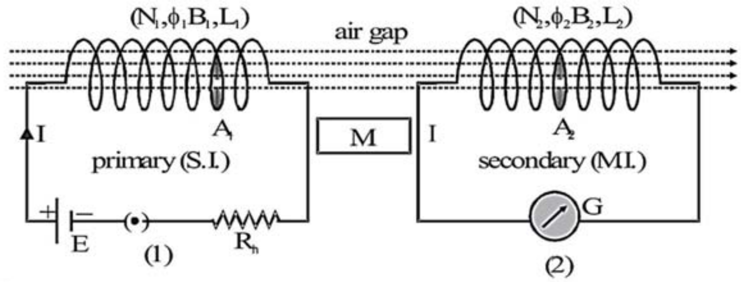
MUTUAL INDUCTION AND COMBINATION OF INDUCTORS

Mutual Induction

When the current flowing through the primary coil or circuit changes, it causes a change in the magnetic flux around the secondary coil or circuit. According to Lenz, to resist this flux change, an electromotive force (emf) is induced in the nearby coil or circuit. This occurrence is known as 'Mutual induction.' In mutual inductance, when two coils are positioned close to each other, the flux linked with the secondary coil changes due to the current in the primary coil.



Due to Air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1 A_1 (\theta = 0°)$.

Case - I When current through primary is constant Total flux of secondary is directly proportional to current flow through the primary coil

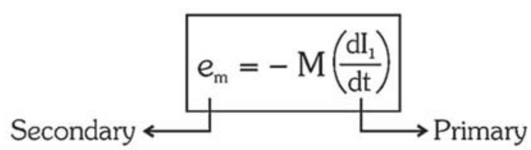
$$N_2\phi_2 \propto I_1 \Rightarrow N_2\phi_2 = MI_1, M = \frac{N_2\phi_2}{I_1} = \frac{N_2\ B_1\ A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$$

where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt} \rightarrow \frac{dB_1}{dt} \rightarrow \frac{d\phi_1}{dt} \rightarrow \frac{d\phi_2}{dt} \Rightarrow$ Static EMI

$$N_2\phi_2 = MI_2 - N_2\frac{d\phi_2}{dt} = -M\frac{dI}{dt}, \left[-N_2\frac{d\phi}{dt}\right]$$



Called total mutual induced emf of secondary coil emf.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

'M' depends on :

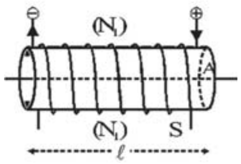
- Number of turns (N_1, N_2).
- Area of cross section.
- Distance between two coils (As d↓ = M↑).
- Coupling factor 'K' between two coils.
- Coefficient of self-inductance (L_1, L_2).
- Magnetic permeability of medium (μr).
- Orientation between two coils.

Coefficient of Mutual Inductance

- In terms of their no of turns
- In terms of their nos of turns (N_1 , N_2)
- In terms of their coefficient of self-inductances

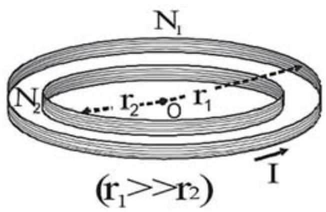
Two co–axial solenoids : (MS_1S_2)

Coefficient of mutual inductance between two solenoids



$$M_{s_1s_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{\ell}\right]A \Rightarrow M_{s_1s_2} = \left[\frac{\mu_0N_1N_2A}{\ell}\right]$$

Two plane concentric coils : (MC_1C_2)

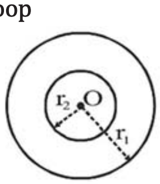


$$M_{c_1c_2} = \frac{N_2\ B_1\ A_2}{I_1} \text{ where } B_1 = \frac{\mu_0\ N_1I_1}{2r_1},\ A2 = \pi r22$$

$$M_{c_1c_2} = \frac{N_2}{I_1}\left[\frac{\mu_0\ N_1I_1}{2r_1}\right](\pi r22) \Rightarrow M_{c_1c_2} = \frac{\mu_0\ N_1\ N_2\pi r_2^2}{2r_1}$$

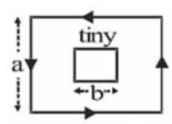
Two concentric loop

$M \propto \frac{r_2^2}{r_1}(r1 \gg r2$



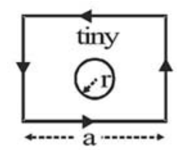
Two concentric square loops

$M \propto \frac{b^2}{a}$



A square and a circular loop

$M \propto \frac{r^2}{a}$



In terms of L_1 and L_2 : For two magnetically coupled coils :-

$M = K\sqrt{L_1\ L_2}$ here 'K' is coupling factor between two coils and its range $0 \leq K \leq 1$

- For ideal coupling Kmax = 1 $\Rightarrow$ $M_{max} = \sqrt{L_1\ L_2}$ (where M is geometrical mean of L1 and L2)
- For real coupling (0 < K < 1) $M = K\sqrt{LL_2}$
- Value of coupling factor 'K' decided from fashion of coupling.

Induced EMF in secondary coil

A transformer transforms the strength of an alternating electrical force from one level to another, either higher or lower. This transformation occurs through the principle of mutual induction. The transformer comprises two coils, named the primary and secondary windings, both wound around a continuous soft iron core. These coils are not physically connected in any manner.

When an alternating voltage (E) is applied to the primary winding, it generates a substantial alternating magnetic flux. This flux links with the secondary winding, causing an induced electromotive force (emf) in it, denoted as E. In the realm of ideal transformers, specific relationships and behaviors can be demonstrated.

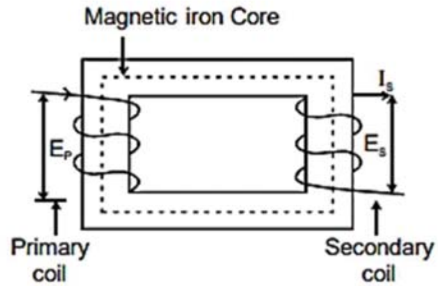
$$\frac{E_s}{E_0} = \frac{N_s}{N_0} = \frac{I_p}{I_s};$$

$\frac{N_s}{N_p} =$ turns ratio of the transformer.

E_sN and I are the emf, number of turns and current in the coils.

$N_s > N_p \Rightarrow E_s > E_p \rightarrow$ step up transformer.

$N_s < N_p \Rightarrow E_s < E_p \rightarrow$ step down transformer.



Series and parallel combination of Inductor

Two coil are connected in series : Coils are lying close together (M)

If $M = 0$, $L = L_1 + L_2$ If $M \neq 0$ $L = L_1 + L_2 + 2M$

- When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$
- When current flow in two coils are mutually in opposite directions.

$$L = L_1 + L_2 - 2M$$

Two coils are connected in parallel :

(a) If $M = 0$ then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1 L_2}{L_1 + L_2}$

(b) If $M \neq 0$ then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

Ex. How does the mutual inductance of a pair of coils change when :

(i) the distance between the coils is increased ?

(ii) the number of turns in each coil is decreased ?

(iii) A thin iron rod is placed between the two coils, other factors remaining the same ? Justify your answer in each case .

Sol. (i) The mutual inductance of two coils, decreases when the distance between them is increased. This is because the flux passing from one coil to another decreases.

(ii) Mutual inductance $M = \frac{\mu_0 N_1 N_2 A}{\ell}$ i.e., $M \propto N_1 N_2$ Clearly, when the number of turns N_1 and N_2 in the two coils is decreased, the mutual inductance decreases.

(iii) When an iron rod is placed between the two coils the mutual inductance increases, because $M \propto$ permeability (μ)

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$