms on the screen.

Height of the object: h_1

Distance of the object from the lens: $u = -x$

Distance of the image from the lens: $v = +y$

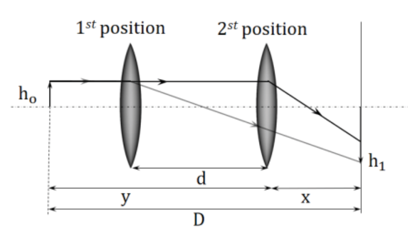
Applying lens formula, we get:

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{f} \qquad \ldots.(1)$$

Applying the formula of longitudinal magnification of lens, we get:

$$\frac{h_1}{h_0} = \frac{y}{x} \qquad \ldots..(2)$$

Second position



As the lens is further moved towards the screen by a distance d from the 1^{st} position, let another image of height h_2 is formed. This will happen only if the value of u and v of 1^{st} position gets interchanged at 2^{nd} position.

Height of the object: h_o

Height of the object: h_2

Distance of the object from the lens: $u = -\text{y}$
Distance of the image from the lens: $v = +\text{x}$
Applying lens formula, we get

$$\frac{1}{y} + \frac{1}{x} = \frac{1}{f}$$

Applying the formula of longitudinal magnification of lens, we get:

$$\frac{h_2}{h_o} = \frac{x}{y} \qquad \ldots..(3)$$

Multiplying equation (2) with equation (3), we get,

$$\frac{h_1}{h_o} \times \frac{h_2}{h_o} = \frac{y}{x} \times \frac{x}{y} \Rightarrow \frac{h_1 h_2}{h_o^2} = 1$$

$$h_o = \sqrt{h_1 h_2}$$

From the figure, we get,

$$D = x + y \qquad \ldots..(4)$$

$$d = y - x \qquad \ldots..(5)$$

Adding equation (4) and (5), we get,

$$y = \frac{D+d}{2}$$

Subtracting equation (5) from (4), we get,

$$x = \frac{D-d}{2}$$

Substituting the value of x and y in equation (1), we get,

$$\frac{1}{f} = \frac{2(D+d)-2(D-d)}{D^2-d^2}$$

$$\frac{1}{f} = \frac{4D}{D^2-d^2} \quad = \quad \frac{D^2-d^2}{4D}$$