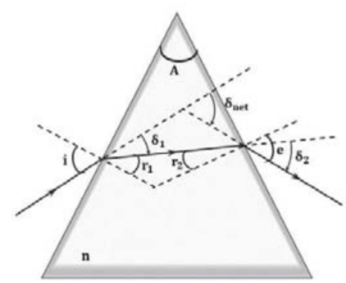
ANGLE OF DEVIATION IN PRISM

Net Angle Of Deviation In Prism



i : Angle of incidence
e: Angle of emergence
δ_1 : Angle of deviation at the 1^{st} refracting surface (C.W.)
δ_2 : Angle of deviation at the 2^{nd} refracting surface (C.W.)
r_1 : Angle of refraction on the 1^{st} refracting surface
r_2 : Angle of incidence on the 2^{nd} refracting surface
n : Refractive index of the prism
A : Angle of prism

$$\text{For } \triangle BCD, \angle DBC = r_1 \text{ and } \angle BDC = r_2$$
$$\angle BCD = [180^\circ - (r_1 + r_2)]$$
$$\square\ ABCD, \angle ABC = 90^\circ, \angle ADC = 90^\circ$$
$$\angle BCD = [180^\circ - (r_1 + r_2)]$$
$$\text{Therefore, for } \square\ ABCD,$$
$$\angle BAC + \angle ABC + \angle BCD + \angle ADC = 360^\circ$$
$$A + 90^\circ + [180^\circ - (r_1 + r_2)] + 90^\circ = 360^\circ$$
$$A + 180^\circ - (r_1 + r_2) = 180^\circ$$
$$A = (r_1 + r_2) \ldots\ldots\ldots (i)$$

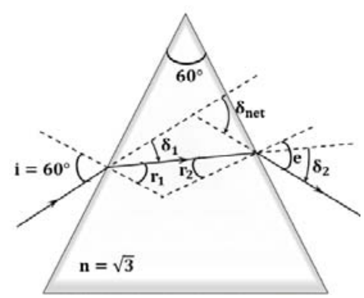
Angle of deviation at the 1 st refracting surface: $\delta_1 = (i - r_1)$ (C.W.) Angle of deviation at the 1^{st} refracting surface: $\delta_2 = (e - r_2)$ (C.W.) Therefore, net angle of deviation in clockwise direction is,

$$\delta_{nd} = \delta_1 + \delta_2$$
$$= i - r_1 + e - r_2$$
$$= i + e - (r_1 + r_2)$$
$$\delta_{net} = i + e - A$$

From equation (i), we have $[A = (r_1 + r_2)]$

Note: In the majority of cases, the values of i, A, and n will be provided. The subsequent steps must be followed to determine the total angle of deviation of the prism.

Step 1: Determine r_1 by applying Snell's law ($n_s \sin i = n_p \sin r_1$) where ns and dp represent the refractive indices of the surroundings and the prism respectively.
Step 2: Calculate r_2 using $[A = (r_1 + r_2)]$
Step 3: Calculate e using Snell's law ($n_p \sin r_2 = n_s \sin e$)
Step 4: Use $\delta_{net} = i + e - A$



Ex. If a ray of light passes through a triangular glass prism of refractive index $n = \sqrt{3}$ at an angle $i = 60^\circ$ as shown, find the net deviation of the ray.

Sol. Given, $i = 60^\circ A = 60^\circ n = \sqrt{3}$

Step 1: Calculate r_1 using Snell's law ($n_s \sin i = n_p \sin r_1$) [where n_s and n_p are R.I of surroundings and the prism] Here, assume $n_s = 1$. Thus

$$1 \cdot \text{Sin}60^\circ = \sqrt{3}\text{Sin}r_1$$
$$r_1 = 30^\circ$$

Step 2: Calculate r_2 using $[A = (r_1 + r_2)] r_1 + r_2 = A$

$$\mathbf{r}_2 = 30^\circ \text{ [Since } A = 60^\circ \text{ and } r_1 = 30^\circ \text{]}$$

Step 3: Calculate e using Snell's law ($n_p \sin r_2 = n_s \sin e$)

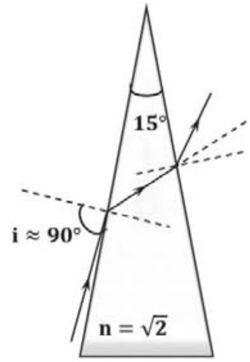
$$\sqrt{3}\sin 30^\circ = 1 \times \sin e \Rightarrow \frac{\sqrt{3}}{2} = \sin e \Rightarrow e = 60^\circ$$

Step 4: Use $\delta_{net} = i + e - A$

$$\mathcal{S} = 60^\circ + 60^\circ - 60^\circ = 60^\circ C \cdot \omega$$

Ex If a ray of light goes through a triangular glass prism of refractive index $n = \sqrt{2}$ at grazing incidence as shown, find the net deviation of the ray.

Sol. Given, $i \approx 90^\circ A = 15^\circ n = \sqrt{2}$



Step 1: Calculate r_1 using Snell's law ($n_s \sin i = n_p \sin r_1$) [where ns and np are R.I of surroundings and the prism] Here, assume $n_s = 1$. Thus,

$$1 \cdot \sin 90^\circ = \sqrt{2} \cdot \sin r_1$$
$$r_1 = 45^\circ$$

Step 2: Calculate r_2 using $[A = (r_1 + r_2)]$

$$r_1 + r_2 = 15^\circ$$
$$45^\circ + r_2 = 15^\circ$$
$$r_2 = -30^\circ$$

r_2 negative means the angle forms at the opposite side to the normal than the usual one. Usually r_2 makes the angle above the normal. So, here it will make the angle below the normal, as shown in the figure.

Step 3: Calculate e using Snell's law ($n_p \sin r_2 = n_s \sin e$)

$$\sqrt{2} \cdot \text{Sin}30^\circ = 1 \cdot \text{Sin}e$$
$$e = 45^\circ$$

Angle of deviation at the 2^{nd} refracting surface:

$$\delta_2 = (45^\circ - 30^\circ) = 15^\circ \text{ (A.C.W.)}$$

Angle of deviation at the 1^{st} refracting surface:

$$\delta_1 = (i - r_1) = (90^\circ - 45^\circ) = 45^\circ \text{ (C.W.)}$$

Therefore, net angle of deviation in clockwise direction is,

$$S_{net} = \delta_1 + \delta_2$$
$$\delta_{net} = 4S - 15^\circ = 30^\circ (c \cdot 0 \cdot)$$

Ex. If a ray of light goes through a triangular glass prism of refractive index $n = \sqrt{3}$ at an angle $i = 60^\circ$ as shown, find the net deviation of the ray.

Sol. Given, $i = 60^\circ n = \sqrt{3}$

Step 1: Calculate r_1 using Snell's law $\left(n_s \sin i = n_p \sin r_1\right)$

$$1 \cdot \sin 60^\circ = \sqrt{3} \cdot \sin r_1 r_1 = 30^\circ$$

Angle of deviation at the 1^{st} refracting surface:

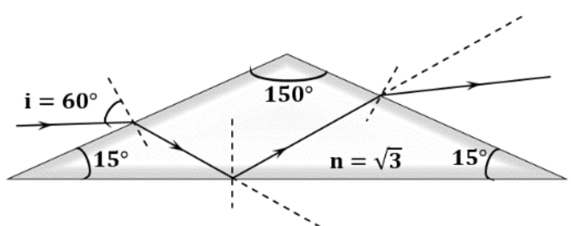
$$\delta_1 = (i - r_1) = (60^\circ - 30^\circ) = 30^\circ \text{ (C.W.)}$$

For $\triangle ABD, \angle ABC = 15^\circ$ and $\angle BAD = (90^\circ + 30^\circ) = 120^\circ$

$$\angle BDA = [180^\circ - (120^\circ + 15^\circ)] = 45^\circ$$

The critical angle at point D is,

$$\theta_c = \sin^{-1}\left(\frac{n_r}{n_d}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) < \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\theta_c < 45^\circ$$



Since at point D, angle of incidence (45°) is greater than the critical angle, total internal reflection will take place.

At point D, the angle of deviation is given by,

$$\delta_2 = (180^\circ - 2i) = (180^\circ - [2 \times 45^\circ]) = 90^\circ \text{ (A.C.W.)}$$

For $\triangle PDQ, \angle PQD = 15^\circ$ and $\angle PDQ = 45^\circ$

$$\angle DPQ = [180^\circ - (45^\circ + 15^\circ)] = 120^\circ$$

Therefore, angle of incidence on the surface RQ is,

$$r_2 = (120^\circ - 90^\circ) = 30^\circ$$

Hence, angle of emergence from the surface RQ is,

$$\sqrt{3} \sin 30^\circ = 1 \times \sin e$$
$$\frac{\sqrt{3}}{2} = \sin e \Rightarrow e = 60^\circ$$

At point P, the angle of deviation is given by,

$$\delta_3 = (60^\circ - 30^\circ) = 30^\circ \text{ (C.W.)}$$

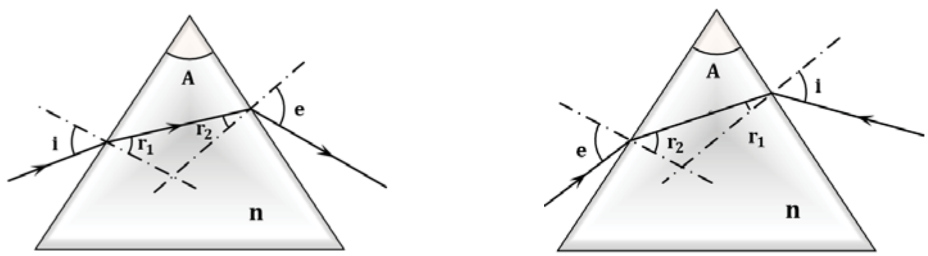
Therefore, net angle of deviation in clockwise direction is,

$$\delta_{net} = \delta_1 + \delta_2 + \delta_3$$
$$\delta_{net} = 30^\circ \text{ (C.W.)} + 90^\circ \text{ (A.C.W.)} + 30^\circ \text{ (C.W.)}$$
$$\delta_{net} = 30^\circ \text{ (A.C.W.)}$$

Angle of deviation

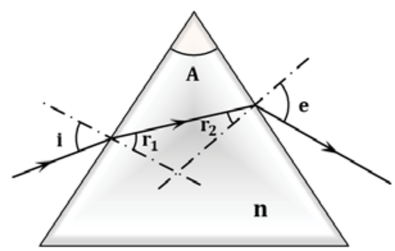
If *i* and *e* switch places, the angle of deviation remains unchanged.

$$\delta_{net} = i + e - A$$



Let's presume the prism's surrounding medium is air, with a refractive index of 1, while the prism itself has a refractive index of n.

Apply Snell's law on the 1st refracting surface:



$$1 \sin i = n \sin r_1$$
$$r_1 = \sin^{-1}\left(\frac{1}{n}\sin i\right)$$

We know: $A = (r_1 + r_2) \Rightarrow r_2 = (A - r_1)$

Apply Snell's law on the 2^{nd} refracting surface:

$$n \sin r_2 = 1 \sin e$$
$$e = \sin^{-1}(n\sin(A - r_1))$$
$$e = \sin^{-1}\left(n\sin\left(A - \sin^{-1}\left(\frac{\sin i}{n}\right)\right)\right)$$

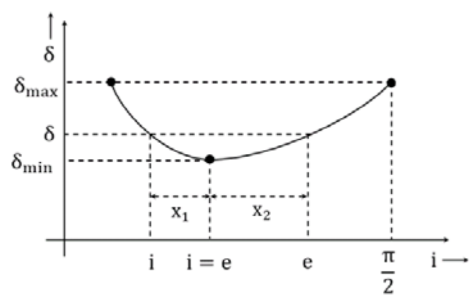
Therefore, net angle of deviation is

$$\delta = i + e - A$$
$$\delta = i + \sin^{-1}\left(n\sin\left(A - \sin^{-1}\left(\frac{\sin i}{n}\right)\right)\right) - A$$

Angle of deviation: Graphical Representation of δ and i

- Not parabola
- Doesn't extend to infinity
- Range: $\delta_{max} < \delta < \delta_{min}$
- $\delta = \delta_{min}$ at $i = e$
- $\delta = \delta_{max}$ at $i = \frac{\pi}{2}$ or $e = \frac{\pi}{2}$
- Asymmetric graph $(x_2 > x_1)$

$$\delta = i + \sin^{-1}\left(n\sin\left(A - \sin^{-1}\left(\frac{\sin i}{n}\right)\right)\right) - A$$



Ex. For a prism given below, angle of incidence for angle of minimum deviation is 35°. If the angle of incidence of the ray on the prism is 31° then the angle of emergence (e) of the ray is:

(a) 37° (b) 38° (c) 39° (d) 40

Sol. Given,

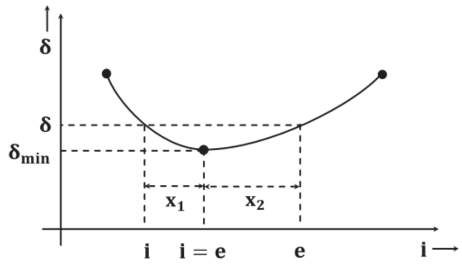
The angle of incidence for minimum angle of deviation, $i_{min} = 35^\circ$

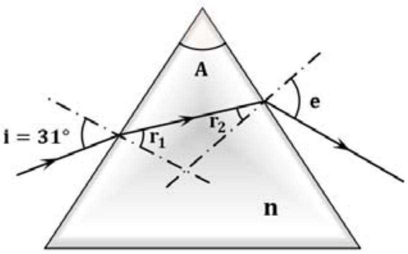
The angle of incidence, $i = 31^\circ$

Therefore, $x_1 = 4^\circ$

Since δ vs i graph is an asymmetric graph, $x_2 > x_1$. Thus,

$$x_2 > (35^\circ + 4^\circ)$$
$$x_2 > 39^\circ$$
$$x_2 > 39^\circ$$



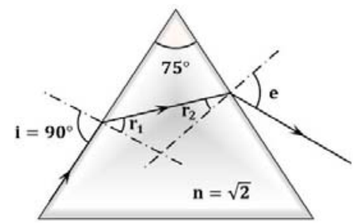


Hence, the angle of emergence, $e > 39°$

In these type of questions, we can use elimination of options method. As $e > 39°$ and there is only one option above 39°, option (d) is the correct answer.

Ex. If angle of prism is 75° for given prism, then find out the angle of maximum deviation (δ_{max}) for the prism.

Sol. We know that the maximum deviation occurs when i or e becomes equal to 90°. Let i = 90° i.e., grazing incidence



For grazing incidence of a ray from rarer to denser medium, the angle of refraction becomes equal to the critical angle.

Therefore, $$\sin\theta_c = \sin r_1 = \frac{1}{\sqrt{2}}$$

$$r_1 = 45°$$

$$r_1 + r_2 = A$$

$$r_2 = A - r_1 = 75° - 45° = 30°$$

Apply Snell's law on the 2^{nd} refracting surface:

$$n_p \sin r_2 = n_s \sin e$$

$$\sqrt{2}\sin 30° = 1\sin e$$

$$\sin e = \frac{1}{\sqrt{2}} \Rightarrow e = 45°$$

The angle of maximum deviation (δ_{max}) for the prism is,

$$\delta_m = 90° + 45° - 75°$$

$$= 60°(C \cdot W)$$

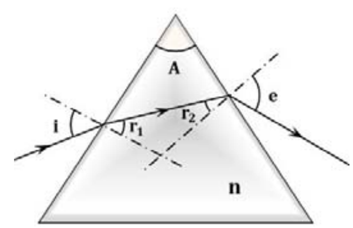
Angle of deviation

For $\delta_{minimum}$ $$i = e \Rightarrow r_1 = r_2 \Rightarrow r_1 = r_2 = \frac{A}{2}$$

$$\delta = i + e - A\delta_m$$

$$\delta_n = i + i - A = 2i - A$$

$$i = \frac{\delta_m + A}{2}$$



$$n_s \cdot \sin i = n_p \cdot \sin r_1$$

$$n_s \cdot \sin(\frac{\delta_m + A}{2}) = n_p \cdot \sin\frac{A}{2}$$

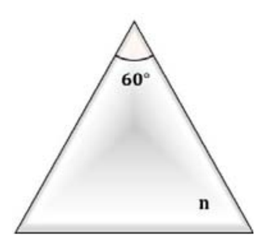
$$\frac{n_p}{n_s} = \frac{\sin(\frac{\delta_m + A}{2})}{\sin A/2}$$

If surroundings medium of prism is air (i.e., $n_s = 1$) and R.I. of the prism is $n_s = n$, then the above formula becomes.

$$n = \frac{\sin(\frac{A + \delta_{min}}{2})}{\sin(\frac{A}{2})}$$

Ex. If angle of minimum deviation is 30° for given equilateral prism, then find out the refractive index (n) of the prism.

Sol. Given,



The angle of minimum deviation, $\delta_{min} = 30°$

The angle of prism, A = 60°

The refractive index (n) of the prism

$$n = \frac{\sin(\frac{A + \delta_{min}}{2})}{\sin(\frac{A}{2})} = \frac{\sin(\frac{60° + 30°}{2})}{\sin(\frac{60}{2})} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$