Chapter 6

Electromagnetic Induction

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  - Faraday's Law of EMI
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  - Faraday's Law of EMI
  - Motional EMF
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INTRODUCTION

In the chapters before, we learned that an electric current creates a magnetic field. Now, let's explore the reverse scenario: can a magnetic field generate an electric current? The answer is yes. When closed coils experience changing magnetic fields, currents are induced in them.

This process of generating an electric current through changing magnetic fields is called electromagnetic induction. The resulting current is referred to as induced current.

If there's a current in the circuit, it must be because of some electromotive force (emf) generated in the circuit. This emf, produced due to a change in the magnetic field, is known as induced emf.

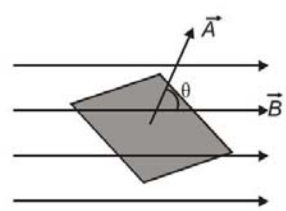
The phenomenon of electromagnetic induction (EMI) is crucial for the functioning of power generators, dynamos, transformers, and more.

MAGNETIC FLUX AND LENZ'S LAW

Magnetic Flux

In a magnetic field, we measure the magnetic flux passing through a surface by counting the total number of magnetic field lines that cross it. Magnetic flux is a scalar quantity, meaning it has only magnitude and no direction.

Picture a flat surface, represented as area A, situated in a magnetic field denoted as B. In this visual, notice that the area vector creates an angle (θ) with the direction of the magnetic field.



Now, we describe magnetic flux as the quantity of magnetic field that goes through a specific area.

$$\phi = \vec{B} \cdot \vec{A} \qquad \text{[for uniform]}$$

- Mutual Induction and Combination of Inductors
  - Mutual Induction
  - Coefficient of Mutual Inductance
  - Induced EMF in secondary coil
  - Series and parallel combination of Inductor

or $\phi = BA\cos\theta = (B\cos\theta)A = B_{\perp}A$

where is the part of the magnetic field B that is perpendicular to the surface of the area. The direction of the area vector is normal to the surface of the area.

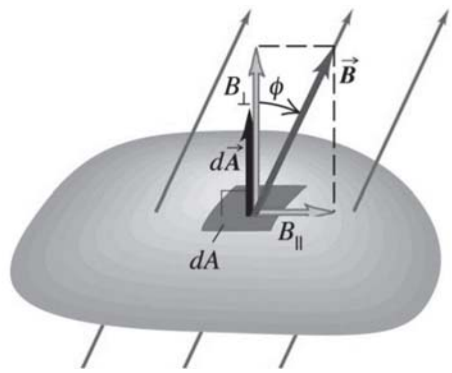
Special Cases

(i) if $\theta = 0°$, then $\phi = BA\cos 0° \Rightarrow \phi = BA$ (maximum flux)

(ii) if $\theta = 90°$, then $\phi = BA\cos 90° \Rightarrow \phi = 0$ (maximum flux)

(iii) if $\theta < 90°$, then $\cos\theta > 0 \Rightarrow \phi > 0$ (positive flux)

(iv) if $\theta > 90°$, then $\cos\theta < 0 \Rightarrow \phi < 0$ (negative flux)



If the surface isn't flat, we can break it down into small areas (dA, as illustrated in the figure). For each of these areas, we figure out the perpendicular component ($B_{\perp}$) at the element's position, as shown in the figure above. The formula for $B_{\perp}$ is $B\cos\phi$, where ϕ represents the angle between the direction of B and a line that is perpendicular to the surface. Generally, this component changes as we move from one point to another on the surface.

We define magnetic flux through the area element $d\Phi_B$ as

$$d\Phi = B_{\perp}dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\phi = \int B_{\perp}dA = \int B\cos\phi\, dA = \int \vec{B}\cdot d\vec{A}$$

The magnetic flux is measured in units called tesla-metre^2, and it's given the name "weber" (Wb) in honor of Wilhelm Weber. One weber is equivalent to one tesla-metre^2 ($1\text{Wb} = 1\text{Tm}^2$). So, it's clear that the magnetic field strength (B) can be expressed in Weber's per square meter (Wb/m^2), which is the same as teslas (T). This is sometimes called flux density

Induced current in loop due to flux change

The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. We shall discuss this further on how it is in accordance with conservation of energy.

Lenz's Law

The magnetic field's flow caused by the induced current resists the change in flow that creates the induced current.

Application of Lenz's Law

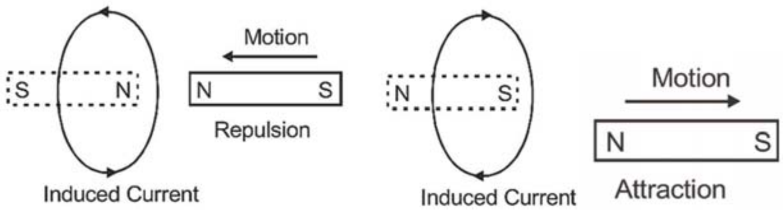
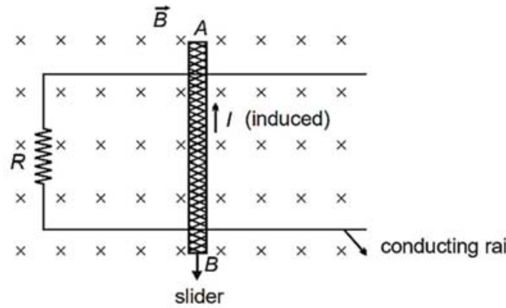


Fig. 1 Fig.2

1. If loop ABCD is brought closed, anticlockwise current is induced. (Fig. 1)
2. If ABCD is moved away, clockwise current is induced. (Fig. 2)
3. As shown, if magnetic field starts increasing, an anticlockwise current starts flowing. Due to this, slider AB moves leftward.



Ex. A closed loop with the geometry shown in figure (a) is placed in a uniform magnetic field directed into the plane of the paper. If the magnetic field decreases with time, determine the direction of the induced e.m.f. in this loop.

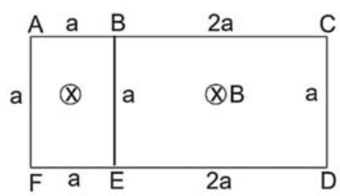


Figure (a)

Sol. There are two loops that are immersed in the magnetic field, namely, ABEFA and BCDE

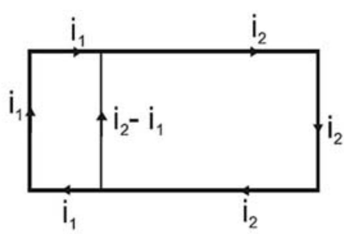


Figure (b)

Consider loop ABEFA. magnetic flux is into the plane of the paper. For loop BCDEB too, the magnetic flux is into the plane of the paper. In both loops, the magnetic flux is decreasing with time. Therefore, the induced current in loop ABEFA, i_1 , will be in a direction so as to induce a flux into the paper. The direction of i_1 is clockwise. Likewise in loop BCDEB the current i_2 will be in a direction so as to induce a flux into the paper. The direction of i_2 is also clockwise. The final induced current in all the arms of the loop are shown Fig. (b)

Ex. In figure there is a constant magnetic field in a rectangular region of space. This field is directed perpendicularly into the page. Outside this region there is no magnetic field. A copper ring moves through the region from position 1 to position 5. Find the induced current in the ring at it passes through positions.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Sol. a) Since the field is zero outside the rectangular region, no flux passes through the region in position 1, there is no change in the flux through the ring, and there is no induced emf or current in the ring.

b) In position 2 the flux increases. According to Lenz's law, the induced current must create an induced magnetic field that opposes the increase. To oppose the increase, the induced field must point opposite to the external field and, therefore, must point out of the page, for which the induced current must be counterclockwise.

c) Here the field is not zero. hence nonzero flux passes through the ring in position 3 but the flux through the ring is constant, and there is no induced emf or current in the ring.

d) In position 4 the flux decreases. According to Lenz's law, the induced current must create an induced magnetic field that opposes the decrease. To oppose the decrease, the induced field must point in the direction of external field and, therefore, must point into the page. For which the induced current must be clock-wise.

e) Since the field is zero outside the rectangular region, no flux passes through the ring in position 5, there is no change in the flux through the ring, and there is no induced emf or current in the ring.