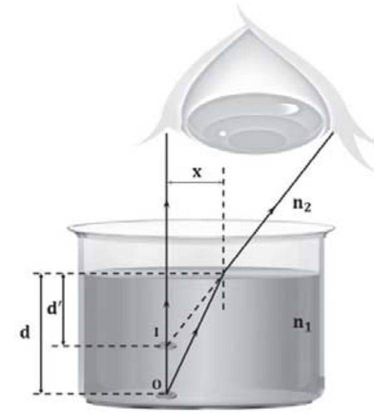
APPARENT DEPTH

Apparent Depth & speed

- Rays are Paraxial i.e. δ is small
- $d =$ Real depth
- $d' =$ Apparent depth

When the object and observers are situated in different mediums, the object appears to be located at a distinct position, and this perceived depth is referred to as the apparent depth.

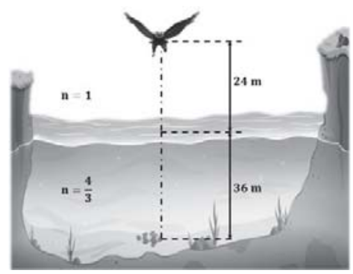
Ex. In the depicted figure, the coin seems to be at a higher position than its true location.



$$\tan i = \frac{x}{d}; \tan r = \frac{x}{d'}$$
$$n_1 \cdot \sin i = n_2 \cdot \sin r$$
$$n_1 \cdot \tan i = n_2 \cdot \tan r$$
$$\begin{Bmatrix} i \approx \sin i \approx \tan i \\ r = \sin r \approx \tan r \end{Bmatrix}$$
$$n_1 \cdot \frac{x}{d} = n_2 \cdot \frac{x}{d'}$$
$$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{\text{Apparent depth}}{\text{Real depth}} = \frac{n_2}{n_1} = \frac{d'}{d}$$

Note: Rays need to be paraxial.
n_2: Represents the refractive index of the medium into which the ray is entering.
n_1 : Denotes the refractive index of the medium from which the ray originates.
d and d' are measured from the surface.

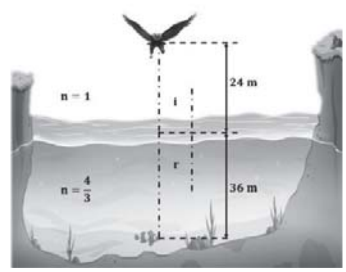
Ex. If the actual height of the eagle and the actual depth of the fish from the free surface of the water are 24 m and 36 m respectively as shown, find
(a) The height of the eagle as viewed by the fish
(b) The depth of the fish as viewed by the eagle



Sol. (a) The height of the eagle as viewed by the fish
Here, the ray is going from rarer to denser medium.
We know that, if $n_1 < n_2, r < i$.

We know that,

$$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{n_2}{n_1} = \frac{d'}{d}$$
$$\frac{4/3}{1} = \frac{d'}{24}$$
$$d' = 32 \text{ m}$$

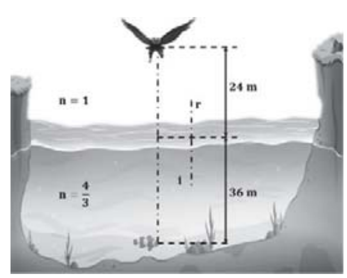


Thus, height of the eagle as viewed by the fish:
$$= 32 + 36 = 68 \text{ m}$$

(b) The depth of the fish as viewed by the eagle
Here, the ray is going from denser to rarer medium.
We know that, if $n_1 > n_2, r > i$.

We know that,

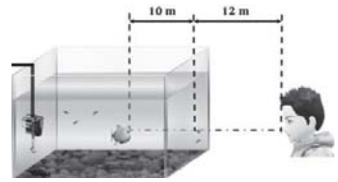
$$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{1}{4/3} = \frac{d'}{36}$$
$$d' = 27 \text{ m}$$



Thus, depth of the fish as viewed by the eagle:
$$= 27 + 24 = 51 \text{ m}$$

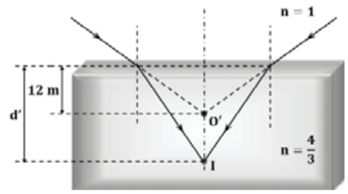
Ex. The actual distance of the fish and the nose of balak from the wall of the aquarium are 10 m and 12 m respectively. Find the distance of the nose as viewed by the fish.

Sol. Here, the ray is going from rarer to denser medium.
We know that, if $n_1 < n_2, r < i$.



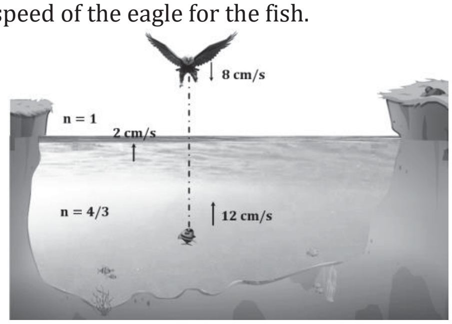
We know that, $$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{4/3}{1} = \frac{d'}{12}$$
$$d' = 16 \text{ m}$$
Thus, the distance of the nose as viewed by the fish:
$$10 + 16 = 26\text{m}$$

Ex. Converging light rays incident at an interface separating the two media having refractive indices $n = 1$ and $n = 4/3$ as shown, find the distance of the point from the interface where the image is formed. Also find the nature of the image.



Sol. Here, the ray is going from rarer to denser medium.
We know that, If $n_1 < n_2, r < i$.
We know that, $$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{4/3}{1} = \frac{d'}{12}$$
$$d' = 16 \text{ m} > 12 \text{ m}$$
As the rays meet in real to form the image after refraction, it is a real image.

Ex. A fish approaches the water surface of the pond with speed 12 cm/s and a eagle approaches the fish with speed 8 cm/s as shown. If the water level is rising with rate 2 cm/s, find.
(a) The apparent speed of the fish for the eagle
(b) The apparent speed of the eagle for the fish.



Sol. **(a)** The apparent speed of the fish for the bird
Let, x = Distance between fish and the surface; y = Distance between image and the surface
$\frac{dx}{dt}$ = velocity of fish w.r.fo surface $\frac{dx}{dt}$ = velocity of image w.r.fo surface
$v_1 =$ Velocity of the image $v_e =$ Velocity of the eagle
$v_s =$ Velocity of the water surface $v_f =$ Velocity of the fish
We know that $$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{1}{\frac{4}{3}} = \frac{y}{x}$$
$$y = \frac{3}{4}x$$
Differentiating w.r.t. time, $$\left(\frac{dy}{du} = \frac{3}{4} \cdot \frac{dx}{du}.\right)$$
$$v_1 - v_s = \frac{3}{4}(v_f - v_s).$$
$$v_1 - 2 = \frac{3}{4}(12 - 2) = \frac{15}{2}$$
$$v_1 = 9.5 \text{ cm/sec}$$
This is the velocity of image that would be observed by a steady observer in the air. As the observer (eagle) is moving with velocity 8 cm/s, the apparent speed of the fish for the eagle is.
$$V_{I\varepsilon} = 9.5 - (-8)$$
$$= 17.5 \text{ cm/sec}$$
$$\frac{35}{2} \text{ cm/s}$$

(b) The apparent speed of the bird for the fish
We know that $$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{\frac{4}{3}}{1} = \frac{y}{x}$$

Differentiating w.r.t. time,

$$\frac{dy}{dt} = \frac{4}{3}\left(\frac{dx}{du}\right)$$
$$v_1 - 2 = \frac{4}{3}(-8 - 2).$$
$$v_1 = -\frac{40}{3} + 2$$
$$v_1 = -\frac{34}{3}$$

This is the velocity of image that would be observed by a steady observer in the water.
As the observer (fish) is moving with velocity 12 cm/s, the apparent speed of the bird for the fish is:

$$V_{If} = \frac{-34}{3} - 12$$
$$= -\frac{70}{3}\text{cm/s}$$

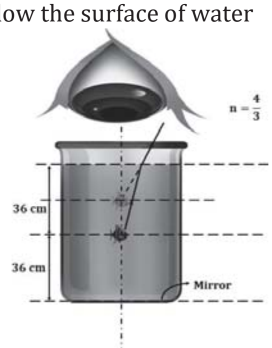
Ex. Determine the number of images of fish visible to observer. Also find the distance between these images.

Sol. Two images of the fish will be visible to the observer, one will be due to difference in the refractive index of the surrounding and water the image will appear slightly above than the actual position of the fish and, second will be due to the reflection in the mirror.

First image:
Here, the first image will form at elevated location due to refraction.
We know that

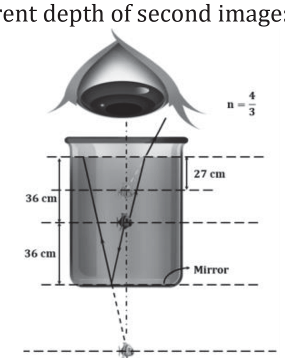
$$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{n_2}{n_1} = \frac{d'}{d} \Rightarrow \frac{1}{4/13} = \frac{d'}{36} d' = 27c_n$$

Thus, the first image will form at 27 cm below the surface of water



Second image:
The second image will form after reflection from the mirror.
After reflection from the mirror, image will form at 36 cm below the mirror.

But, as the rays will pass through water, it won't be visible at the same location for the observer in air.
Apparent depth of second image:



We know that

$$\frac{d'}{d} = \frac{n_2}{n_1}$$
$$\frac{n_2}{n_1} = \frac{d'}{d}$$
$$\frac{1}{4/3} = \frac{d'}{108}$$
$$= 81 \text{ cm}$$

2 Images, 54 cm
Thus, two images will form at distance of: 81 - 27 = 54 cm.