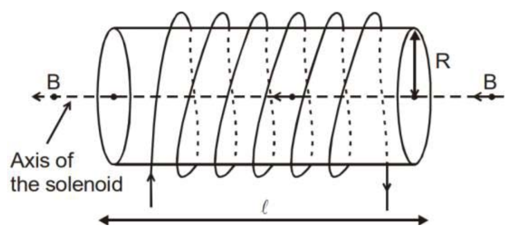
MAGNETIC FIELD DUE TO SOLENOID AND PROPERTIES OF MAGNETIC FIELD LINE

Solenoid

1. Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



2. The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

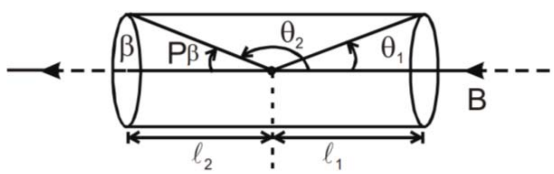
3. $\vec{B}$ on axis (turns should be very close to each others).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$

Where n : number of turns per unit length.

$$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2+R^2}}; \cos\beta = \frac{\ell_2}{\sqrt{\ell_2^2+R^2}} = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right] = \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$



Note: Use right hand rule for direction (same as the direction due to loop)

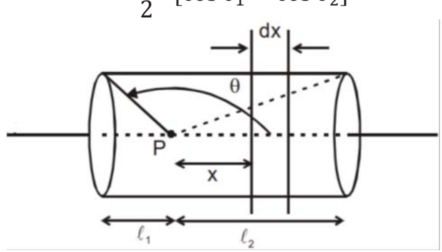
Derivation

Consider a differential element of width dx located at a distance x from point P, where P is the point on the axis for which we are determining the magnetic field. The total number of turns in the element is denoted by dn and is given by ndx, where n represents the number of turns per unit length.

$$dB = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 iR^2 ndx}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$



4. For 'Ideal Solenoid':

Inside (at the midpoint)

$\ell \gg R$ or length is infinite $\quad \theta_1 \to 0$

$$\theta_2 \to \pi$$

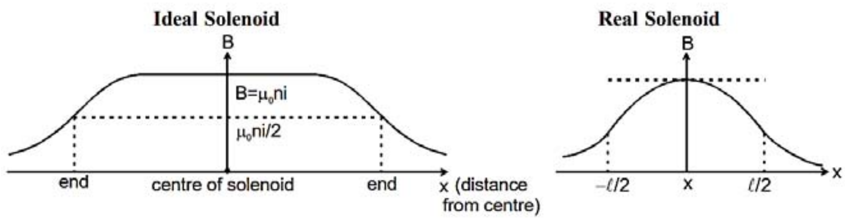
$$B = \frac{\mu_0 ni}{2}[1 - (-1)]$$

$$B = \mu_0 ni$$

If material of the solid cylinder has relative permeability ' μ_r ' then $B = \mu_0 \mu_r ni$

At the ends $\mathbf{B} = \frac{\mu_0 ni}{2}$

5. Comparison between ideal and real solenoid



Ex. A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid. (Given $\mu_0 = 4\pi \times 10^{-7} \frac{V-s}{A-m}$).

Sol. $B = \frac{1}{2}\mu_0 ni[\cos\theta_1 - \cos\theta_2]$

$$n = \frac{1000}{0.4} = 2500 \text{ per meter}$$

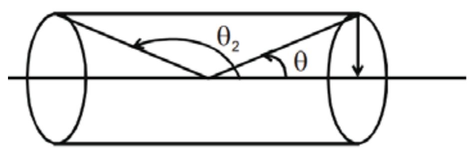
$$i = 5 \times 10^{-3} A.$$

1. $\text{os}\,\theta_1 = \frac{0.2}{\sqrt{(0.3)^2+(0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$

$$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}}$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\,\frac{2\times 0.2}{\sqrt{0.13}}$$

$$\frac{\pi\times 10^{-5}}{\sqrt{13}}\,T$$



2. At the end

$$\cos\theta_2 = \cos 90^\circ = 0$$

$$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2+(0.4)^2}} = 0.8$$

$$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$$

$$B = 2\pi \times 10^{-6}\ Wb/m^2$$