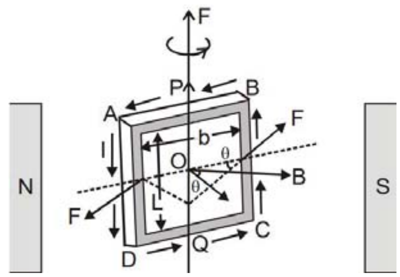
TORQUE ON CURRENT CARRYING LOOP IN UNIFORM B-FIELD

Torque on a current loop

When a coil carrying an electric current is situated in a consistent magnetic field, the resultant force acting on it is consistently zero. Nevertheless, since various sections of the coil encounter forces in distinct directions, the coil may undergo torque or a couple, contingent on the orientation of the coil and the axis of rotation. To illustrate this, contemplate a rectangular coil positioned in a uniform magnetic field B, capable of rotating around a vertical axis PQ, perpendicular to the coil's plane, forming an angle θ with the field direction, as depicted in figure (A).



Axis of rotation (A)

The arms AB and CD will encounter vertical forces, B(NI)b, acting upwards and downwards, respectively. The combination of these two forces results in a net force and torque of zero, as they are collinear with the axis of rotation. Therefore, they do not impact the motion of the coil.

Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value.

$$\tau = F \times \text{Arm} = BINL \times (b\sin\theta)$$

$$\tau = BIA\sin\theta \text{ with } A = NLb \quad \ldots (1)$$

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

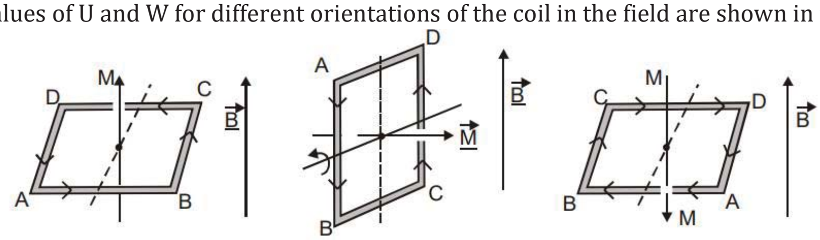
$$\vec{\tau} = \vec{M} \times \vec{B} \qquad [\text{ with } \vec{M} = I\vec{A} = NIA\,\vec{n} \ldots (2)]$$

1. The torque will be at its minimum value (equal to 0) when sinθ is at its minimum (equal to 0). This occurs when θ is 0° or 180°, meaning the plane of the coil is perpendicular to the magnetic field. In other words, the normal to the coil is collinear with the field, as depicted in figures (A) and (C).
2. The torque will reach its maximum value (equal to BINA) when sinθ is at its maximum (equal to 1). This occurs when θ is 90°, indicating that the plane of the coil is parallel to the magnetic field. In other words, the normal to the coil is perpendicular to the field, as illustrated in figure (B).
3. By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$$U = -\vec{M} \cdot \vec{B} \text{ with } F = -\frac{dU}{dr}$$

$$W = MB(1 - \cos\theta)$$

The values of U and W for different orientations of the coil in the field are shown in fig.



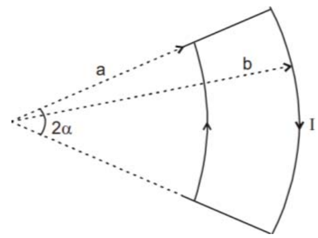
θ = 0° ($\vec{M}$ is parallel to $\vec{B}$)
τ = 0 = min
W = 0 = min
U = –MB = min
Stable equilibrium
(A)

θ = 90° ($\vec{M}$ is ⊥ to $\vec{B}$)
τ = MB = max
W = MB
U = 0
No equilibrium
(B)

θ = 180° ($\vec{M}$ is antiparallel to $\vec{B}$)
τ = 0
W = 2MB = max
U = MB = max
Unstable equilibrium
(C)

4. Devices like electric motors, moving coil galvanometers, and tangent galvanometers operate on the principle that a coil carrying current in a uniform magnetic field undergoes torque or a couple.

Ex. A loop with current I is in the field of a long straight wire with current I_0. The plane of the loop is perpendicular to the straight wire. Find torque acting on the loop.



Sol. Here

$$d\vec{s} = (rd\theta dr) \quad \text{(Inwards)}$$

$$d\vec{M} = (rId\theta dr) \quad \text{(Inwards)}$$

$$\vec{B} = \frac{\mu_0 I_0}{2\pi r} \text{ (inwards)} \quad \text{(Tangential clockwise)}$$

$$d\tau = |d\vec{M} \times \vec{B}| = \frac{\mu_0 I_0 d\theta dr}{2\pi}$$

$$\tau = \int_{-\alpha}^{\alpha} \int_a^b d\,\tau \cos\theta$$

$$= \frac{\mu_0 I I_0}{2\pi} \int_{-\alpha}^{\alpha} \int_a^b \cos\theta d\theta dr$$

$$= \frac{\mu_0 I_0 (b-a) \sin\alpha}{\pi} \quad \text{(To the left)}$$