## MAGNETIC FIELD DUE TO STRAIGHT CURRENT CARRYING WIRE

### Biot - Savart Law

Magnetic fields originate from currents resulting from the movement of charges. When charges in a conducting wire are in motion, creating a current I, the magnetic field at any point P due to the current can be computed by aggregating the contributions of the magnetic fields. $d\vec{B}$ from small segments of the wire $d\vec{s}$ (Figure).

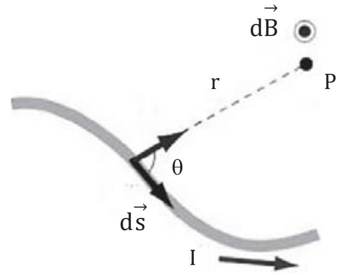



Figure: Magnetic field $d\vec{B}$ at point P due to a current – carrying element $Id\vec{s}$

The segments can be conceptualized as a vector quantity with a magnitude equal to the length of the segment and oriented in the direction of the current flow. The infinitesimal current source can thus be expressed as: $I\, d\vec{s}$.Let r denote as the distance from the current source to the field point P and $\vec{r}$ the corresponding unit vector. The Biot–Savart law gives an expression for the magnetic field contribution $d\vec{B}$, from the current source, $I_d\vec{s}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times r}{r^2}$$

Where $\mu_0$ is constant called the permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T.M/A here Tesla (T) is SI unit of } \vec{B}$$

Notice that the expression is remarkably similar to the Coulomb's law for the electric field due to a charge element dq:

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

Adding up these contributions to find the magnetic field at the point P requires integrating over the current source.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s}\times\hat{r}}{r^2}$$

The integral is a vector integral, which means that the expression for B is really three integrals, one for each component of $\vec{B}$ the vector nature of this integral appears in the cross product. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Bio - Savart law.

**Magnetic Field due to a Finite Straight Wire**

Assess the magnetic field at point P generated by the depicted segment in the figure, where a slender, straight wire carrying a current I is positioned along the x-axis.

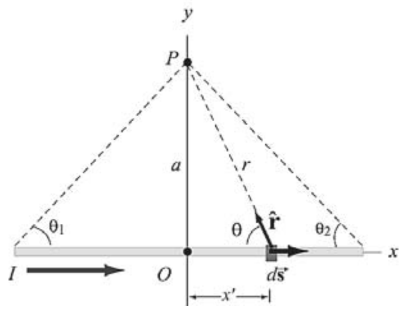



Figure: A thin straight wire carrying a current I

The contribution to the magnetic field due to I $d\vec{s}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\times\sin\theta}{r^2}\hat{k}$$

Which shows that the magnetic field at P will in the $+\hat{k}$ direction, or out of the page.
Simplify and perform the integration. The variables θ, x, and r are interdependent. To facilitate the integration, let's express the variables x and r in terms of θ. Referring to the figure, we have.

$$\begin{cases} r = a/\sin\theta = a\operatorname{cosec}\theta \\ x = a\cot\theta \Rightarrow dx = -a\operatorname{cosec}^2\theta d\theta \end{cases}$$

Upon substituting the above expressions, the differential contribution to the magnetic field is obtained as.

$$dB = \frac{\mu_0 I}{4\pi}\frac{(-a\operatorname{cosec}^2\theta d\theta)\sin\theta}{(a\operatorname{cosec}\theta)^2} = \frac{\mu_0 I}{4\pi a}\sin\theta d\theta$$

Integrating across all angles spanning from $-\theta_1$ to $\theta_2$ (employing a negative sign for $\theta_1$ to account for the segment extending along the negative x-axis from the origin), we acquire.

$$dB = \frac{\mu_0 I}{4\pi a}\int_{-\theta_1}^{\theta_2}\sin\theta d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_2 + \cos\theta_1)$$

The initial term incorporating $\theta_2$ represents the contribution from the segment along the +x axis, whereas the subsequent term involving $\theta_1$ encompasses the contribution from the segment along the –x axis. The sum of these two terms yields the overall contribution.

**Special scenarios:**

**1. Magnetic field along the perpendicular bisector of a finite straight wire with a length of 2L**

In this instance, where $\theta_2 = \theta_1 = \theta$, point P is situated along the perpendicular bisector. If the length of the rod is 2 L, then $\cos\theta = L\sqrt{L^2 + a^2}$ and the magnetic field is.

$$B = \frac{\mu_0 I}{2\pi a}\cos\theta = \frac{\mu_0 I}{2\pi a}\frac{L}{\sqrt{L^2+a^2}}$$

**2. Magnetic field due to semi-infinite straight wire**

Here $\theta_1 = 90°, \theta_2 = 0°$ or $\theta_1 = 0°, \theta_2 = 90°$

$$B = \frac{\mu_0 I}{4\pi a}$$

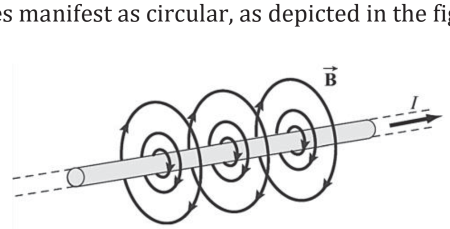
**3. Magnetic field due to infinite straight wire**

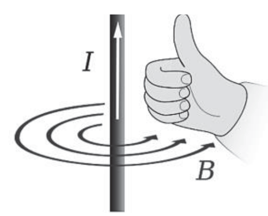
Here $\theta_1 = \theta_2 = 0°$

$$B = \frac{\mu_0 I}{2\pi a}$$

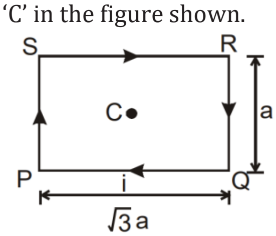
**Direction of magnetic field of a straight wire**

Keep in mind that in this limit, the system exhibits cylindrical symmetry, and the magnetic field lines manifest as circular, as depicted in the figure.




**Ex.** Find resultant magnetic field at 'C' in the figure shown.



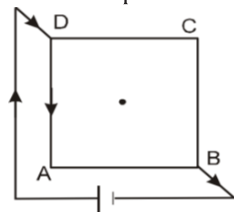

**Sol.** It is clear that 'B' at 'C' due all the wires is directed ⊗. Also B at ' C ' due PQ and SR is same.
Also due to QR and PS is same

$$B_{res} = 2(B_{PQ} + B_{SP})$$

$$B_{PQ} = \frac{\mu_0 i}{4\pi\frac{a}{2}}(\sin 60° + \sin 60°), \Rightarrow B_{sp} = \frac{\mu_0 i}{4\pi\frac{\sqrt{3}a}{2}}(\sin 30° + \sin 30°)$$

$$B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

**Ex.** Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure.

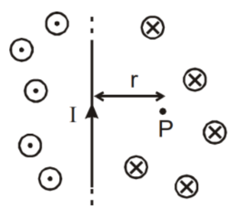



**Sol.** The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero. Hence, the net field at the centre will be zero.

**Special scenarios:**

1. For an infinitely long wire, the magnetic field at point 'P' (illustrated in the figure with r perpendicular to the wire) is determined by utilizing $\theta_1 = \theta_2 = 90°$ and applying the formula for 'B' due to a straight wire.

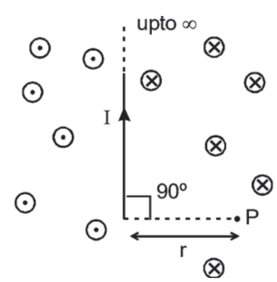
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$




The direction of $\vec{B}$ the magnetic field at different points is illustrated in the figure. The magnetic lines of force form concentric circles around the wire, as depicted earlier.

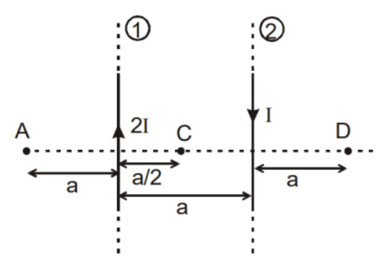
2. For an infinitely long wire with 'P' situated as depicted in the figure, the orientation of $\vec{B}$ at various points is as shown in the figure. At 'P'

$$B = \frac{\mu_0 I}{4\pi r}$$




**Ex.** Ex. In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2I and I consider points A, C, D on the line per pendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find

1. $\vec{B}$ at A, C, D
2. Position of point on line A C D where $\vec{B}$ is 0.



**Sol.** **1.** Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

At A: $\vec{B}_1$ is $\odot$ and $\vec{B}_2$ is $\otimes$

$$B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a} \odot$$

At C: $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ also $\otimes$

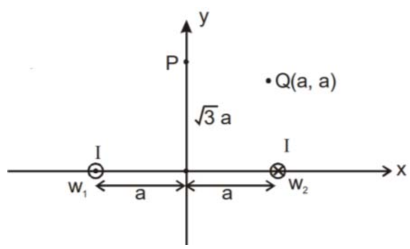
$$B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a} \otimes \text{ Ans.}$$

At D: $\vec{B}_1$ is $\otimes$ and $\vec{B}_2$ is $\odot$ and both are equal in magnitude.

$$B = 0 \text{ Ans.}$$

**2.** It is clear from the above solution that $B = 0$ at point ' D '.

**Ex.** Ex. In the figure shown two long wires $W_1$ and $W_2$ each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in $W_1$ is outward and in W it is inwards. Find the $\vec{B}$ at ' P ' and ' Q '.

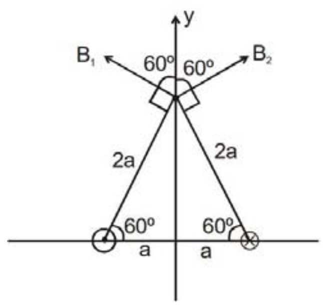



**Sol.** Let $\vec{B}$ due to $W_1$ be $\vec{B}_1$ and due to $W_2$ be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$

$$B_p = 2B\cos 60^\circ = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\vec{B}_p = \frac{\mu_0 I}{4\pi a}\hat{j} \text{ Ans}$$

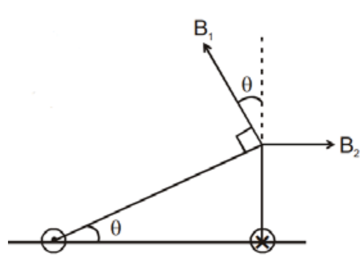



For θ $\quad B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}, \Rightarrow B_2 = \frac{\mu_0 I}{2\pi a}$

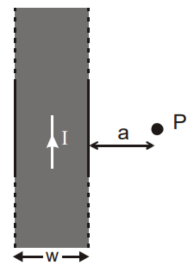
$$\tan\theta = \frac{a}{2a} = \frac{1}{2} \Rightarrow \vec{B} = (B_1\cos\theta\hat{j}) + (B_2 - B_1\sin\theta)\hat{i}$$

$$\sin\theta = \frac{1}{\sqrt{5}} \Rightarrow \vec{B} = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$$

$$\cos\theta = \frac{2}{\sqrt{5}} \Rightarrow \vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$$

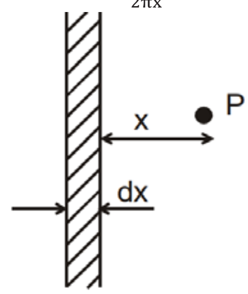


**Ex.** In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'. Find the magnetic field at point 'P' which lies in the plane of the sheet.




**Sol.** To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

$$dB = \frac{\mu_0(\frac{I}{w}dx)}{2\pi x} \otimes$$




due to each such wire $\vec{B}$ will be directed inwards

$$B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x}\int_{x=a}^{a+w}\frac{dx}{x}$$

$$= \frac{\mu_0 I}{2\pi w} \cdot \ln\frac{a+w}{a} \text{ Ans.}$$