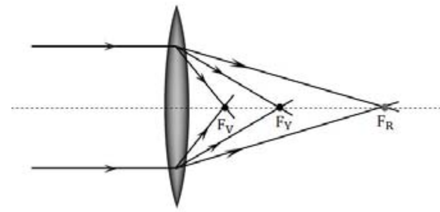
CHROMATIC ABERRATION AND OBLIQUE VISION

Chromatic Aberration

Lens can't focus all the colors at a single point due to the variation in refractive index with the wavelength of light.



Converging lens

For converging lens, $f_R > f_V$

Practically, $f_R > f_V$ (defect in focal length) is very small as the change in refractive index itself is small.

Hence, we chose yellow colored wavelength as average value for calculating refractive index.

$f_R > f_V$: Positive for converging lens

$f_Y =$ Mean focal length

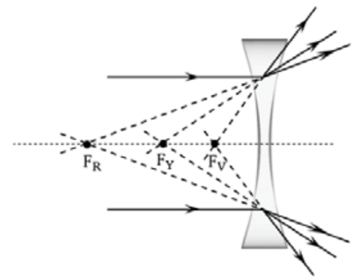
Lens can't focus all the colors at a single point due to the variation in refractive index with the wavelength of light.

Diverging lens

$f_R > f_V$: Negative for converging lens

As the defect is positive for converging lens and negative for diverging lens, we can combine both the lenses to have zero defect.

Chromatic Aberration makes the image blurry



Condition for Achromatism or Achromatic Combination

For red colored light, $\frac{1}{f_R} = (n_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ….(1)

For red colored light, $\frac{1}{f_v} = (n_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ …..(2)

Normal focal length, $\frac{1}{f} = (n_y - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Subtracting equation (1) from (2),

$$\frac{1}{f_v} - \frac{1}{f_R} = (n_v - n_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_R - f_v}{f_v - f_R} = \left(\frac{(n_V - n_R)}{(n_y - 1)}\right)(n_y - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

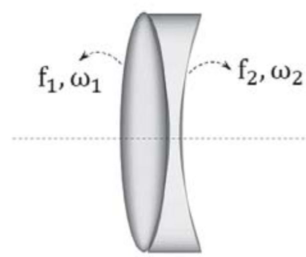
$$\frac{dt}{f^2} = \frac{\omega}{f}$$

Here,

Dispersive power of medium: $\omega = \frac{n_y - nR}{n_Y - 1}$

$f_R = f_v = f$ [Since, $f_R - f_v$ is very small]

Now consider combination of two lenses as shown in the figure. We will need the combination in which defect is not present.



We have, $\frac{df}{f^2} = \frac{\omega}{f}$

Focal length of equivalent lens, $\frac{1}{f_E} = \frac{1}{f_1} + \frac{1}{f_2}$ … (1)

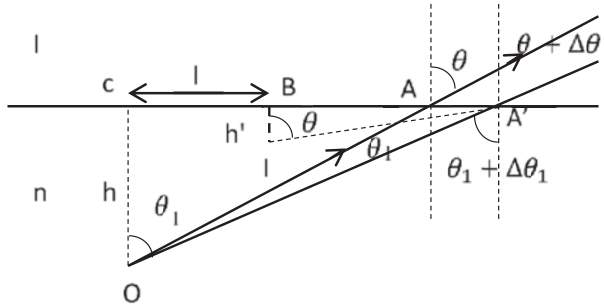
Differentiate equation (1) and equate defect to zero

$$\frac{-df_1}{f_\varepsilon^2} = 0 = \frac{-df_1}{f_1^2} - \frac{dt_2}{f_2^2}$$

$$\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

Zero defect is possible only for combination of converging and diverging lens as the defect is positive for converging lens and negative for diverging lens.

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$



Lenses kept with separation

When the two lenses of different focal lengths are placed as shown in the figure, equivalent focal length is given by,

$$\frac{1}{f_E} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Oblique vision

We know that when an observer perpendicularly sees the image of an object immersed in a liquid of R.I. n_1, the formula of apparent depth becomes:

$$\frac{d'}{d} = \frac{n_2}{n_1} = \frac{\text{Apparent Depth}}{\text{Real Depth}}$$

If the viewer looks at the image straight on in relation to the liquid's surface and if the rays reaching the observer's eye are close to the optical axis, then the formula mentioned earlier will remain valid.

Now, let's imagine a situation where the observer is positioned at an angle relative to the object submerged in the liquid. As a result, the angle at which the light ray approaches the observer is not small. Therefore, these rays cannot be regarded as paraxial rays.

Let's imagine that the object is submerged in a medium with a refractive index of n, while the observer is situated in air, where the refractive index is 1.

Assume that the height of the object is h, and the angle of observation of the object lies in the neighborhood of the angle θ.

Let the two extreme rays of the light beam that come to the eye of the observer make the angles θ and $(\theta + \Delta\theta)$, as shown in the figure.

Due to this oblique vision of the observer, let the image is formed at I and assume that the height of the image is h'

The horizontal distance between the object and image i.e., the distance between C and B is l.

From ΔIAB, we get: $\tan\theta = \frac{AB}{h'} \Rightarrow AB = h'\tan\theta \ldots\ldots (1)$

From ΔOCA, we get $\tan\theta_1 = \frac{AC}{h} \Rightarrow AC = h\tan\theta_1 \ldots\ldots (2)$

From the figure, we can write that:

$$AC - AB = l \Rightarrow l\tan\theta_1 - h'\tan\theta = l \ldots\ldots (3)$$

From Snell's law, we get: $n\sin\theta_1 = 1 \cdot \sin\theta \ldots\ldots (4)$

Since the change in angle remains small as we shift from A to A' , differentiating the above equation, we get:

$$n \cdot \text{Cos}\,\theta_1 \cdot d\theta_1 = \text{Cos}\,\theta d\theta$$

$$\frac{d\theta_1}{d\theta} = \frac{\text{Cos}\,\theta}{n\text{Cos}\,\theta_1} \ldots\ldots (5)$$

Now, h , h' and l will not be changed as we shift from A to A'. Thus, differentiating equation (3) , we get:

$$h\text{Sec}^2\theta_1 d\theta_1 - h'\sec^2\theta d\theta = 0$$

$$h\sec^2\theta_1 \cdot \frac{d\theta_1}{d\theta} = h'\sec^2\theta.$$

$$h' = \frac{h\,\text{Sec}^2\theta_1}{\sec^2\theta} \cdot \frac{\text{Cos}\,\theta}{n\text{Cos}\,\theta_1}$$

[$\because$ From equation (5), we have: $\frac{d\theta_1}{d\theta} = \frac{\cos\theta}{n\cos\theta_1}$]

$$h' = \frac{h\sec^3\theta_1}{n\sec^2\theta} \ldots\ldots (6)$$

From equation (4), we have:

$$\sin\theta_1 = \frac{\sin\theta}{n}$$

$$\cos\theta_1 = \sqrt{1-\sin^2\theta_1}$$

$$\text{Cos}\,\theta_1 = \frac{\sqrt{n^2-\sin^2\theta}}{n}$$

$$\sec\theta_1 = \frac{n}{\sqrt{n^2-\sin^2\theta}}$$

Substituting the value of equation (6), we get:

$$h' = \frac{h\sec^3\theta_1}{n\sec^2\theta}$$

$$h' = \frac{h\cdot n^3}{n\sec^2\theta\cdot(n^2-\sin^2\theta)^{3/2}}.$$

$$h' = \frac{hn^2\cos^2\theta}{(n^2-\sin^2\theta)^{3/2}}.$$