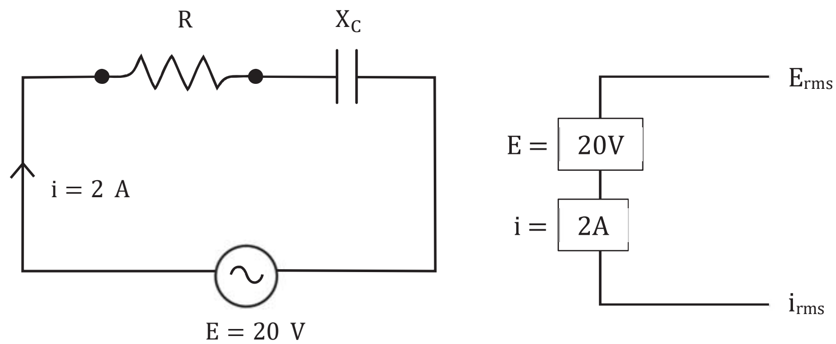
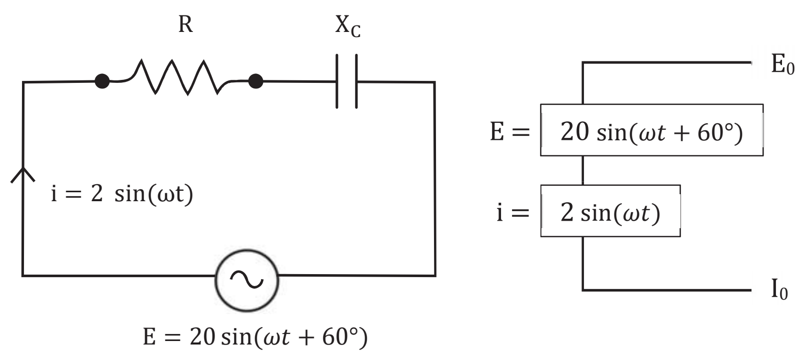
LCR CIRCUIT

Basic terminology in a circuit

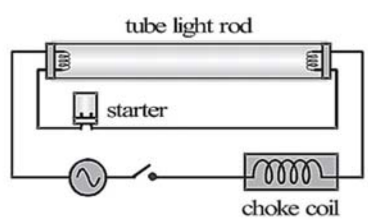




- There is 2 A Current in A.C circuit
  ⇒ 2 A is r.m.s value
- 20 V Supply in A-circuit
  ⇒ 20 V is r.m.s value.

Choke Coil

In a circuit with direct current (DC), we use resistance to lower the flow of current, causing a loss of electrical energy in the form of heat (I^2 R) through the resistance. However, in an alternating current (AC) circuit, we can decrease the current using a choke coil, which results in minimal energy loss. A choke coil is a copper coil wrapped around a soft iron laminated core. Placing this coil in series with the circuit helps control the current and is also called a ballast.



Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)
The current in the circuit

$$I = \frac{E}{Z} \text{ with } Z = \sqrt{(R + r)^2 + (\omega L)^2}$$

So due to large inductance L of the coil, the current in the circuit is decreased appreciably.
However, due to small resistance of the coil r,
The power loss in the choke

$$P_{av} = V_{rms} I_{rms} \cos\phi \to 0$$

$$\because \cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$$

- Choke coil is a high inductance and negligible resistance coil.
- Choke coil is used to control current in A.C. circuit at negligible power loss
- Choke coil used only in A.C. and not in D.C. circuit
- Choke coil is based on the principle of wattless current.
- Iron cored choke coil is used generally at low frequency and air cored at high frequency.
- Resistance of ideal choke coil is zero

Ex. A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil

Sol.: $V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120V$

Ex. An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

Sol.: Resistance of lamp $R = \frac{V}{I} = \frac{80}{10} = 8\Omega$

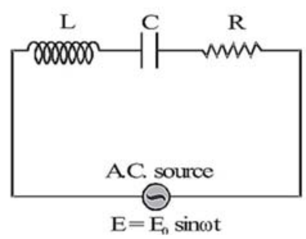
Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then.

$$Z = \frac{V}{I} = \frac{100}{10} = 10\Omega \text{ but } Z = \sqrt{R^2 + (\omega L)^2}$$

$$\Rightarrow (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02H$$

LCR Circuit

In a circuit shown in the diagram, there's a lineup of a resistance R, an inductor coil with inductance L, and a capacitor with capacitance C. All of these are connected to a source of alternating power with a peak value of E_0.

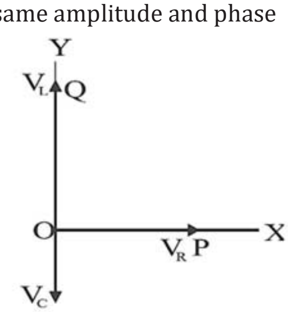


Phasor Diagram For Series L-C-R circuit

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$. As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase

However voltage across each element bears a different phase relationship with the current.

Let at any instant of time t the current in the circuit is I Let at this time t the potential differences across L, C, and R

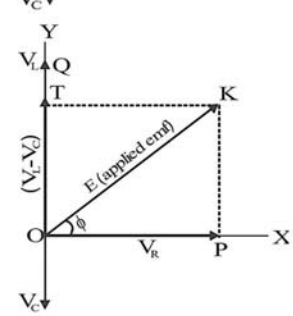


$$V_L = I\,X_L, V_C = IX_C \text{ and } V_R = IR$$

Now, V_R is in phase with current I but VL leads I by 90°

While V_C legs behind I by 90°.

The vector OP represents V_R (which is in phase with I) the vector



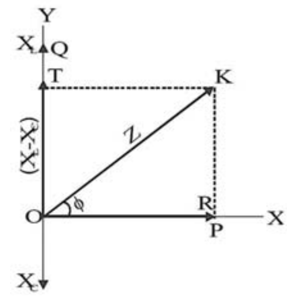
OQ represent V_L (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°)

V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be (V_L – V_C) which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.



Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

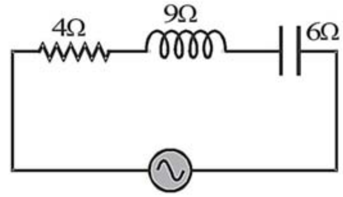
Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ $\tan\phi = \frac{X_L - X_C}{R}$

Ex. Find out the impedance of given circuit.

Sol.: $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (9-6)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5\Omega$

$(\because X_L > X_c \therefore$ Inductive)



Ex. In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

Sol.: $$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + [\omega L - \frac{1}{\omega C}]^2}} = \frac{50}{\sqrt{300^2 + [2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}}]^2}}$$

$$\Rightarrow I_{max} = \frac{50}{\sqrt{(300)^2 + [100 - \frac{10^3}{2}]^2}} = \frac{50}{100\sqrt{9+16}} = \frac{1}{10} = 0.1\text{ A}$$