Chapter 8

Electromagnetic Waves

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INTRODUCTION TO EM WAVE AND FOUR BASIC EQUATIONS OF ELECTRICITY AND MAGNETISM

An Introduction to Electromagnetic Wave

Electromagnetic waves manifest when charges undergo acceleration. Charges that remain stationary or move at a constant velocity do not give rise to these waves. Instead, stationary charges establish an electrostatic field, while steady currents generate magnetic fields that remain constant over time.

Maxwell's theory elucidates that the emission of electromagnetic waves occurs when charges experience acceleration. Though a rigorous demonstration of this concept entails complexity, a qualitative understanding is attainable. Imagine a charge oscillating to and Fro at a particular frequency. Such oscillation necessitates changes in velocity, indicating acceleration. Throughout this oscillatory motion, the charge engenders an oscillating electric field within its surrounding environment.

Consequently, this electric field generates an oscillating magnetic field, perpetuating the cycle. The oscillating magnetic field serves as a source for yet another oscillating electric field, thus perpetuating the sequence as the waves traverse through space.

The frequency of these electromagnetic waves aligns with the frequency of the charge's oscillation. Put simply, if the charge oscillates rapidly, the resulting waves exhibit a high frequency.

Significantly, the energy conveyed by these waves originates from the energy of the source, i.e., the accelerating charge. As the charge undergoes acceleration, it dissipates a portion of its energy, which subsequently transfers to the propagating electromagnetic waves.

In essence, electromagnetic waves emerge when charges undergo acceleration, initiating a cyclical pattern of oscillating electric and magnetic fields that perpetuate each other. The frequency of these waves mirrors the frequency of the charge's oscillation, while the energy of the waves derives from the accelerating charge's energy.

Basic Equations for Electricity and Magnetism

Maxwell's equations elucidate the relationship between the electric and magnetic fields within an electromagnetic wave, demonstrating that they are orthogonal to each other and to the wave's direction of propagation.

This observation resonates with our earlier discourse on displacement current. To illustrate, consider the electric field between the plates of a capacitor, aligned perpendicular to the plates. Due to the presence of displacement current, it induces a magnetic field circling a loop parallel to the capacitor plates. Consequently, in this configuration, the magnetic field (B) and the electric field (E) are perpendicular.

This orthogonality principle extends universally. In the depicted diagram, we showcase a typical instance of a planar electromagnetic wave advancing along the z-axis (where the fields are functions of the z-coordinate at a specific time, t). The electric field (E_x) aligns along the x-axis and undergoes sinusoidal modulation concerning z, at a given instant. Simultaneously, the magnetic field, labeled as B_y, aligns along the y-axis and also manifests sinusoidal variation concerning z. In this scenario, the electric field E_x and the magnetic field B_y stand perpendicular to each other, as well as to the propagation direction along the z-axis.

These fields, denoted as E_x and B_y, can be represented by the following equations:

$$E_x = E_0 \sin(kz - \omega t) \quad \text{(Equation 7(a)}$$

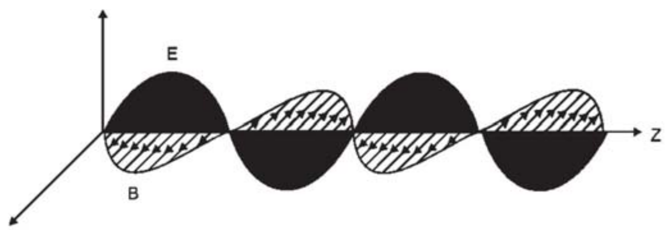
$$B_y = B_0 \sin(kz - \omega t) \quad \text{(Equation 7(b)}$$

Here, the symbol k is related to the wavelength λ of the wave through the standard relationship:

$$K = \frac{2\pi}{\lambda} \qquad \text{(Equation 8)}$$

The symbol ω represents the angular frequency, while k signifies the magnitude of the wave vector, also referred to as the propagation vector. The direction of k indicates the wave's direction of propagation. The speed of propagation of the wave is given by ω/k.

Through an analysis of the expressions for Ex and By provided in equations (a) and (b), along with the utilization of Maxwell's equations, we can derive that:



$$\omega = cK, \text{where}, c = 1/\sqrt{\mu_0 \varepsilon_0} \qquad \text{(Equation 9(a)}$$

The equation ω = cK establishes a fundamental connection for describing waves. This relationship is often alternatively articulated in terms of frequency.
ν (=ω/ 2π) and wavelength. λ (= 2π / k) as

$$2\pi v = c\left(\frac{2\pi}{\lambda}\right)$$

$$\nu\lambda = c \qquad \text{(Equation 9(b)}$$

Maxwell's equations illuminate the interrelation between the magnitudes of electric and magnetic fields within electromagnetic waves, often expressed as $B_0 = \frac{E_0}{c}$. Electromagnetic waves epitomize self-sustaining oscillations of electric and magnetic fields occurring in free space, such as a vacuum. This sets them apart from other wave phenomena previously explored, as they do not necessitate a material medium for their propagation.

For instance, sound waves in air manifest as compressions and rarefactions, constituting longitudinal waves. Conversely, water waves are transverse, traveling horizontally and radially outward. Transverse elastic waves, analogous to sound waves, can also propagate through rigid solids.

In the 19th century, scientists adhered to a mechanical model that posited a hypothetical substance called ether permeating all space and matter, believed to respond to electric and magnetic fields akin to any elastic medium. It was postulated that this ether served as the conduit for the propagation of electromagnetic waves. However, the Michelson-Morley experiment of 1887 conclusively debunked the existence of ether. Presently, we comprehend that electromagnetic waves can perpetuate themselves in a vacuum devoid of any physical medium.

Yet, when a material medium is present, electromagnetic waves, such as light, can traverse through substances like glass.

Previously discussed was how the total electric and magnetic fields within a medium are delineated in terms of permittivity (ε) and magnetic permeability (μ), representing the factors by which the total fields deviate from the external fields. In the context of describing electric and magnetic fields within a material medium, these values replace ε_0 and μ_0 in Maxwell's equations. Consequently, the speed of light within this medium is defined by:

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{...... (10)}$$

The velocity of light is contingent upon the electric and magnetic properties of the medium through which it traverses. When electromagnetic waves travel through free space or a vacuum, the velocity of light represents an immutable constant. Numerous experiments involving electromagnetic waves of varying wavelengths have consistently demonstrated that this velocity remains invariant, irrespective of the wavelength, with only slight deviations of a few meters per second recorded. The established constancy of the velocity of electromagnetic waves in a vacuum finds robust support in experimental evidence, and the precise value of this velocity is widely recognized. Its reliability is such that it serves as a standard reference for defining length.

Hertz made significant contributions not only by corroborating the existence of electromagnetic waves but also by showcasing their capacity to undergo phenomena such as diffraction, refraction, and polarization. This groundbreaking work culminated in the unequivocal conclusion that electromagnetic radiation behaves as waves. Furthermore, Hertz successfully generated stationary electromagnetic waves and determined their wavelength by measuring the distance between successive nodes. Armed with the knowledge of the wave's frequency (which equaled the oscillator's frequency), he computed the wave's speed using the formula $v=\nu\lambda$ and discovered that the waves traveled at the same velocity as light.

Indeed, electromagnetic waves carry both energy and momentum, analogous to other wave forms. In a region of free space where an electric field E exists, an associated energy density emerges, expressed as $\varepsilon_0 E^2/2$. Similarly, a magnetic field B is linked with magnetic energy density $B^2/2\mu_0$. As electromagnetic waves comprise both electric and magnetic fields, they inherently possess a non-zero energy density.

Consider a plane perpendicular to the direction of propagation of the electromagnetic wave. If electric charges reside on this plane, they will experience a force and undergo motion due to the influence of the electric and magnetic fields of the electromagnetic wave. Consequently, these charges acquire energy and momentum from the waves, underscoring that electromagnetic wave, akin to other wave forms, transport both energy and momentum.

Due to the momentum, they carry, electromagnetic waves exert a pressure known as radiation pressure. For an absorptive surface, the total momentum imparted to the surface over a time interval t is given by:

$$P = \frac{U}{c} \qquad \text{...... (11)}$$

This connection illustrates how light, functioning as an electromagnetic wave, transfers energy from the sun to the Earth, a pivotal process essential for sustaining life on our planet.

Example:

Imagine a planar electromagnetic wave traversing through a vacuum. At a specific location and moment in time, the intensity of the electric field (E) measures 6 volts per meter (V/m). Our inquiry pertains to determining the magnitude of the magnetic field (B) at this precise location.

Solution.

Eq the magnitude of B is

$$\mathrm{B} = \frac{\mathrm{E}}{\mathrm{c}}$$

$$= \frac{6.3\,\mathrm{V/m}}{3\times10^{8}\,\mathrm{m/s}} = 2.1\times10^{-8}\,\mathrm{T}$$

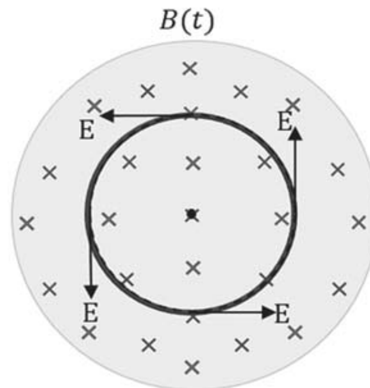
To determine the direction, we observe that the electric field (E) aligns with the y-direction, while the wave propagates along the x-axis. Consequently, the magnetic field (B) must be oriented perpendicular to both the x- and y-axes. Employing vector algebra, we can deduce.

E × B should be along x-direction. Since, $(+\hat{j}) \times (+\hat{K}) = \hat{i}$, B is along the z-direction
Thus, B = 2.1×10^{-8} kT

Faraday's Law for time varying Magnetic field

Faraday's Law describes the phenomenon of electromagnetic induction, specifically concerning the generation of an electromotive force (emf) in a closed circuit due to a time-varying magnetic field. This law is crucial in understanding the interplay between magnetic fields and electric currents. Mathematically, Faraday's Law can be expressed as:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$



The presence of a time-varying magnetic field B(t) gives rise to the generation of an electric field E.

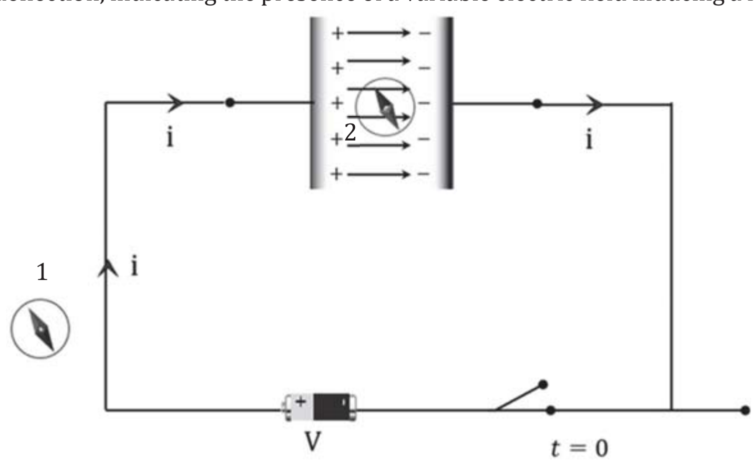
Just as a time-varying magnetic field induces an electromotive force (emf), a time-varying electric field gives rise to the generation of a Magneti field.

The electric field within a capacitor fluctuates correspondingly with the variation in charge on it over time, particularly when it is connected within an alternating current (AC) circuit.

Concept of Displacement Current

The notion of displacement current stands as a pivotal element in the realm of electromagnetism, first elucidated by James Clerk Maxwell in his development of Maxwell's equations. It assumes a foundational role in comprehending the dynamics of electric fields and their interplay with evolving magnetic fields, particularly in scenarios involving fluctuating electric fields.

Consider a scenario where two magnetic needles are positioned within a circuit, as illustrated. As an electric current flows through the conductor, it instigates the generation of a magnetic field, prompting needle 1 to deflect. Interestingly, careful observation reveals that needle 2 also experiences deflection, indicating the presence of a variable electric field inducing a magnetic field.



Ampere's Law as applied to the needle positioned at point 1: $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c$

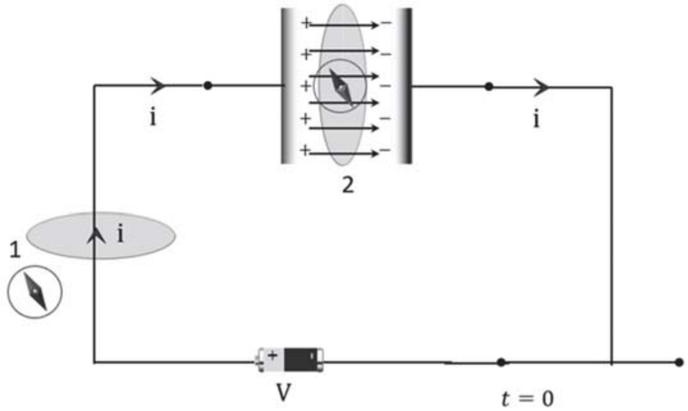
Applicable in fields that do not vary with time.

Ampere's Law applied to the needle situated at point 2:

The presence of a magnetic field occurs even in the absence of any current flow due to the existence of a time-varying electric field.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 i_d$$

Maxwell introduces a modified law to account for the magnetic field arising from a time-varying electric field.



$\phi_E =$ Electric flux

Here, $i_d =$ Displacement current is reliant on the rate at which the electric flux changes.

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

Displacement current is imperceptible but arises as a consequence of a time-varying electric field.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c \oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

The overarching concept proposed by Maxwell is that the origin of a magnetic field extends beyond solely the conduction of electric current; it also encompasses the temporal change of the electric field. Consequently, the overall current (i) comprises both the conduction current (i_c) and the displacement current (i_d), constituting the total current flow in a system.

Beyond the capacitor's confines: $i = i_c, i_d = 0$

Within the capacitor's enclosure: $i = i_d, i_c = 0$

Two mechanisms generate a magnetic field: One arises from the flow of electrons, termed conduction current. The other results from a changing electric field, identified as displacement current.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 (i_c + i_d)$$

Ampere's Law applied to loop 1:

$$B \cdot 2\pi r = \mu_0 i_c$$

$$B = \frac{\mu v i}{2\pi r}$$

The amended version of Ampere's Law pertaining to loop 2:

Here, $l_c = 0 l_d = 1$

$$B \cdot 2\pi r = \mu_0 (\frac{d\varphi_\varepsilon}{dt^-})^{-l'_d}$$

Differentiating, $\frac{d\phi_E}{dt} = \frac{1}{\epsilon_0} \frac{dq}{dt} = \frac{1}{\epsilon_0} i_C$

Based on this examination, displacement current is equivalent to conduction current.

$$B = \frac{\mu_0 i_c}{2\pi\gamma}$$

Maxwell's Equations

Maxwell's equations are a set of four fundamental equations that describe how electric and magnetic fields interact and propagate. They were formulated by James Clerk Maxwell in the 19^{th} century and are central to the theory of classical electromagnetism. The equations are typically written in terms of electric field (E) and magnetic field (B) vectors, electric charge (ρ), and electric current (J).

In the absence of magnetic monopoles, the equations are:

(i.) Gauss's law for electrostatic:

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

This equation relates the electric field to the electric charge density (ρ).

It states that the electric flux through any closed surface is proportional to the charge enclosed by that surface, with ε_0 being the permittivity of free space.

(ii.) Gauss's law for magnetism:

This equation states that there are no magnetic monopoles. The magnetic field lines neither begin nor end—they always form closed loops.

$$\oint \vec{B} \cdot d\vec{S} = 0$$

(iii.) Faraday's law of emf:

This equation describes how a changing magnetic field induces an electric field. It states that the electromotive force (E) around a closed loop is equal to the negative rate of change of the magnetic flux through the loop.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

(iv.) Ampere - Maxwell law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$$

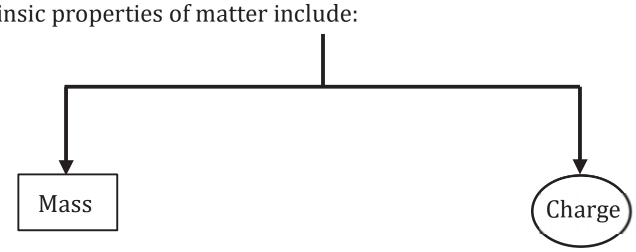
This equation relates magnetic fields to electric currents and the rate of change of the electric field. The first part ($\mu_0 J$) describes how the magnetic field is generated by electric currents, while the second part ($\varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$) describes how a changing electric field can also induce a magnetic field. μ_0 is the permeability of free space.

These four equations together encapsulate the behavior of electric and magnetic fields, and they form the foundation of classical electromagnetism. They have been crucial in understanding various phenomena such as electromagnetic waves, optics, and the behavior of electric circuits.

Intrinsic Properties of Matter

The intrinsic properties of matter are characteristics or qualities that are inherent to a substance and define its fundamental nature. These properties are not dependent on the amount of the substance or its external conditions.

Some of the key intrinsic properties of matter include:



Mass:

Mass is the measure of the amount of matter in an object. It is a fundamental property that remains constant regardless of the object's location or environment.

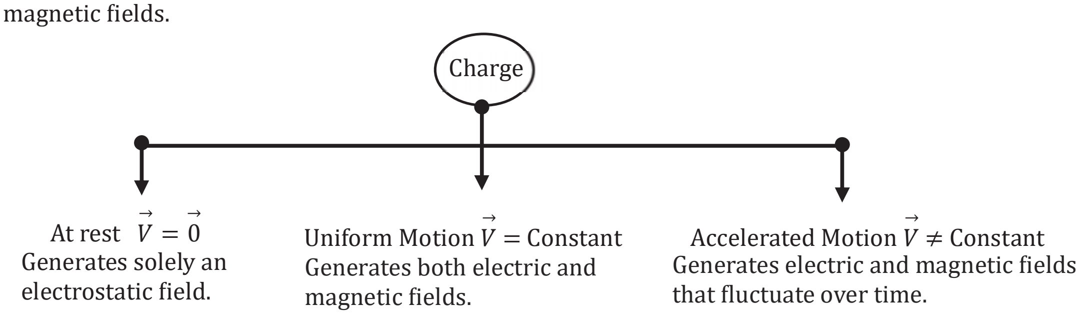
Charge:

Generates and undergoes phenomena associated with both electric and magnetic fields.

These intrinsic properties play a crucial role in determining the behavior and characteristics of different substances and are essential for understanding their physical and chemical properties.

Motion of Charged particle

The motion of a charged particle can be influenced by several factors, including electric and magnetic fields.

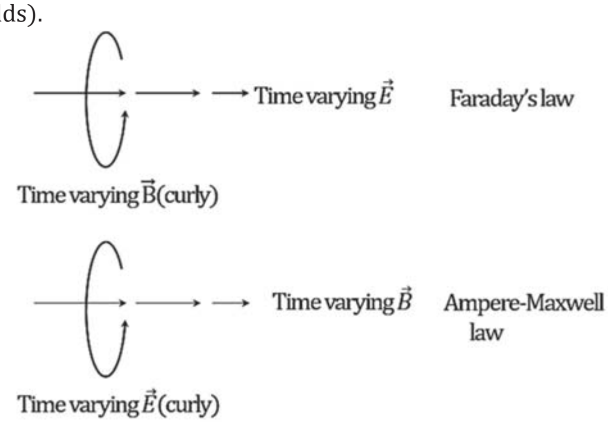


These are some fundamental principles governing the motion of charged particles in electric and magnetic fields. Understanding these concepts is crucial in various fields such as electromagnetism, particle physics, and plasma physics.

Accelerated Motion of Charged Particle

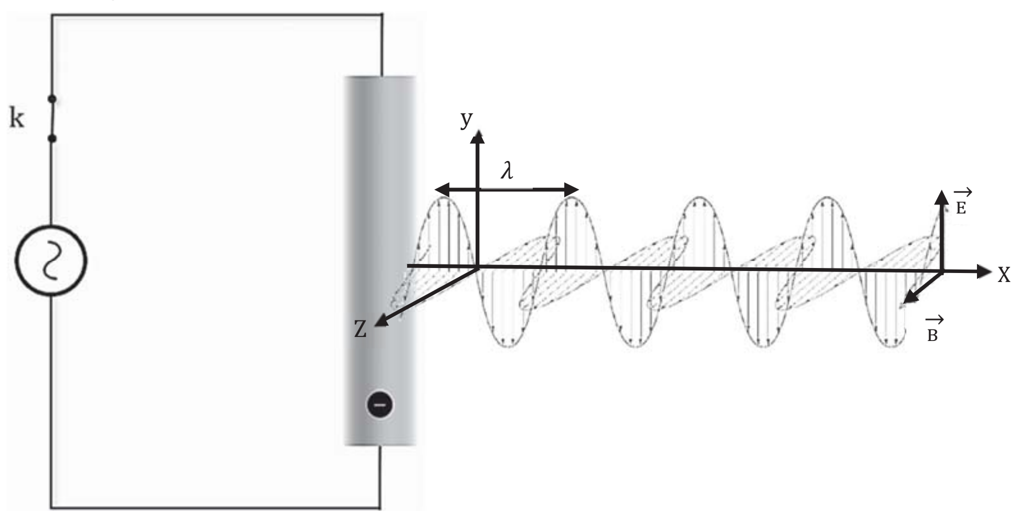
Accelerated motion of a charged particle refers to the movement of the particle in a way that its velocity changes over time. In this context, "accelerated" specifically denotes a change in velocity, whether it involves an increase or decrease in speed or a change in direction.

When a charged particle experiences accelerated motion, it undergoes changes in its kinetic energy and momentum. This acceleration can occur in various scenarios, such as when the particle is subjected to an electric field, a magnetic field, or both simultaneously (as in the case of electromagnetic fields).



For instance, in an electric field, a charged particle will experience a force that accelerates it according to Newton's second law of motion (F = ma), where F is the force exerted on the particle, m is its mass, and a is its acceleration. Similarly, in a magnetic field, a charged particle experiences a force proportional to its velocity and the strength of the magnetic field.

Accelerated motion of charged particles is fundamental in many phenomena and applications, including the operation of electric circuits, particle accelerators, and the generation of electromagnetic radiation.

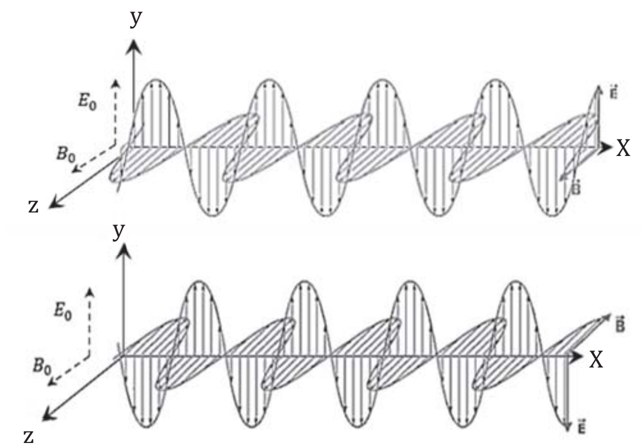


The oscillations of electric and magnetic fields mutually regenerate, initiating the propagation of electromagnetic energy through space, forming what is known as an electromagnetic wave. The energy linked to the advancing wave is obtained at the cost of the energy from the source.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

The equations describing the electric and magnetic fields that fulfill the aforementioned differential equations are as follows:

$$E = E_y = E_0 \sin\ \omega(t - \frac{x}{c})$$
$$B = B_z = B_0 \sin\ \omega(t - \frac{x}{c})$$



In this particular scenario, a wave or energy is moving and spreading along the x-direction.