RESONANCE IN LCR CIRCUITS AND TRANSFORMER

Resonant

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit. There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

Series Resonance

(a) At Resonance

(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

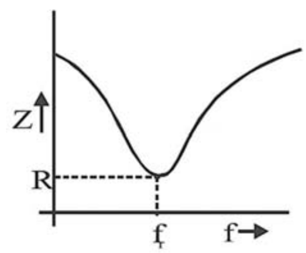
(iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

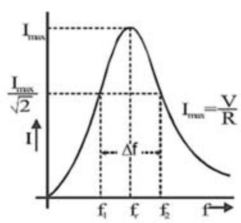
$\because X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$

(c) Variation of Z with f

i. If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)
ii. At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero
iii. If $f > f_r$ then $X_L > X_C$ circuit nature is inductive,ϕ (positive)



(d) Variation of I with f as f increase, Z first decreases then increase



as f increase, I first increase then decreases

When a series resonant circuit is at resonance, its impedance becomes the lowest, making it an "acceptor circuit." This means it easily takes in the current that matches its own natural frequency, out of various currents. In radio or TV tuning, we tune the circuit to the same frequency as the station we want to receive, just like how the acceptor circuit readily accepts the right current at its natural frequency.

Band width $= \Delta f = f_2 - f_1$

Quality factor Q : Q-factor of AC circuit basically gives an idea about stored energy & lost energy

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

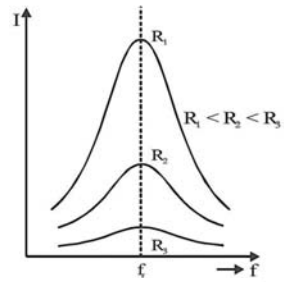
(i) It represents the sharpness of resonance.

(ii) It is unit less and dimension less quantity

(iii) $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{\text{band width}}$

Magnification

At resonance V_L or $V_C = QE$ (where E = supplied voltage)
So at resonance Magnification factor = Q-factor



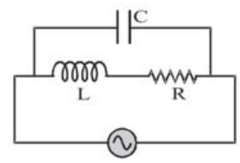
Sharpness of Resonance

Sharpness ∝ Quality factor ∝ Magnification factor R decrease
⇒ Q increases ⇒ Sharpness increases

Parallel Resonance

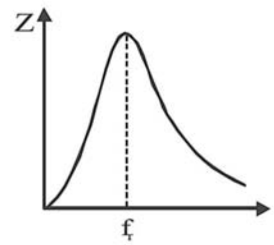
(a) **At resonance**

(i) $S_L = S_C$ (ii) $I_L = I_C$ (iii) $\phi = 0$
(iv) $Z_{max} = R$ (impedance maximum)
(v) $I_{min} = \frac{V}{R}$ (current minimum)



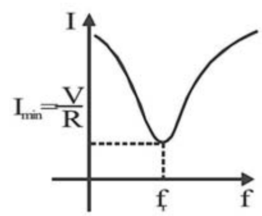
(b) **Resonant frequency** fr = $= \frac{1}{2\pi\sqrt{LC}}$

(c) **Variation of Z with f** as f increases , Z first increases then decreases



- If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive
- If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.

(d) **Variation of I with f as f increases** , I first decreases then increases



Note :

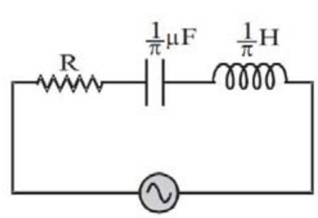
For this circuit $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \Rightarrow Z_{max} = \frac{L}{RC}$ For resonance $\frac{1}{LC} > \frac{R^2}{L^2}$

- Series resonance circuit gives voltage amplificaltion while parallel resonance circuit gives current amplification.
- At resonance current does not depend on L and C, it depends only on R and V.
- At half power frequencies : net reactance = net resistance.
- As R increases ,bandwidth increases
- To obtain resonance in a circuit following parameter can be altered :
- (i) L (ii) C (iii) frequency of source.
- Two series LCR circuit of same resonance frequency f are joined in series then resonance frequency of series combination is also f
- The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.
- Unit of $\sqrt{LC}$ is second

Ex. For what frequency the voltage across the resistance R will be maximum.

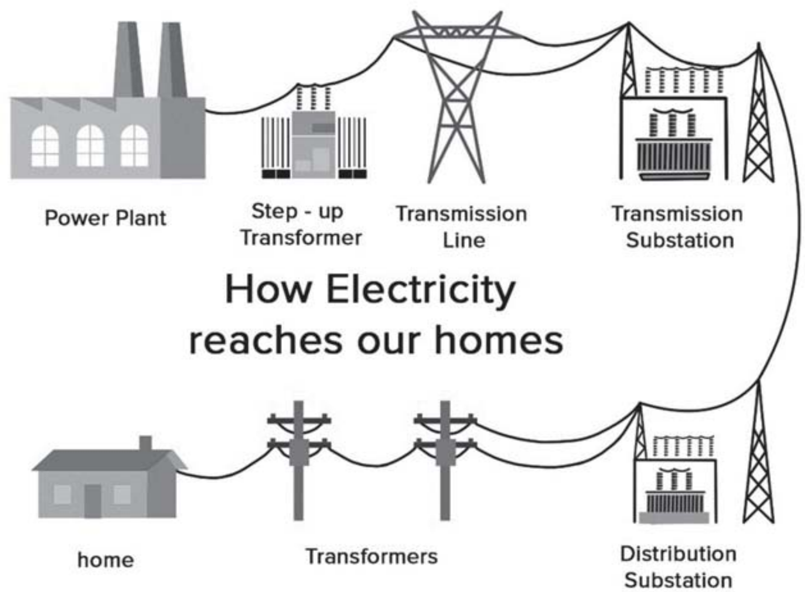
Sol.: It happens at resonance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times 10^{-6}\times\frac{1}{\pi}}} = 500\text{Hz}$$



How does electricity reach our house?

- Electricity is created at a power station using large generators. These stations can use different sources like sunlight, wind, coal, natural gas, or water.
- The electric current is sent through transformers to increase its voltage for efficient long-distance travel.
- The electric charge moves through high-voltage transmission lines that span across the country.
- When it reaches a substation, the voltage is reduced so that it can be sent through smaller power lines.
- The electricity then travels through distribution lines to your neighborhood. Smaller transformers further decrease the voltage to make it safe for use in our homes, often found on poles or as big green boxes on the ground (called pad mount transformers).
- It connects to your house and passes through a meter that measures the amount of electricity your family uses.
- The electricity reaches the service panel in your basement or garage, where breakers or fuses protect the wires in your house from overloading. (Safety tip: Only let your parents or a professional operate the service panel, and never touch it yourself!)
- Finally, the electricity moves through wires inside the walls to reach outlets and switches throughout your house.



Basis Discussion of Transformer- Working, Power Relation, Types, Efficiency, Major Power Losses

Working principle

Mutual induction

A transformer consists of two main parts:

(a) **Shell:** This section includes primary and secondary coils made of copper. The effective resistance between these coils is infinite because the electric circuit between them is open ($R_{ps} = \infty$).

(b) **Core:** Located between the two coils, it magnetically connects them. Both coils are wound around the same core. As alternating current flows through the primary coil, it creates a changing magnetic field in the core, inducing alternating voltage in the secondary coil.

Work:

The transformer regulates the voltage of alternating current (AC) and transfers electrical power without changing the frequency of the input supply, which naturally fluctuates.

Special Points:

- A transformer doesn't work with a direct current (DC) supply. If a battery is connected to its primary, there is no output across the secondary (output is always zero).
- It's not referred to as an 'Amplifier' because it lacks power gain like a transistor.
- Since transformers have no moving parts, there are no mechanical losses in the device.

Types : According to voltage regulation it has two –

(i) Step up transformer : $N_S > N_P$ (ii) Step down transformer $N_S < N_P$

Step up transformer : Converts low voltage high current in to High voltage low current

Step down transformer : Converts High voltage low current into low voltage high current.

When we transmit power, we always use "high voltage, low current." This way, we minimize voltage drop and reduce power losses in the transmission line.

voltage drop = I_L , R_L , I_L = line current R_L = total line resistance,

$$I_L = \frac{\text{power to be transmission}}{\text{line voltage}} \quad \text{power losses } = I_L^2 R_L$$

High voltage coil having more number of turns and always made of thin wire and high current coil having less number of turns and always made of thick wires.

Ideal Transformer : (η = 100%)

(a) **No flux leakage**

$$\phi_s = \phi_p \Rightarrow \frac{-d\phi_s}{dt} = \frac{-d\phi_p}{dt}$$

$e_s = e_p$ = e induced emf per turn of each coil is also same.

total induced emf for secondary $E_s = N_s\, e$

total induced emf for primary $E_p = N_p\, e$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = n \text{ or } p$$

where n : turn ratio , p : transformation ratio

(b) **No load condition**

$$V_p = E_p \text{ and } E_S = V_S \frac{V_S}{V_p} = \frac{N_S}{N_p} \text{ from (i) and (ii) } \frac{V_S}{V_p} = \frac{N_S}{N_p} = n \text{ or } p$$

(c) **No power loss**

$$P_{out} = P_{in} \text{ and } V_s I_s = V_p I_p \frac{V_S}{V_p} = \frac{I_p}{I_S} \text{ valid only for ideal transformer}$$

$$\text{from equation (iii) and (iv) } \frac{V_S}{V_p} = \frac{I_p}{I_S} = \frac{N_S}{N_p} = n \text{ or } p$$

Note

Generally transformers deals in ideal condition i.e. $P_{in} = P_{out}$, if other information are not given.

Real transformer (η ≠ 100%)

Some power is always lost due to flux leakage, hysteresis, eddy currents, and heating of coils

hence $P_{out} < P_{in}$ always. efficiency of transformer $\eta = \frac{P_{out}}{P_{in}} = \frac{V_S}{V_p} \cdot \frac{I_S}{I_p} \times 100$