PROPERTIES OF EM WAVES, ENERGY DENSITY, INTENSITY AND POYNTING VECTOR

Faraday's Law and Ampere's Law

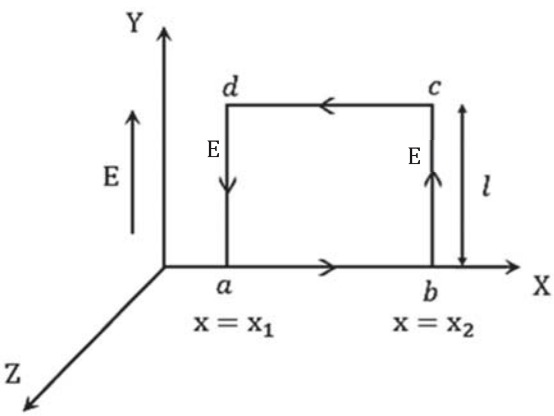
Faraday's Law and Ampere's Law are two fundamental principles in electromagnetism that play pivotal roles in understanding the behavior of electric and magnetic fields.

Faraday's Law:

Faraday's law of electromagnetic induction, formulated by British physicist Michael Faraday in the 1830s, describes how a changing magnetic field induces an electromotive force (EMF) and thus an electric current in a nearby conductor. Faraday's Laws of Electromagnetic Induction comprise two fundamental principles. The initial law elucidates the process by which an electromotive force (emf) is induced within a conductor when subjected to a changing magnetic field. Meanwhile, the second law provides a quantitative understanding of the magnitude of the emf generated within the conductor under such circumstances.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$
$$E = E_y = E_0 \sin\ \omega(t - \frac{x}{c})$$
$$B = B_z = B_0 \sin\ \omega(t - \frac{x}{c})$$

Faraday's law can be effectively utilized by dividing the loop into four distinct segments, as outlined below.



For part *ab*: $\vec{E} \perp \vec{ab} \Rightarrow \int_{ab} \vec{E} \cdot dl = 0$

For part *bc*: $\int_{bc} \vec{E} \cdot dl = E_0 \sin\,(\omega t - \frac{\omega x_2}{c}) l$

For part *cd*: $\vec{E} \perp \vec{cd} \Rightarrow \int_{cd} \vec{E} \cdot dl = 0$

For part *da*: $\int_{da} \vec{E} \cdot dl = E_0 \sin\,(\omega t - \frac{\omega x_1}{c}) l \operatorname{Cos}\ 180^\circ$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$
$$E = E_y = E_0 \sin\ \omega(t - \frac{x}{c})$$
$$B = B_z = B_0 \sin\ \omega(t - \frac{x}{c})$$

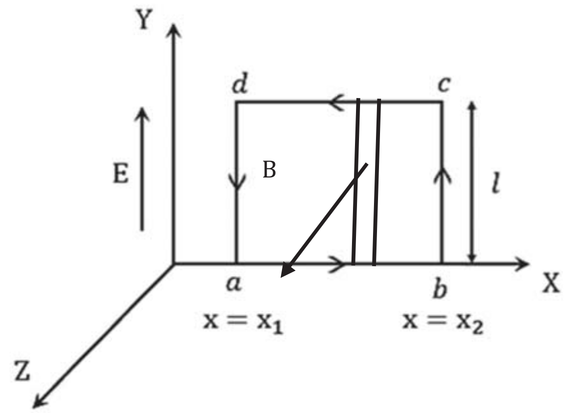
Therefore, the integral over the entire loop yields the following result:

$$\oint \overline{E} \cdot d\overline{d} = E_0 l[\sin\,(\omega t - \frac{\omega x_2}{c}) - \sin\,(\omega t - \frac{\omega x_1}{c})]$$

To calculate the right-hand side (RHS) of Faraday's law, let's examine a strip located at position x with a width of dx, as illustrated below: The flux passing through the strip can be determined as follows: $d\phi = \beta_0 \operatorname{Sin}\,(\omega t - \frac{\omega x}{c}) l dx$

By performing integration over the range from x_1 to x_2, we obtain the total flux passing through the entire loop.

$$Q_B = -B_0 l(\frac{c}{-\omega})[\operatorname{Cos}\,(\omega t - \frac{\omega x}{c})]_{x_1}^{x_2}$$
$$\phi_B = \frac{\beta_0 l}{\omega} c[\operatorname{Cos}\,(\omega t - \frac{\omega x_2}{c}) - \operatorname{Cos}\,(\omega t - \frac{\omega x_1}{c})]$$



$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

$$E = E_y = E_0 \sin\,\omega(t - \frac{x}{c})$$

$$B = B_Z = B_0 \sin\,\omega(t - \frac{x}{c})$$

When we differentiate the flux with respect to time (t),

$$-\frac{d\phi_B}{dt} = \frac{\beta_0 lc}{\psi}[\phi\text{Sin}\,(\omega t - \frac{\omega x_2}{c}) - \phi\text{Sin}\,(\omega t - \frac{\omega x_1}{c})]$$

By setting the derived expressions of the left-hand side (LHS) and right-hand side (RHS) of Faraday's law equal to each other, we obtain:

$$\frac{B_0}{C} = E_0\boldsymbol{X}$$

$$\Rightarrow \quad \frac{E_0}{B_0} = C = \text{ Velocity of electromagnetic radiation}$$

Therefore, for an electromagnetic wave, if one of the electric fields (E) or magnetic field (B) is determined, the other quantity can also be determined using the relation stated above.

$$E_0 = cB_0$$

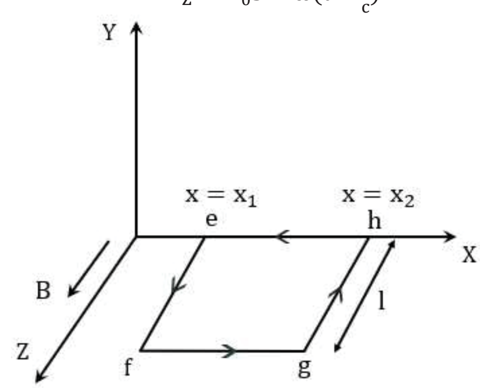
Ampere's Law:

Ampere's law delineates the intricate relationship between magnetic fields and the electric currents that engender them. It precisely defines the magnetic field strength associated with a given electric current or conversely, the electric current engendered by a specific magnetic field, under the condition that the electric field remains constant over time. This law serves as a cornerstone in understanding the dynamic interplay between electric currents and magnetic fields, offering invaluable insights into electromagnetic phenomena.

$$\oint \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$$

$$E = E_y = E_0 \sin\,\omega(t - \frac{x}{c})$$

$$B = B_Z = B_0 \sin\,\omega(t - \frac{x}{c})$$



Let's contemplate a loop labeled as $efghe$ situated in a vacuum, as depicted, with the absence of conduction current.

Ampere's law can be effectively employed by subdividing the loop into four distinct segments, as outlined below.

For part ef: $\int_{ef} \vec{B} \cdot d\vec{l} = B_0 l \text{Sin } (\omega t - \frac{\omega x_1}{c})$

For part fg: $\{\vec{B} \perp \vec{l}\} \Rightarrow \int_{fg} \vec{B} \cdot d\vec{l} = 0$

For part gh: $\int_{gh} \vec{B} \cdot d\vec{l} = \beta_0 l \text{Sin } (\omega t - \frac{\omega *_2}{c}) \text{Cos } 180°$

For part he: $\{\vec{B} \perp \vec{l}\} \Rightarrow \int_{he} \vec{B} \cdot d\vec{l} = 0$

Therefore, the integral over the entirety of the loop yields:

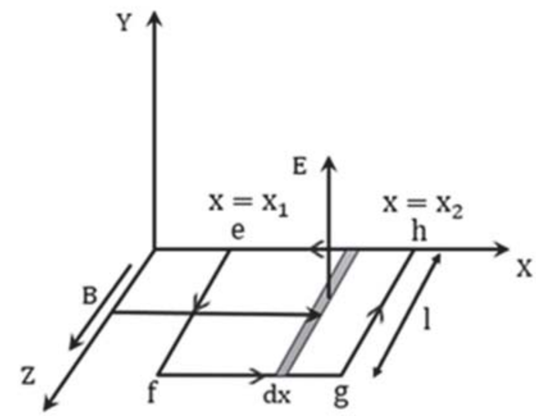
$\oint \vec{B} \cdot \overline{d} = B \cup l[\text{Sin } (\omega t - \frac{\omega x_1}{c}) - \text{Sin } (\omega t - \frac{\omega x_2}{c})]$ $\overset{L \cdot HS}{=}$

$$\oint \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$$

$$E = E_y = E_0 \sin \ \omega(t - \frac{x}{c})$$

$$B = B_Z = B_0 \sin \ \omega(t - \frac{x}{c})$$

To determine the right-hand side (RHS) of Ampere's law, let's focus on a strip located at position x, illustrated below, with a width of dx:



The amount of magnetic flux passing through the strip. $d\phi = E_0 \text{Sin } (\omega t - \frac{\omega x}{c}) \cdot l dx$

By performing integration over the range from $x1$ to $x2$, we obtain the total magnetic flux passing through the entire loop.

$$\phi_E = \int_{x_1}^{x_2} E_0 \sin \ (\omega t - \frac{\omega x}{c}) \cdot l dx = \frac{tE_0 lc}{t\omega} [\cos \ (\omega t - \frac{\omega x}{c})]_{x_1}^{x_2}$$

$$Q_\varepsilon = \frac{E_0 lc}{\omega} [\text{Cos } (\omega t - \frac{\omega x_2}{C}) - \text{Cos } (\omega t - \frac{\omega x1}{c})]$$

$$\oint \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$$

$$E = E_y = E_0 \sin \ \omega(t - \frac{x}{c})$$

$$B = B_z = B_0 \sin \ \omega(t - \frac{x}{c})$$

RHS of the Ampere’s law:

The right-hand side of Ampère's law refers to the mathematical expression representing the total magnetic field generated by the current enclosed within a closed loop. This side of the equation quantifies the magnetic field intensity in relation to the electric current flowing through the loop, providing a crucial aspect of understanding the magnetic effects of electric currents.

$$\mu_0 r_0 \frac{dQ_e}{dt^t} = \frac{E_0 l_0}{\omega_0} \mu_0 s_0 \mu [\text{Sin } (\nu t - \frac{\omega x_1}{c}) - \text{Sin } (\omega t - \frac{\omega x_t}{c})]$$

The right-hand side of Ampère's law refers to the mathematical expression representing the total magnetic field generated by the current enclosed within a closed loop. This side of the equation quantifies the magnetic field intensity in relation to the electric current flowing through the loop, providing a crucial aspect of understanding the magnetic effects of electric currents.

$$B_0 l = E_0 l'(\mu_0\varepsilon_0 \Rightarrow \frac{\beta_0}{E_0} = \mu_0\varepsilon_0 \Rightarrow \frac{1}{C^2} = \mu_0\varepsilon_0 \Rightarrow C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Furthermore, there exists another pertinent consideration or factor.

$$\frac{E_0}{B_0} = C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Speed of EM Waves

The velocity of electromagnetic waves, commonly referred to as the speed of light in a vacuum, represents the rate at which electromagnetic radiation propagates through empty space, denoted by the symbol 'c'.

Velocity of electromagnetic waves in a vacuum

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$
$$= 2.997\cdots\times10^8 \text{ m/sec}$$

Velocity of light in a vacuum

$$2.99793 \times 10^8 \text{ m/sec}$$

Velocity of electromagnetic waves in a medium

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$
$$\frac{E_0}{B_0} = C$$

μ : permittivity of the medium

ε : permeability of the medium

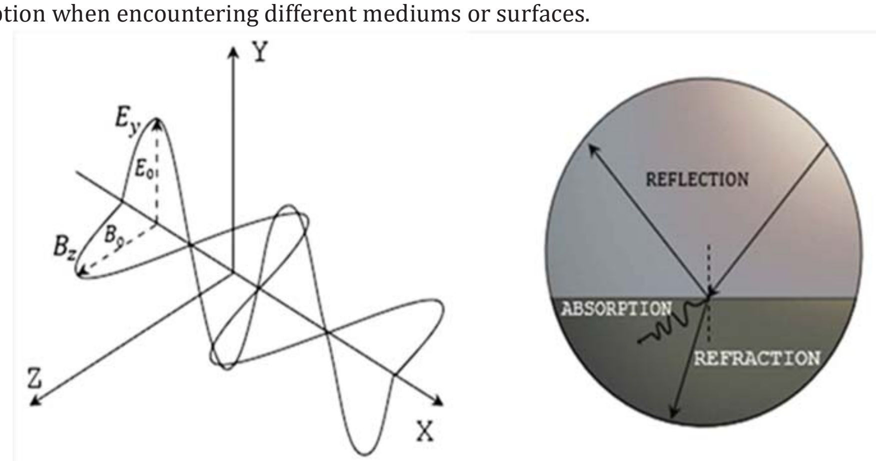
Refractive index of the medium (n): $n = \sqrt{\mu_1\varepsilon_r}$

$$C_m = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \frac{c}{\sqrt{\mu_1\varepsilon_r}} \quad C_n = \frac{C}{n}$$

Light exhibits electromagnetic characteristics.

Properties of EM Waves

These entities possess the capability to traverse through the vacuum of space. Their fundamental characteristic is their transverse nature. Their primary function is to convey energy from one location to another. Furthermore, they have the ability to undergo reflection, refraction, or absorption when encountering different mediums or surfaces.

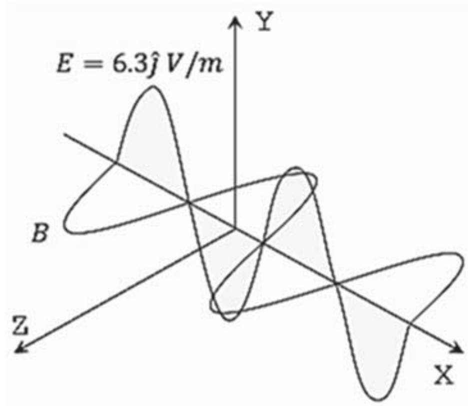


Frequency represents its intrinsic attribute. When it moves from one medium to another, only its velocity and wavelength undergo alterations.

Ex. At a specific point in space and time, an electromagnetic wave propagating in the x-direction with a frequency of 25 MHz exhibits an electric field of magnitude 6.3 j V/m. What is the magnetic field strength (B) at this location?

Sol. We have,

$$\frac{E}{B} = c$$
$$B = \frac{6.3}{3\times10^8}$$
$$B = 2.1 \times 10^{-8} \text{ Tesla}$$



Given that E is oriented in the y-direction, the magnetic field (B) is oriented in the z-direction. The electric field (E) perpendicular to the magnetic field (B) is also perpendicular to the direction of the flow of energy.

$B = 2.1 \times 10^{-8}\hat{k}T$

Energy Density:

The concept of electric field energy and magnetic field energy involves analyzing the energy contained within a specific volume, typically denoted as dV, which represents a small segment of space. Specifically, the energy density within the electric field pertains to the amount of energy present within this volume due to the electric field's influence.

Electric field energy density $= \frac{1}{2}\pi E^2$.

Density of energy in the magnetic field,

The magnetic field energy is situated $= \frac{=B^2}{2\mu_0}$

The energy density in the electric field at time *t*.

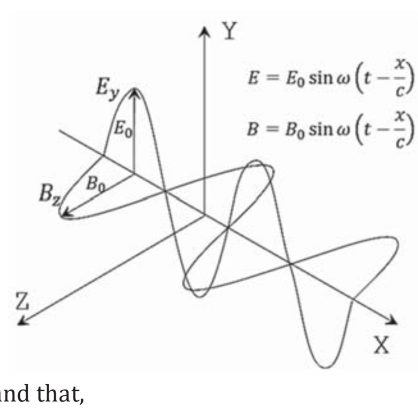
$$\varepsilon \cdot d/e_f = \frac{1}{2}E_0^2 \sin^2\left(\omega t - \frac{\omega x}{c}\right)$$

The mean energy density in the electric field:

$$\varepsilon \cdot d/\varepsilon \cdot f \text{ a werage value } = \frac{1}{2}\varepsilon_0 \frac{\int_0^{2\pi} E_0^2 \sin^2 /\omega t - \frac{\omega t}{c}) dt}{\int_0^{\pi/\omega} dt} = \frac{1}{2}\frac{\varepsilon_0 E_0^2}{2} = \frac{\varepsilon_0 E_0^2}{4}$$

Comparable to the electric field, the average energy density in the magnetic field is:

$$\varepsilon \cdot d|_{m \cdot f} = \frac{1}{2}\frac{\beta^2}{\mu_0} \Rightarrow \varepsilon \cdot dh_f \text{ aurage } = \frac{\beta_O^2}{4\mu_0}$$



At present, we understand that,

$$\frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 t_0}} \Rightarrow \frac{E_0^2}{B_0^2} = \frac{1}{\mu_0 \varepsilon_0} \Rightarrow \varepsilon_0 E_0^2 = \frac{B_0^2}{\mu_0}$$

Derived from the average energy density in the electric field:

$$\frac{\varepsilon_0 E_0^2}{4} = \frac{B_0^2}{4\mu_0}$$

This indicates that the average energy density in both the electric and magnetic fields is equal for an electromagnetic wave.

Energy density in electro-magnetic wave is the sum of energy densities in electric and magnetic field.

$$\mathcal{E} \cdot d_{\text{average in}}\, \varepsilon \cdot H \cdot \omega = \frac{E_0 E_0^2}{4} + \frac{B_0^2}{4\mu_0}$$

$$= 2 \times \frac{\varepsilon_0 E_0^2}{4} = 2 \times \frac{B_0^2}{4\mu_0}$$

$$u_{\text{avg}} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2$$

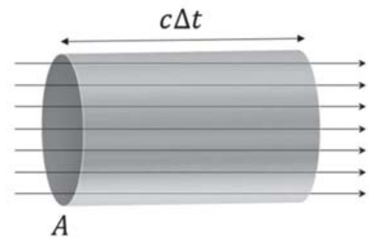
The energy conveyed by an electromagnetic wave is evenly distributed between the electric and magnetic fields.

Intensity:

$$= \frac{\text{Energy}}{\text{Area} \times \text{fime}} = \text{watt}/\text{m}^2$$

The intensity of a wave is defined as the amount of energy that passes through a specified area per unit time, with the area being perpendicular to the direction in which the wave propagates. Specifically, it represents the energy flow per unit time through a unit area perpendicular to the wave's propagation direction.

Alternatively, one can conceptualize intensity as the energy passing through a cylindrical area (denoted as A) within a given timeframe.



Energy $= \frac{1}{2} E_0^2 \times A \times c$

$= \frac{B_0^2}{2\mu_0} \times A \times C$

Intensely $= \frac{\text{Enerag in 1sec}}{\text{Area } (A)}$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \times C = \frac{\varepsilon_0 E_0}{2} B_0 C \times C = \frac{\varepsilon_0 E_0 B_0 C^2}{2} = \frac{E_0 B_0}{2\mu_0} = \frac{B_0^2 C}{2\mu_0}$$

$$I = \frac{U}{A\Delta t} = \frac{1}{2}\varepsilon_0 E_0^2 c = \varepsilon_0 E_{rms}^2 c$$

$$I = \frac{1}{2}\frac{B_0^2}{\mu_0} c = \frac{B_{rms}^2}{\mu_0} c$$

Poynting Vector:

The direction of the cross-product of $\vec{E} \times \vec{B}$ is aligned with the direction of the electromagnetic wave propagation or the direction of energy travel.

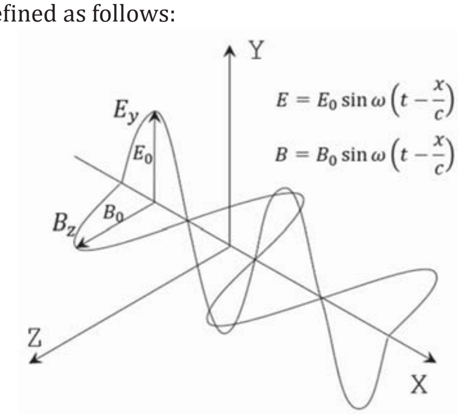
The Poynting vector is a physical quantity that is defined as follows:

$$\vec{P} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$= \frac{E_0 B_0 (\sin^2 /\omega t - \frac{vx}{c})}{\mu_0}$$

auerage $= \frac{1}{2}$

$|\vec{P}|_{\text{awreye}} = \frac{E_0 B_0}{2\mu_0}$



A Poynting vector is a mathematical construct used in physics to describe the directional energy flux density within an electromagnetic field. Essentially, it quantifies the rate at which energy is transferred through a given area within the field.