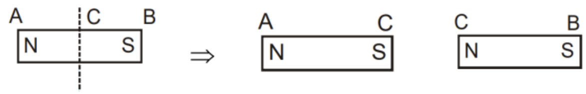
MAGNETIC DIPOLE

Pole Strength Magnetic Dipole and Magnetic Dipole Moment

Every magnet possesses two poles, 'N' and 'S'. Similar poles of two magnets repel each other, while opposite poles attract each other, forming an action-reaction pair.

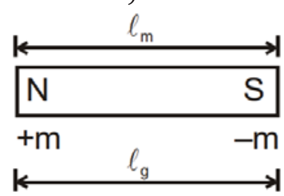
(i) (ii)

The poles of a single magnet do not converge due to mutual attraction, and efforts to obtain two isolated poles by cutting the magnet in the middle are futile. Instead, the other end becomes a pole of opposite nature. As a result, 'N' and 'S' poles persist together.



These poles are identified as positive and negative poles. The North Pole is considered the positive pole, equivalent to positive magnetic charge, while the South Pole is regarded as the negative pole, akin to negative magnetic charge. Their quantitative representation involves "POLE STRENGTH," denoted as +m for the North Pole and -m for the South Pole, analogous to the charges +q and -q in electrostatics. Pole strength is a scalar quantity and signifies the potency of the pole and, consequently, of the magnet.

A magnet can be viewed as a dipole, as it invariably possesses two opposing poles (similar to an electric dipole with opposite charges -q and +q). This is referred to as a MAGNETIC DIPOLE, and it possesses a MAGNETIC DIPOLE MOMENT, symbolized by $\vec{M}$ It is a vector quantity. Its direction is from –m to +m that means from 'S' to 'N')



$M = m \cdot \ell_m$ here $\ell_m =$ magnetic length of the magnet. ℓ_m is slightly less than ℓ_gIt is the geometric length of the magnet, measured from end to end). The positions of 'N' and 'S' are not precisely at the ends of the magnet. For computational purposes, we can assume $\ell_m = \ell_g$[Actually $\ell_m/\ell_g \simeq 0.84$]. The units for 'm' and 'M' will be discussed later for your recollection and comprehension.

Magnetic Field and Strength of Magnetic Field.

The region surrounding a magnetic pole has a unique impact, causing the other pole to undergo a force. This distinctive influence is termed the MAGNETIC FIELD, and the resulting force is referred to as 'MAGNETIC FORCE.' The qualitative representation of this field is conveyed through the 'STRENGTH OF MAGNETIC FIELD,' also known as "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY," symbolized by $\vec{B}$. It is essential to note that the magnetic field strength is a vector quantity.

Definition of $\vec{B}$

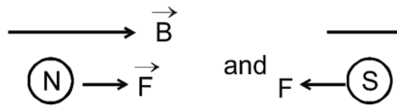
The force encountered by a north pole with unit pole strength at a specific location, arising from other poles (referred to as the source), is termed the magnetic field strength at that point due to the source.

$$\vec{B} = \frac{\vec{F}}{m}$$

Here $\vec{F}$ = Magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is Tesla or Weber /m^2 (abbreviated as T and Wb/m^2).

We can also write $\vec{F} = m\vec{B}$ According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$

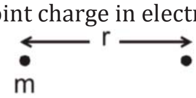


The field produced by sources is independent of the test pole, regardless of its magnitude or sign.

1. **$\vec{B}$ Due to various source**

(a) Due to a single pole:

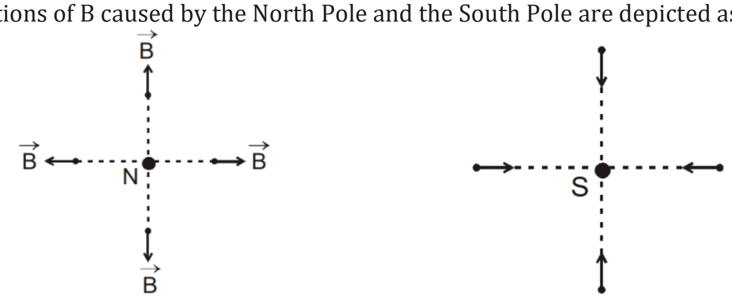
(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

This is magnitude

The directions of B caused by the North Pole and the South Pole are depicted as follows:

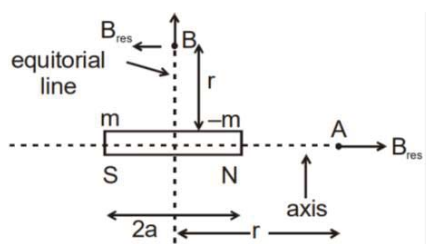


In vector form $\vec{\mathbf{B}} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$

Here m is with sign and $\vec{r}$= position vector of the test point with respect to the pole.

(b) Due to a bar magnet:

(Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.



At A (on the axis) $= 2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a $\lll$ r

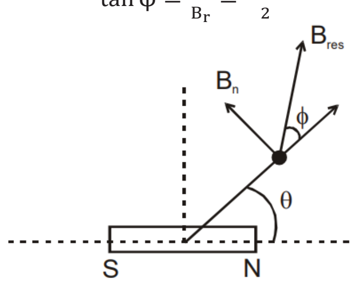
At B(on the equatorial) $= -\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a $\lll$ r

At General point $B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$

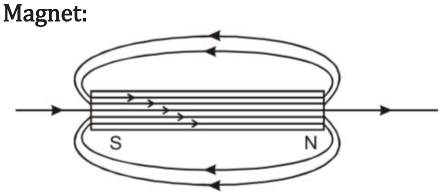
$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1 + 3\cos^2\theta}$$

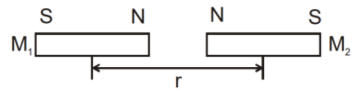
$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$



Magnetic Lines of Force of a Bar Magnet:



Ex. Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



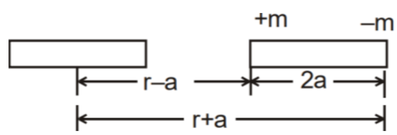
Sol. To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and –m as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and –m respectively. They experience magnetic forces F_1 and F_2 as shown.

$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m$$

$$F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$

$$F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right) - \left(\frac{1}{(r+a)^3}\right)\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3} - \left(1+\frac{a}{r}\right)^{-3}\right]$$



Through the utilization of acceleration, binomial expansion, and neglecting terms of high power, we obtain.

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1 + \frac{3a}{r} - 1 + \frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4}$$

$$= 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

Direction of F_{res} is towards right.

Alternative Method

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \Rightarrow \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2 \times \frac{dB}{dr} \Rightarrow F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1 M_2}{r^4}$$

Ex. A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Sol The pole strength is m = 120 CGS units = 12A-m.

Magnetic length is $2\lambda = 10$ cm or $\ell = 0.05$ m.

Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is.

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12\text{ A}-m)\times(0.05m)\times(0.2m)}{[(0.2m)^2-(0.05\text{ m})^2]^2}$$

$$= 3.4\times 10^{-5}T$$

Ex. Find the magnetic field due to a dipole of magnetic moment 1.2 A-m² at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

Sol. The magnitude of the field is

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2A-m^2}{1m^3}\sqrt{1+3\cos^2 60^\circ} = 1.6\times 10^{-7}T$$

The direction of the field makes an angle α with the radial line where

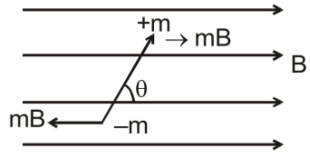
$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Magnet in an External Uniform Magnetic Field

(Same as case of electric dipole)

$$F_{res} = 0 \quad \text{(for any angle)}$$
$$\tau = MB\sin\theta$$

Here θ is angle between $\vec{B}$ and $\vec{M}$



Note

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is

$$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$$

$\theta = 0°$ is stable equilibrium

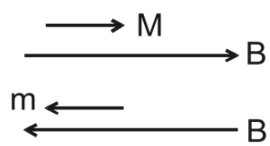
$\theta = \pi$ is unstable equilibrium

For small ' θ ' the dipole performs SHM about $\theta = 0°$ position

$$\tau = -MB\sin\theta;\ I\alpha = -MB\sin\theta$$

For small $\theta, \sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$



Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \Rightarrow T = 2\pi\sqrt{\frac{I}{MB}}$$

Here $I = I_{cm}$ if the dipole is free to rotate $= I_{hinge}$ if the dipole is hinged

Ex. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

Sol. The work done by the external agent = change in potential energy.

$$= (-MB\cos\theta_2) - (-MB\cos\theta_1) = -MB(\cos 60° - \cos 0°)$$
$$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^4\ \text{J/T})(4 \times 10^{-5}\text{T}) = 0.2\ \text{J}$$

Ex. A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Sol. **(a)** The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2) = \frac{100\times10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\text{kg} - \text{m}^2. = \frac{25}{6} \times 10^{-5}\text{Kg} - \text{m}^2$$

We have

$$T = 2\pi\sqrt{\frac{I}{MB}} \text{ or, } M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2\times25\times10^{-5}\ \text{kg/m}^2}{6\times(25\times10^{-6}\ \text{T})\times\frac{\pi^2}{4}\text{s}^2} = 27\text{A} - \text{m}^2.$$

(b) In this case the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b^2) \text{ where } b' = 0.5\ \text{cm}.$$

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \quad \text{... (2)}$$

Dividing by equation (1),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2+b'^2)}}{\sqrt{\frac{m'}{12}(L^2+b^2)}} = \frac{\sqrt{(7\text{cm})^2+(0.5\text{ cm})^2}}{\sqrt{(7\text{ cm})^2+(1.0\text{ cm})^2}} = 0.992 \text{ or, } T' = \frac{0.992\times\pi}{2}\text{ s} = 0.496\pi\text{s}$$

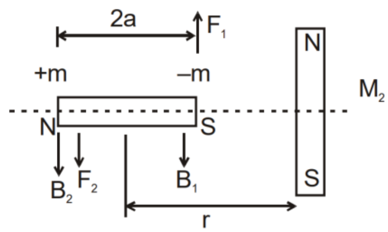
Magnet in an External Non-uniform Magnetic Field

In such problems, no specific formulas are employed. Instead, examine the force on individual poles and compute the resultant torque on the dipole.

Ex. Find the torque on M_1 due to M_2 in Que. 1

Sol. Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2 respectively. The forces on –m and +m are F_1 and F_2 as shown in the figure. The torque (about the centre of the dipole m_1) will be

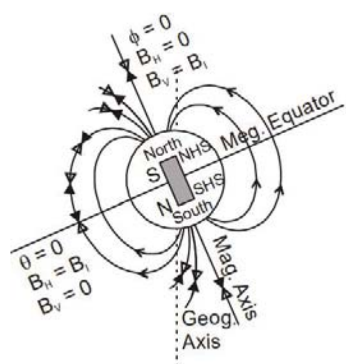
$$F_1a + F_2a = (F_1 + F_2)a$$
$$[(\frac{\mu_0}{4\pi})\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m]a$$
$$\cong \frac{\mu_0}{4\pi}M_2m(\frac{1}{r^3}+\frac{1}{r^3})a \because a < r$$
$$\frac{\mu_0M_2m}{4\pi}\frac{2}{r^3}a = \frac{\mu_0M_1M_2}{4\pi r^3}$$



Terrestrial Magnetism (Earth's Magnetism):

Introduction:

The notion that Earth possesses magnetism was initially proposed in the late sixteenth century by Dr. William Gilbert. Although the origin of Earth's magnetism remains a subject of speculation among scientists, there is consensus that Earth acts as a magnetic dipole with a slight inclination (11.5°) to its axis of rotation, with the south pole pointing north. The figure illustrates the lines of force in Earth's magnetic field, which run parallel to the surface near the equator and perpendicular to it near the poles. When delving into the Earth's magnetism, it is important to bear in mind that:



1. At a specific location, the magnetic meridian is not a mere line but rather a vertical plane that traverses through the axis of a freely suspended magnet. Essentially, it is a plane encompassing both the location and the magnetic axis.

2. The geographical meridian of a place is a vertical plane that intersects the line connecting the geographical north and south. In essence, it is a plane encompassing both the location and the Earth's axis of rotation, commonly known as the geographical axis.

3. The magnetic Equator is a substantial circle, centered at the Earth's core, positioned on the Earth's surface and perpendicular to the magnetic axis. The magnetic equator, passing through Trivandrum in South India, divides the Earth into two hemispheres.

The hemisphere containing the south polarity of Earth's magnetism is referred to as the northern hemisphere (NHS), while the other is termed the southern hemisphere (SHS).

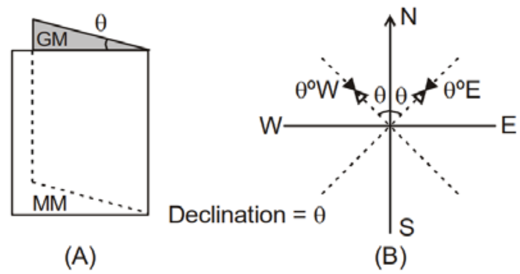
4. Earth's magnetic field exhibits variability, being neither constant nor consistent across its surface. This irregularity is observed from place to place and can also change over time at a given location.

Elements of the Earth's Magnetism:

The Earth's magnetism is fully defined by three parameters known as the elements of Earth's magnetism.

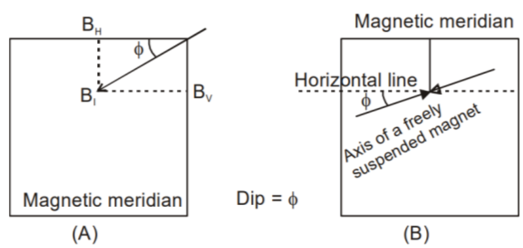
1. **Variation or Declination θ**

The angle between the geographical meridian and the magnetic meridian at a specific location is referred to as declination. In other words, it represents the angle between the north-south direction indicated by a magnetic compass needle and the geographical north-south direction at that location. Declination is denoted as θ° E or θ° W, depending on whether the north pole of the compass needle is to the east (right) or west (left) of the geographical north-south direction. For instance, a declination of 10°W at London indicates that the north pole of a compass needle in London points 10° to the west, or left, of the geographical north.



2. **Inclination or Angle of Dip ϕ**

It is the angle formed between the direction of the resultant intensity of Earth's magnetic field and the horizontal line in the magnetic meridian at a specific location. More precisely, it represents the angle formed between the axis of a freely suspended magnet (either facing up or down) and the horizontal line in the magnetic meridian at that location. It is important to note that in the northern hemisphere, where the south polarity of Earth's magnetism is predominant, the north pole of a freely suspended magnet (or a pivoted compass needle) will dip downward, towards the Earth. Conversely, in the southern hemisphere, the opposite phenomenon occurs.



The measurement of the angle of dip at a specific location is conducted using an instrument known as a Dip-Circle. This device features a magnetic needle that is unrestricted in its rotation within a vertical plane, allowing it to be positioned in any vertical direction. For instance, the angle of dip at Delhi is recorded as 42°.

3. **Horizontal Component of Earth's Magnetic Field B_H:**

At a specific location, it is characterized as the Earth's magnetic field component projected along the horizontal axis in the magnetic meridian. This component is denoted as BH and is quantified using a vibration or deflection magnetometer. In the case of Delhi, the measurement of the horizontal component of Earth's magnetic field is obtained. 35μ T, i.e., 0.35G.

If at a place magnetic field of earth is B_t and angle of dip ϕ, then in accordance with figure (a).

$$B_H = B_I \cos\phi \quad \text{and} \quad B_v = B_I \sin\phi \qquad \text{... (1)}$$

$$\tan\phi = \frac{B_V}{B_H} \quad \text{and} \quad I = \sqrt{B_H^2 + B_v^2} \qquad \text{... (2)}$$