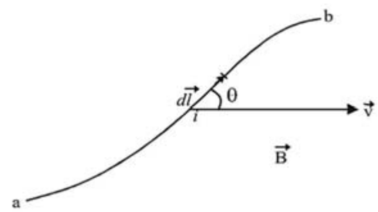
MOTIONAL ELECTROMOTIVE FORCE

When a wire moves in a magnetic field, the charges within the wire feel a magnetic force represented by the equation $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{v}$ is the speed of the wire. Here, we assume that the velocity of the charges inside the wire is the same as the velocity of the wire itself, and we ignore any random motion of charges. Because of this force, the charges start moving in a specific direction within the wire, resulting in the generation of an electromotive force (emf). This type of emf is referred to as motional emf.

Consider a conductor of arbitrary shape as shown in figure moving in some magnetic field $\vec{B}$



The different parts of the conductor may have different velocity. Consider an element $d\vec{l}$ have velocity $\vec{v}$. Force due to magnetic field on a charge in conductor.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Work done by this force on charge q in passing through $d\vec{l}$

$$dW = \vec{F} \cdot d\vec{l} = q(\vec{v} \times \vec{B}) \cdot d\vec{l}$$

So emf induced within $d\vec{l} : d\varepsilon = \frac{dW}{q} = (\vec{v} \times \vec{B}) \cdot d\vec{l}$

Integrate this to find net emf : $\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$

This emf is directed from a to b.

Other Approach

Because of the magnetic force, charges begin to move within the conductor, creating an electric field in the conductor. In a balanced state, the force from the electric field equals the force from the magnetic field. If we denote the induced electric field as $\vec{E}$, then:

$$q\vec{v} \times \vec{B} + q\vec{E} = 0 \Rightarrow \vec{E} = -\vec{v} \times \vec{B}$$

Now, a difference in potential is established across the ends of the conductor:

$$V_b - V_a = -\int_a^b E \cdot d\vec{l} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

If the circuit is not complete, the potential difference across these terminals will be equal to the induced emf. Therefore, the induced emf is:

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Note: We can also write $\varepsilon = \int_a^b \vec{B} \cdot (d\vec{l} \times \vec{v})$

As $d\vec{l} \times \vec{v}$ is the area swept per unit time by length $d\vec{l}$ and hence $\vec{B}.\left(d\vec{l} \times \vec{v}\right)$ is the flux of induction through the area. Therefore, the motional e.m.f. is equal to the flux of induction cut by the conductor per unit time. * We can write the expression for induced emf in various forms :

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{l} = \int_a^b (\vec{B} \times d\vec{l}) \cdot \vec{v} = \int_a^b (d\vec{l} \times \vec{v}) \cdot \vec{B}$$

If any two out of $\vec{v}, \vec{B}$ and $d\vec{l}$ become parallel or antiparallel, ε will become zero