LR, RC CIRCUITS, POWER AND POWER FACTOR

Impedance

Impedance is like a team of obstacles in an electrical circuit, made up of resistance and reactance. Imagine it as everything that tries to slow down the flow of tiny electrons in the circuit. This teamwork affects how electrical current is produced. You can find impedance in all parts of the circuit and in any electrical setup. We use the letter Z to talk about impedance in math, and it's measured in ohms. Think of it as a big group that includes both resistance and reactance.

In phasor terms, we show impedance (Z) as a mix of resistance (R) and reactance (X), kind of like combining two friends to make a strong team.

$$Z = R + jX$$

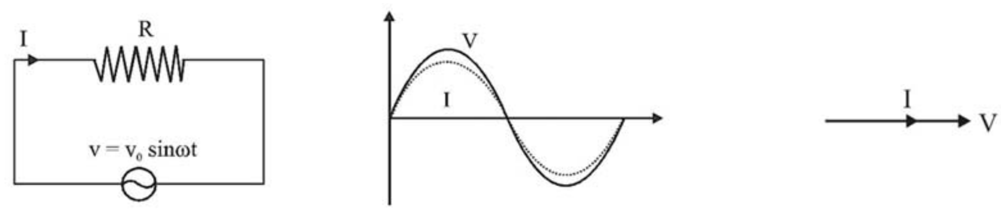
Where reactance X is a combination of Inductive X_1 and capacitive X_c

$$X = X_l + X_C$$

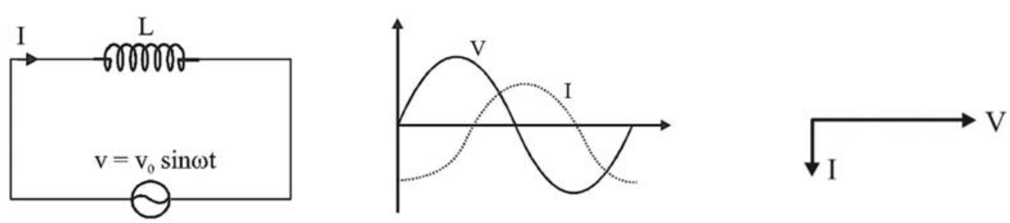
Circuit	$Z = \frac{\varepsilon_0}{i_0}$
Purely resistive	R
Purely capacitive	$\frac{1}{\omega C}$
Purely inductive	ωL

$$Z = \sqrt{R^2 + X^2} \text{ where } X = \text{reactance}$$

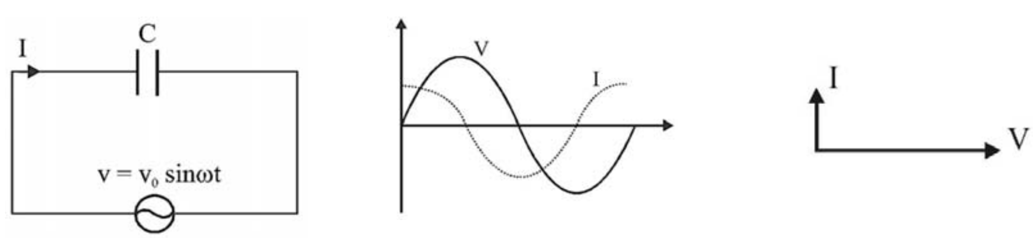
AC Circuit - Combinations



$$I = \frac{V_0}{R} \sin \omega t$$

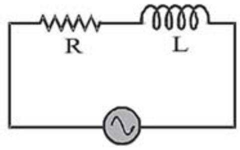
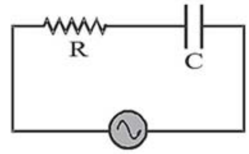
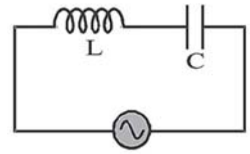
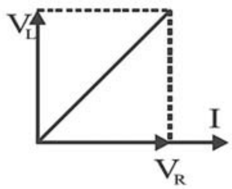
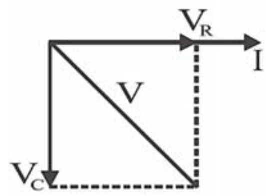
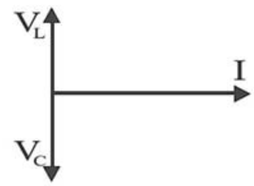
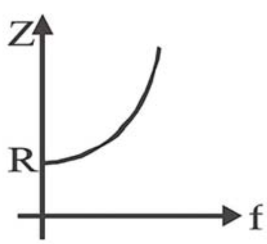
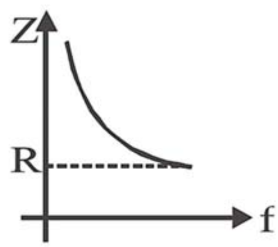
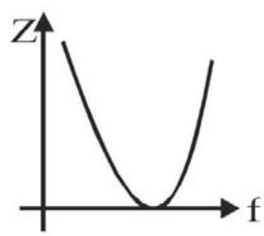


$$I = \frac{V_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$



$$I = V_0 \omega C \sin\left(\omega t + \frac{\pi}{2}\right)$$

LR- Circuit, RC – Circuit and LC Circuit

Term	R-L	R-C	L-C
Circuit	 I is same in R & L	 I is same in R & C	 I is same in L & C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C(V_L > V_C)$ $V = V_C - V_L(V_C > V_L)$
Phase difference in between V & I	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I $\left(\phi = -\frac{\pi}{2} \text{ to } 0\right)$ V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$) Impedance $Z = \sqrt{R^2 + X_L^2}$
$Z = \sqrt{R^2 + (X_C)^2}$ Variation of Z with f	$Z = \|X_L - X_c\|$ as f↑,Z ↑ 	as f↑,Z 	as f↑, Z first ↓ then↑ 
At very low f At very high f	$Z \simeq R(X_L \to 0)$ $Z \simeq X_L$	$Z \simeq X_C$ $Z \simeq R(X_C \to 0)$	$Z \simeq X_c$ $Z \simeq X_L$

Power and power factor
Power in AC Circuit

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi) = V_0 I_0 \sin\omega t(\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

Average power $<P>= \frac{1}{T}\int_0^T (V_0 I_0 \sin^2 t\,\omega t \cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0 \times \sin\phi\right]$$

$$\Rightarrow < P >= \frac{V_0 I_0 \cos\phi}{2} = V_{rms} I_{rms} \cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = VI\cos\phi$	$P = VI$	$P = V_\circ I_\circ$

- $I_{rms}\cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.

- I_{rms} $\sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\vec{p} = E_{rmc}I_{rmc}\cos\phi$ = rms power $\times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{rms Power}}$ and $\cos\phi = \dfrac{R}{Z}$

Power factor : (i)is leading if I leads V

(ii) is lagging if I lags V

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1

Pure/Ideal	ϕ	V	Power factor = $\cos\phi$	Average power
R	0	V, I same Phase	1 (maximum)	$V_{rms} \cdot I_{rms}$
L	$+\frac{\pi}{2}$	V leads I	0	0
C	$-\frac{\pi}{2}$	V lags I	0	0
Choke coil	$+\frac{\pi}{2}$	V leads I	0	0

- At resonance power factor is maximum($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}I_{rms}$

Ex. A voltage of 10 V and frequency 103 Hz is applied to $\frac{1}{\pi}$ μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Sol.: $\because X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times10^3\times\frac{10^{-6}}{\pi}} = 500\Omega \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\Omega$

Power factor $\cos\phi = \dfrac{R}{Z} = \dfrac{500}{500\sqrt{2}} = \dfrac{1}{\sqrt{2}}$, Power dissipated $= V_{ms}\, I_{ms}\, \cos\phi = \dfrac{V_{ms}^2}{Z}\cos\phi = \dfrac{(10)^2}{500\sqrt{2}} \times \dfrac{1}{\sqrt{2}}$

$= \frac{1}{10}$ W