AC CIRCUITS

Root Mean Square Value

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} \quad \text{rms value} = \text{Virtual value} = \text{Apparent value}$$

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t dt}$$

$$= I_0\sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

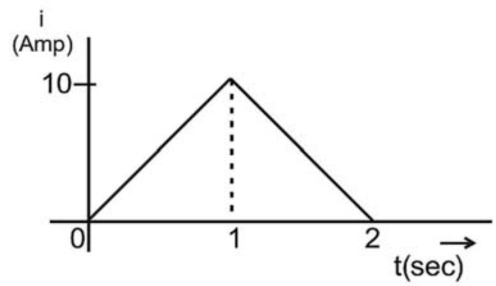
If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

For above varieties of current rms $= \frac{\text{Peak value}}{\sqrt{2}}$

Root Mean Square Value of a function, from t_1 to t_2 , is defined as

$$f_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$$

Ex. Find the average value of current shown graphically, from t = 0 to t = 2 sec.



Sol. From the i – t graph, area from t = 0 to t = 2 sec

$= \frac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

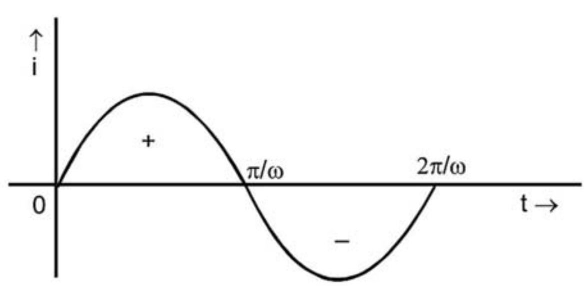
$\therefore$ Average Current $= \frac{10}{2} = 5$ Amp.

Ex. Find the average value of current from t = 0 to $t = \frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

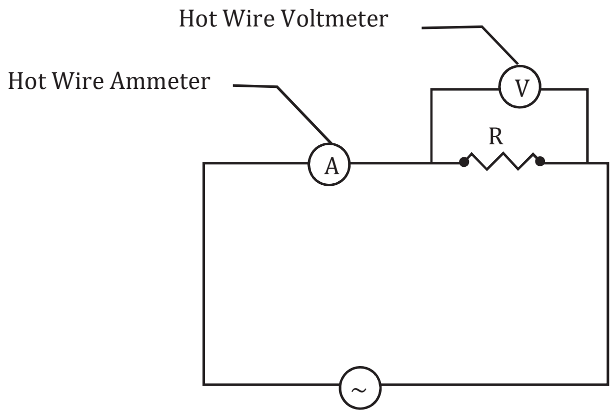
Sol. $<i> = \frac{\int_0^{\frac{2\pi}{\omega}} I_m \sin \omega t dt}{\frac{2\pi}{\omega}} = \frac{\frac{I_m}{\omega}(1-\cos \omega \frac{2\pi}{\omega})}{\frac{2\pi}{\omega}} = 0$

It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore$ $< i >$ in one cycle = 0.



Hot wire Ammeter and voltmeter



- Moving coil galvanometers, ammeters, and voltmeters are designed to measure direct current (DC). They display a zero reading for alternating current (AC) because of their inertia.
- Hot wire ammeters and hot wire voltmeters, on the other hand, are specifically used for measuring AC supply.

Ac Circuits

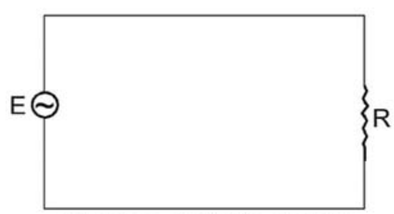
In AC circuits, we have three basic elements: resistors, indicators, and capacitors. Let's explore how each of them behaves when connected in AC circuits.

Purely Resistive circuit

In a circuit diagram, there's a resistor linked to an alternating current (AC) source labeled as ε. The AC source is represented by the symbol "~" on the diagram. To keep things simple, let's focus on a source that generates a smoothly changing voltage across its terminals. We'll refer to this voltage as AC voltage, and it can be described as follows:

$$V = V_0 \sin \omega t \qquad \text{……(i)}$$

where V_0 is the amplitude of the sinusoidal voltage and ω is its angular frequency.



AC voltage applied to a resistor

The instantaneous potential drop across the resistor R is

$$V_0 \sin \omega t = IR$$

Or

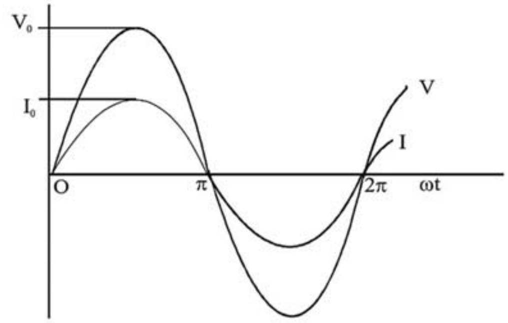
$$I = \frac{V_0}{R} \sin \omega t$$

$$I = I_0 \sin \omega t \qquad \text{…….(ii)}$$

where I is the instantaneous current and the current amplitude I_0 is given by

$$I_0 = \frac{V_0}{R} \qquad \text{……(iii)}$$

Equation (iii) is basically Ohm's law, and it works just as effectively for both AC and DC voltages when it comes to resistors. The voltage across a pure resistor and the current flowing through it, as described by equations (i) and (ii), are displayed over time in the accompanying figure. It's important to observe that both the voltage (V) and current (I) hit zero, reach minimum, and reach maximum values simultaneously. This clearly indicates that in a circuit with only resistance, the voltage and current are synchronized or "in phase."



We observe that, similar to the applied voltage, the current changes in a smooth, wave-like pattern, showing positive and negative values during each cycle. This means that when we add up all the instantaneous current values over one complete cycle, the total is zero, resulting in an average current of zero. It's important to note, however, that even though the average current is zero, it doesn't imply zero average power or no dissipation of electrical energy. As you may already know, Joule heating is determined by the formula I^2R and relies on I^2 (which is always positive, regardless of whether I is positive or negative), not just I.

Hence, there is Joule heating and dissipation of electrical energy when an AC current flows through a resistor.

The instantaneous power dissipated in the resistor is

$$P = I^2R = I_0{}^2R\sin^2 \omega t \qquad \text{.......(iv)}$$

The average value of Power P over a cycle is

$$\vec{P} = \frac{1}{2}I_0^2R = I_{rms}^2R \qquad \text{........(v)}$$

Where the bar over a letter (here, P) denotes its average value.
To express ac power in the same fprm as dc power ($P=I^2 R$), as special value of current is used. It is called, root mean square (rms) or effective current and is denoted by I_{rms}. similarly, we define the rms voltage or effective voltage From equation (iii), we have

Or $\quad V_0 = I_0R \qquad$ .....(vi)

Or $\quad \frac{V_0}{\sqrt{2}} = \frac{I_0}{\sqrt{2}}R \qquad$ .....(vii)

Or $\quad V_{rms} = I_{rms}R \qquad$ .....(viii)

When we talk about root mean square (rms) values, the formulas for power and the connection between current and voltage in AC circuits are essentially identical to those used in DC circuits. To put it simply, the I_{rms} or rms current is like the equivalent DC current that would result in the same average power loss as the alternating current. Equation (v) can be expressed in another way as well.

$$\vec{P} = \frac{V_{rms}^2}{R} = I_{rms}^2R \text{ (since } V_{rms} = I_{rms}R) \qquad \text{......(ix)}$$

Ex. A bulb is rated 60 W at 220 V/30 Hz. Find the maximum value of instantaneous current through the filament ?

Sol. $V_{max} = 220\sqrt{2} = 311 \text{ V}$

$$R = \frac{220^2}{P} = \frac{220 \times 220}{60} = \frac{2420}{2} = 806.67\Omega$$

$$I = \frac{V_{max}}{R} = \frac{311}{806.67} = 0.39 \text{ A}$$

Ex. A light bulb is rated at 200 W for a 220 V supply. Find
(a)The resistance of the bulb
(b) The peak voltage of the source; and
(c) The rms current through the bulb.

Sol. (a) We are given $P = 100\ \text{W}$ and $V = 220\ \text{V}$. The resistance of the bulb is

$$R = \frac{V_{ms}^2}{P} = \frac{(220\ V)^2}{200\ W} = 242\Omega$$

(b) The peak voltage of the source is

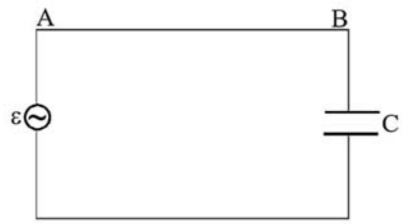
$$V_m = \sqrt{2}V_{max} = 311\ \ V$$

(c) Since, $P = I_{ms}\, V_{rms}$

$$I_{rms} = \frac{P}{V_{mps}} = \frac{200\ W}{220\ V} = 0.90\ \ A$$

Purely capacitive circuit

An ac source ε connected to a capacitor only, a purely capacitive ac circuit is as shown.



An AC source connected to a capacitor

When the capacitor is connected to an ac source, as in figure, it limits or regulates the current, but does not completely prevent the flow of charge. The capacitor is alternately charged and discharged as the current reverses each half cycle. Let 1(t) be the charge on the capacitor at any time t. The instantaneous voltage V(t) across the capacitor is

$$V(t) = \frac{q(t)}{C} \qquad \text{......(xv)}$$

To find the current, we use the relation $I = \frac{dq}{dt}$

$$I = \frac{d}{dt}(V_0 C\sin \omega t) = \omega C V_0 \cos(\omega t)$$

Using the relation, $\cos(\omega t)\ \sin\left(\omega t + \frac{\pi}{2}\right)$, we have

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{......(xvi)}$$

where the amplitude of the oscillating current is

$$I_0 = \frac{V_0}{(I/\omega C\,)}$$

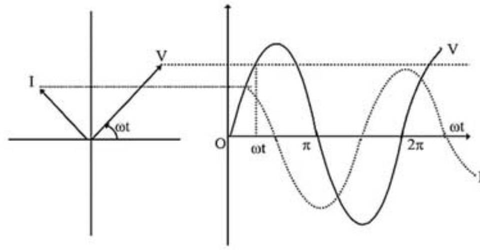
Comparing it to $I_0 = \frac{V_0}{R}$ for a purely resistive circuitmm we find that $(I/\omega C)$ plays the role of resistance. It is called capacitive reactance and is denoted by X_c

$$X = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

So that the amplitude of the current is

$$I_0 = \frac{V_0}{X} \qquad \text{... ... (xviii)}$$

The dimension of capacitive reactance is the same as that of resistance and its SI unit is Ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as does the resistance in a purely resistive circuit. But it is inversely proportional to the frequency and the capacitance.



AS comparison of equation (xvii) with the equation of source voltage equation (i) shows that the current in a capacitor leads the voltage by $\frac{\pi}{2}$. Figure shows the phasor diagram at an instant t. here the current phasor I is $\frac{\pi}{2}$ rad ahead of the voltage phasor V as they rotate counter clockwise. Figure shows the variation of voltage and current with time. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period. The instantaneous power supplied to the capacitor is

$$P_c = IV = I_0\cos(\omega t).V_0\sin(\omega t)$$
$$= I_0 \ V_0\cos(\omega t)\sin(\omega t)$$
$$= \frac{I_0V_0}{2}\sin(2\omega t) \qquad \text{... ... (xiv)}$$

So, as in the case of an inductor, the average power Since average of sin 2ωt over a complete cycle is zero. As discussed in the case of an inductor, the energy stored by a capacitor in each quarter period is returned to the source in the next quarter period.
Thus, we see that in the case of an inductor. The current lags the voltage by 90° and in the case of a capacitor, the current leads the voltage by 90°.

Ex. 30.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive resistance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current.

Sol. The capacitive reactance is

$$X_c = \frac{1}{2\pi fC} = 106\Omega$$

The rms current is

$$i_{rms} = \frac{V_{rms}}{X_c} = 2.08 \ A$$

The peak current is

$$I_0 = \sqrt{2}I = 2.96 \ A$$

This current oscillates between 2.96A and –2.96A and is ahead of the voltage by 90°. If the frequency is doubled, the capacitive reactance is halved and consequently, the current is doubled.

Purely Inductive circuit

In the circuit diagram, there's an AC source connected to an inductor. Typically, inductors have some resistance in their coils, but for simplicity, let's imagine this inductor as an ideal one with zero resistance. So, the circuit we're dealing with is purely an inductive AC circuit.
Let the voltage across the source be $V = V_0\sin \omega t$. Using the loop equation $\sum\varepsilon(t) = 0$, and since there is no resistor in the circuit

$$V - L\frac{dI}{dt} = 0 \qquad \text{......(x)}$$

where the second term is the self-induced emf in the inductor, and L is the self-inductance of the coil. Combining equation (i) and (x), we have

An AC source connected to an inductor

$$\frac{dI}{dt} = \frac{V}{L} = \frac{V_0}{L}\sin \omega t$$

......(xi)

$$dt = \frac{V_0}{L}\sin\omega t dt \qquad \Rightarrow I = -\frac{V_0}{\omega L}\cos(\omega t)$$

Using $-\cos(\omega t) = \sin(\omega t - \frac{\pi}{2})$, we have

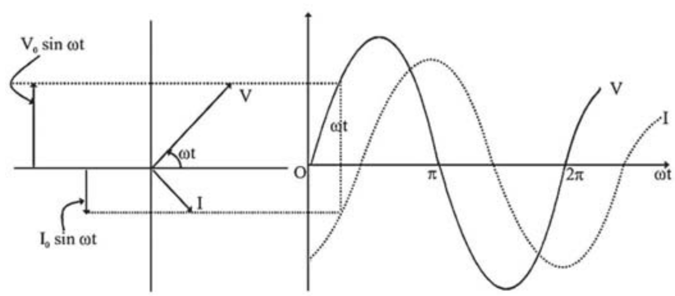
$$i = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad ......(xii)$$

where $I = -\frac{V_0}{\omega L}$ is the amplitude of the current. The quantity ωL is analogous to the resistance and is called inductive reactance, denoted by XL :

$$X_I = \omega L = 2\pi f L \qquad .....(xiii)$$

The dimension of inductive reactance is the same as that of resistance and SI unit is ohm(Ω). The inductive reactance limits the current in a pure inductive circuit in the same way as does the resistance in a pure resistive circuit. The inductive resistance is the directly proportional to the inductance frequency of the voltage source.

A comparisons of equation (i) and (ii) for the source voltage and the current in an inductor shows that the current lags the voltage by $\frac{\pi}{2}$ or one-quarter $\left(\frac{1}{4}\right)$ cycle. Figure a shows the voltage and the current phasors in the present case at instant t. The current phasor is 1 is $\frac{\pi}{2}$ behind the voltage phasor V. When rotated with frequency ω counter clockwise, they generate the voltage and current given by equation (1) and (xii), respectively and as shown in figure (b).



(a) A phasor diagram for the circuit in figure
(b) Graph of V and I versus ωt

We see that current reaches its maximum value later than the voltage by one-fourth of a period $\left[\frac{T}{4} = \frac{\frac{\pi}{2}}{\omega}\right]$.

You have seen that an inductor has reactance that limits current similar to resistance in a dc circuit. Does it also consume power like a resistance ? Let us try to find out. The instantaneous power supplied to the inductor is

$$P_L = IV = I_m \sin\left(\omega t - \frac{\pi}{2}\right) V_0 \sin(\omega t) = -I_0 V_0 \cos(\omega t)\cdot\sin(\omega t)$$

$$= -\frac{I_0 V_0}{2}\sin(2\omega t) \qquad ......\ (xiv)$$

So, the average power over a complete cycle is zero since the average of sin(2ωt) over a complete cycle is zero.

Thus, the average power supplied to an inductor over one complete cycle is zero.

Physically, this result means the follows, During the first quarter of each current cycle, the flux through the inductor builds up and sets uo a magnetic field and energy is stored in the inductor. In the next quarter of cycle, as the current decrease, the flux decreases and the stored energy is returned to the source. Thus, in each half cycle, the energy which is withdrawn from the source is returned to it without any dissipation of power.