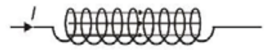
MAGNETIC ENERGY IN INDUCTOR AND L-R CIRCUIT

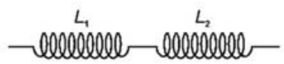
Magnetic Energy in an Inductor



$$\text{Energy } U_B = \frac{1}{2}LI^2, \text{ Energy Density } = \frac{1}{2}\frac{B^2}{\mu_0}$$

Combination of Inductors

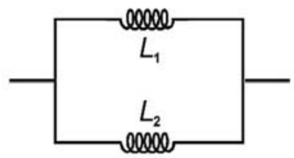
1. Inductor in series



(a) $L = L_1 + L_2$

(b) $L = L_1 + L_2 \pm 2M$ (If mutual inductance is also considered)

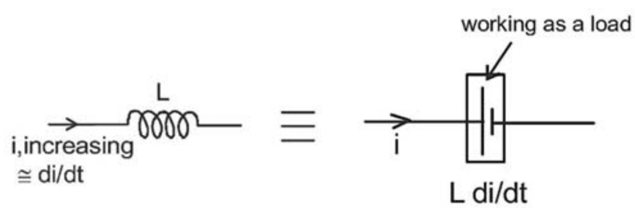
2. Inductor in parallel



$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (Neglecting mutual induction)

Energy Density

When the current in an inductor is i at a particular moment and is rising with a rate di/dt, the induced electromotive force (emf) will resist the current. This behavior is illustrated in the figure.



Power consumed by the inductor = $i\,L\,\frac{di}{dt}$

Energy consumed in dt time = $i\,L\,\frac{di}{dt}dt$

$\therefore$ total energy consumed as the current increases from 0 to I = $\int_0^1 iLdi = \frac{1}{2}L^2$

$$\frac{1}{2}Li^2 \qquad \Rightarrow U = \frac{1}{2}LI^2$$

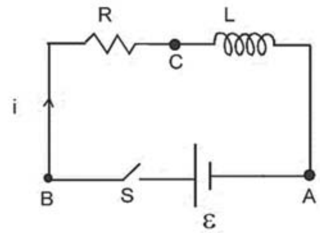
Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $\qquad U = \int \frac{B^2}{2\mu_0\mu_r}dV$

Growth of current in L-R circuit

The diagram depicts a setup with a cell, an inductor (L), and a resistor (R) connected one after another in a circuit. Imagine closing the switch (S) at t = 0. Now, let's say at a certain moment, the current in the circuit is denoted as i, and it's rising at a rate di/dt.



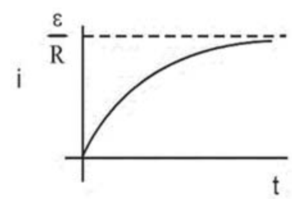
Writing KVL along the circuit , we have

$$\varepsilon - L\frac{di}{dt} - iR = 0$$

On solving we get,

$$i = \frac{\varepsilon}{R}\left(1 - e^{\frac{-Rt}{L}}\right)$$

The quantity $\frac{L}{R}$ is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.



Note

1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.
2. After one time constant , current in the circuit = 63% of the final current (verify yourself)
3. More time constant in the circuit implies slower rate of change of current.
4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor. $L_1 i_1 = L_2 i_2$

Decay of current in L-R circuit

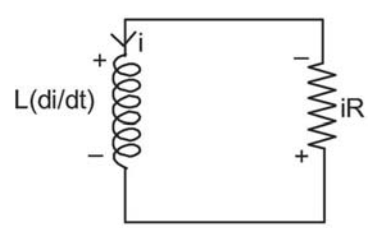
Let the initial current in the circuit be Io .At any time t, let the current be i and let its rate of change at this instant be $\frac{di}{dt}$.

$$L \cdot \frac{di}{dt} + iR = 0$$

$$\frac{di}{dt} = -\frac{iR}{L}$$

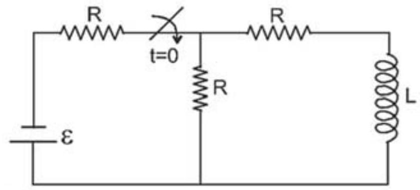
$$\int_{I_0}^{i} \frac{di}{i} = -\int_0^t \frac{R}{L} \cdot dt$$

$$\ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L} \text{ or } i = I_0 e^{\frac{-Rt}{L}}$$

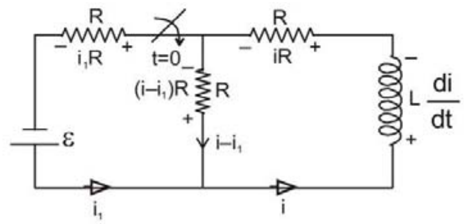


Current after one time constant: $I = I_o e^{-1} = 0.37\%$ of initial current.

Ex. In the following circuit the switch is closed at $t = 0$.Intially there is no current in inductor. Find out current the inductor coil as a function of time.



Sol. At any time t



$-\varepsilon + i_1R - (i - i_1)R = 0$

$-\varepsilon + 2i_1R - iR = 0$

$i_1 = \dfrac{iR + \varepsilon}{2R}$

Now,

$-\varepsilon + i_1R + iR + L \cdot \frac{di}{dt} = 0 \qquad -\varepsilon + \left(\frac{iR+\varepsilon}{2}\right) + iR + i \cdot \frac{di}{dt} = 0$

$-\dfrac{\varepsilon}{2} + \dfrac{3IR}{2} = -L \cdot \dfrac{di}{dt}$

$\left(\frac{-\varepsilon+3iR}{2}\right) dt = -L \cdot di \qquad -\frac{-d}{2L} = \frac{di}{-\varepsilon+3iR}$

$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon+3iR} \qquad -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)$

$-\ln\left(\dfrac{-\varepsilon + 3R}{-\varepsilon}\right) = \dfrac{3Rt}{2L}$

$i = +\dfrac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$