MAGNETIC MOMENT

The magnetic field, particularly at significant distances, arising from the current in a circular loop, exhibits behavior akin to the electric field of an electric dipole. As we understand, this magnetic field is present on the axis of a circular loop with a radius R, carrying a constant current I.

$$B = \frac{\mu_0 I(2\pi a^2)}{4\pi(a^2+x^2)^{3/2}}$$

Its direction is along the axis and given by the right-hand thumb rule. Here, x is the distance along the axis from the centre of the loop.

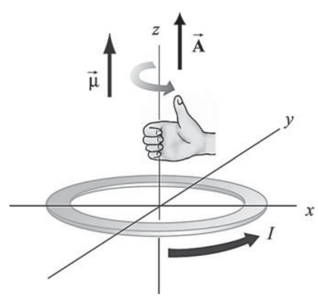
For $x >> R$, we may drop the R^2 term in the denominator. Thus

$$B = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IA}{x^2}\right)$$

Where $A = \pi R^2 =$ area of the loop

The expression closely resembles an earlier expression derived for the electric field of a dipole. This resemblance becomes apparent if we can establish a definition for the magnetic dipole moment $\vec{\mu}$ as. $\vec{\mu} = I\vec{A}$

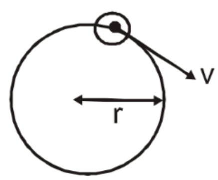
The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the loop's plane) and is ascertained using the right-hand rule (as illustrated in the figure). The SI unit for this is.μαγνετιχδ1πολεμομεντ1σαμπερε − μετερ^2(A. μ^2).



Ex. Find the magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v.

Sol. Magnetic moment $\mu = iA$ we can write

$$i = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$$



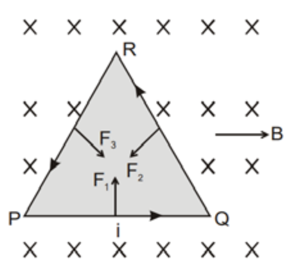
$A =$ area of the loop $= pr^2$

$$\mu = (I)\left(\frac{ev}{2\pi r}\right)(\pi r^2)$$

$$\mu = \frac{evr}{2}$$

Ex. A wire is bent in the form of an equilateral triangle PQR of side 20 cm and carries a current of 2.5 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Sol. Suppose the field and the current have directions as shown in figure. The force on PQ



$$\vec{F}_1 = i\vec{\ell} \times \vec{B} \text{ or, } F_1 = 2.5\text{ A} \times 20\text{ cm} \times 2.0\text{ T} = 1.0\text{ N}$$

The rule of vector product shows that the force F1 is perpendicular to PQ and is directed towards the inside of the triangle.

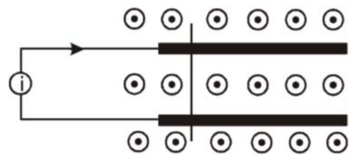
The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpen- dicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalized. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Ex. Figure shows two long metal rails placed horizontally and parallel to each other at a separation y. A uniform magnetic field B exists in the vertically upward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

(a) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part.



Sol. **(a)** The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B}$$
$$F = iyB$$

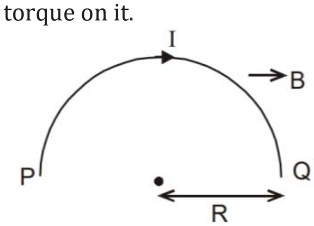
It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

$$\mu_0 mg = iyB$$
$$\mu_0 = \frac{iyB}{mg}$$

(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{iyB}{2mg}$ the wire will slide towards right. The frictional force by the rails is. $f = \mu mg = \frac{iyB}{2}$ towards left.

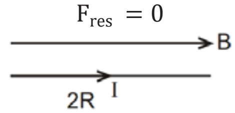
The resultant force is $iyB - \frac{iyB}{2} = \frac{iyB}{2}$ towards right. The acceleration will be $a = \frac{iyB}{2m}$ The wire will slide towards right with this acceleration.

Ex. In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.



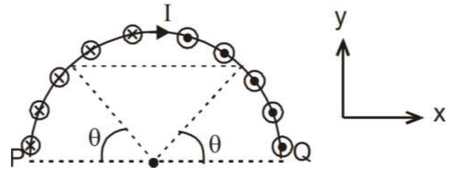
Sol. The wire is equivalent to

$$\theta = 0$$
$$F_{res} = 0$$

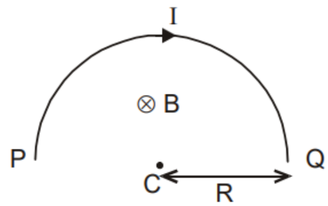


Forces on individual parts are marked in the figure by ⊗ and ⊙. By symmetry their will be pair of forces forming couples.

$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta) \cdot 2R\cos\theta$$
$$\tau = \frac{i\pi R^2}{2}B$$
$$\vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$$

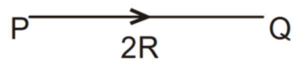


Ex. In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.



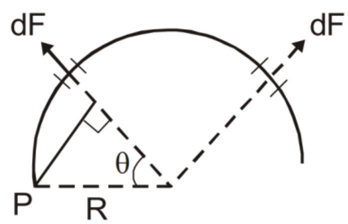
$$\vec{F}_{nett} = I.\,2R.\,B$$

wire is equivalent to

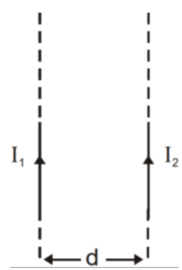


Force on each element is radially outward : $\tau_c = 0$ point about

$$P = \int_0^{\pi} [i(Rd\theta)B\sin 90°]\,R\sin\theta$$
$$= 2IBR^2$$



Ex. Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.



Sol. On (2), B due to (i) is $= \frac{\mu_0 I_1}{2\pi d} \otimes$

F on (2) on 1 m length $I_2 \cdot \frac{\mu_0 I_1}{2\pi d} \cdot 1$

Towards left it is attractive $\frac{\mu_0 I_1 I_2}{2\pi d}$ (Hence proved)

Similarly on the other wire also.

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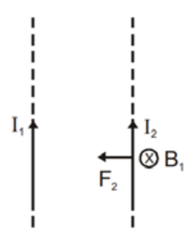
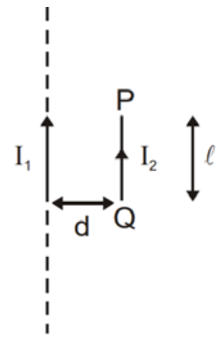


Note

- The definition of the ampere, the fundamental unit of current, is established by the formula provided above. If $I_1 = I_2 = 1$ A, d = 1m then $F = 2 \times 10^{-7}$N
- If two lengthy wires, each carrying an identical current and positioned 1 meter apart, generate a magnetic force of 2×10^{-7} N on a 1-meter length, the current flowing through them is 1 ampere.
- The formula mentioned above is applicable even when one wire is infinitely long, and the other is of finite length. In such a scenario, the force per unit length on each wire will not be equal.
  Force per unit length on PQ $= \frac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)
- When the currents flow in opposite directions, the magnetic force acting on the wires will be repulsive.



Ex. Find the magnetic force on the loop 'PQRS' due to the loop wire.

Sol. $F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a} a(\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$