LORENTZ FORCE ON A MOVING CHARGE WHEN E IS PERPENDICULAR TO B

Lorentz Force

In the existence of both an electric field, $\vec{E}$ and magnetic field $\vec{B}$, the cumulative force acting on a charged particle is $\vec{F} = q(\vec{E} + \vec{v} + \vec{B})$
This is referred to as the Lorentz force.

Velocity Selector

By integrating the two fields, it becomes possible to selectively manipulate particles with a specific velocity. This principle was employed by J.J. Thomson in determining the charge-to-mass ratio of electrons. The schematic diagram of Thomson's apparatus is illustrated in the figure.

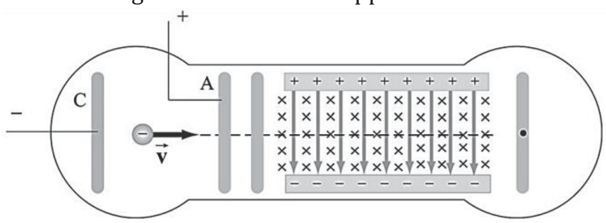


Figure: Thomson's apparatus

Electrons, possessing a charge q = -e and a mass m, are emitted from the cathode C and subsequently accelerated towards slit A. The potential difference between A and C is denoted as: $V_{\Delta} - V_C = \Delta V$ The alteration in potential energy is equivalent to the external work conducted in accelerating the electrons: $\Delta U - W_{ext} = q\Delta V = -e\Delta V$ By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$ Thus, the speed of the electrons is given by.

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Should the electrons traverse a zone featuring a consistent downward electric field, their negatively charged nature will lead to an upward deflection. Nevertheless, if, alongside the electric field, a magnetic field directed into the page is applied, the electrons will encounter an extra downward magnetic force. $-e\vec{v} \times \vec{B}$ When the two forces precisely counterbalance each other, the electrons will follow a linear trajectory. From the equation, it is evident that the criterion for the annulment of the two forces is expressed as: $eE = evB$. which implies.

$$v = \frac{E}{B}$$

Simply put, particles with a speed $v = E/B$ are the only ones capable of moving in a straight line.
By merging the two equations, we derive: $\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}$
By measuring E, ΔV and B The ratio of charge to mass can be easily ascertained. The most accurate measurement thus far is: $e/m = 1.758820174(71) \times 10^{11} C/kg$

Mass Spectrometer

Multiple techniques exist for determining the mass of an atom. One approach involves utilizing a mass spectrometer. The fundamental aspect of a Bainbridge mass spectrometer is depicted in the figure. Initially, a particle with a charge +q is passed through a velocity selector.

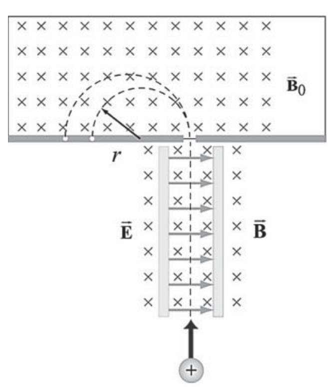


Figure: A Bainbridge mass spectrometer

The electric and magnetic fields applied adhere to the relation $E = vB$, ensuring that the particle's trajectory forms a straight line. Upon entering a zone featuring a second magnetic field, $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq., we have $r = \frac{mV}{qB_0}$

Since $v = E/B$, the mass of the particle can be written as $m = \frac{qB_0r}{v} = \frac{qB_0Br}{E}$

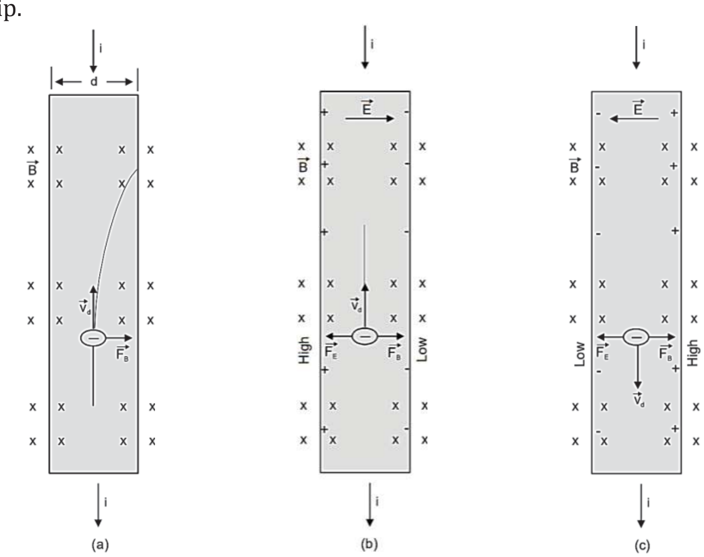
Hall's Effect

In 1879, Edwin H.Hall, a graduate student at Johns Hopkins University, demonstrated that electrons in copper wire can undergo drift in the presence of a magnetic field. This phenomenon, known as the Hall effect, enables the determination of:

1. The sign of charge carriers in a conductor (whether they are positively or negatively charged).
2. The density of charge carriers per unit volume of the conductor.

Consider a strip of current-carrying wire placed in an external magnetic field. Suppose the wire has a width of d, a cross-sectional area of A, and a charge carrier density of n per unit volume.

In Figure (a), there is a copper strip with a width of d carrying a current i, flowing conventionally from the top to the bottom of the figure. The charge carriers are electrons, and they drift (with a drift speed vd) in the opposite direction, from the bottom to the top. At the depicted moment, an external magnetic field B, directed into the plane of the figure, has been recently activated. It's evident that a magnetic force will influence each drifting electron, propelling it toward the right edge of the strip.



Over time, electrons gradually shift to the right, predominantly accumulating on the right edge of the strip, while uncompensated positive charges remain fixed at the left edge. The segregation of positive charges on the left and negative charges on the right induces an electric field E within the strip, oriented from left to right in Fig. b. This field applies an electric force FE to each electron, endeavoring to push it to the left. Consequently, an opposing force to the magnetic force begins to accrue on the electrons.

Eventually, an equilibrium is established where the electric force on each electron intensifies sufficiently to counterbalance the magnetic force. When this equilibrium is achieved, as depicted in Fig. b, the forces attributed to B and E reach a state of equilibrium. The drifting electrons then proceed along the strip towards the top of the page at a velocity vd, with no further accumulation of electrons on the right edge of the strip, and consequently, no further augmentation of the electric field E.

$$eE = ev_dB \quad \text{... (1)}$$

$$v_d = \frac{J}{ne} = \frac{i}{neA} \quad \text{... (2)}$$

A Hall potential difference V corresponds to the electric field across the width d of the strip.

$$V = Ed \quad \text{... (3)}$$

From (1), (2) and (3)

$$\frac{E}{B} = \frac{V}{Bd} = \frac{i}{neA}$$
$$n = \frac{idB}{eAV}$$
$$v_d = \frac{i}{neV} = \frac{V}{Bd}$$

By linking a voltmeter across the width, we can gauge the potential difference between the two edges of the strip. Additionally, the voltmeter can indicate which edge has a higher potential. In the scenario depicted in Fig. B, we would observe that the left edge has a higher potential, aligning with our assumption that the charge carriers are negatively charged.

The Hall effect can be employed to directly measure the drift speed (vd) of the charge carriers, which, as you may remember, is on the order of centimeters per hour. In this ingenious experiment, the metal strip is mechanically moved through the magnetic field in a direction opposing the drift velocity of the charge carriers. The velocity of the moving strip is then fine-tuned until the Hall potential difference diminishes.

Under this condition, where there is no Hall effect, the velocity of the charge carriers concerning the laboratory frame must be zero. Therefore, the velocity of the strip must be equivalent in magnitude but opposite in direction to the velocity of the negative charge carriers.

For a brief consideration, let's entertain the contrary assumption that the charge carriers in current i are positively charged (Fig. c). Take a moment to affirm that as these charge carriers travel from top to bottom in the strip, they are compelled toward the right edge by the magnetic force FB, implying that the right edge is at a higher potential. However, this assertion conflicts with our voltmeter reading. Hence, we conclude that the charge carriers must be negatively charged.

Ex. A non-relativistic proton beam passes without deviation through the region of space where there are uniform transverse mutually perpendicular electric and magnetic fields with E = –120 kV/m and B = 50mT. Then the beam strikes a grounded target. Find the force with which the beam acts on the target of the beam current is equal to I =0.80 mA.

Sol. $F = \frac{dp}{dt} = v\frac{dm}{dt} = v\frac{dm}{dq}\frac{dq}{dt} = \frac{E}{B}\frac{m}{q}I = 20\mu N$

Ex. A particle of mass m and charge q is released from the origin in a region occupied by electric field E and magnetic field B,

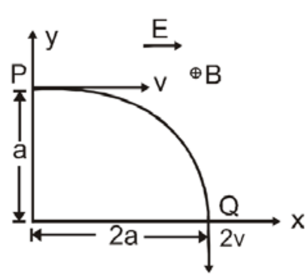
$$B = B_0\hat{j}, E = E_0\hat{i}$$

Determine the velocity of the particle.

Sol. Since the magnetic field does not perform any work, therefore, whatever has been gain in kinetic energy it is only because of the work done by electric field. Applying work-energy theorem.

$$W_E = \Delta K$$
$$qE_0 = \frac{1}{2}mv^2 - 0 \text{ or } v = \sqrt{\frac{2qE_0}{m}}$$

Ex. A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q as shown in figure. The velocities at P and Q are $v\hat{i}$ and $-2v\hat{i}$ Find (a) E (b) rate of work done by the electric field at P. (C) rate of work done by each the fields at Q.



Sol. Increase in Kinetic energy of particle

$$= \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$$

Work done by the uniform electric field, E, in going from P to Q $= (qE) \times 2a = 2qEa$

Hence, $2qEa = \frac{3}{2}mv^2$ or $E = \frac{3mv^2}{4qa}$

Rate of work done by the electric field at

$$P_{at}P = F \cdot v = qE \cdot v$$

$$qE\hat{i} \cdot v\hat{i} = qEv = q \cdot \frac{3mv^2}{4qa} \cdot v = \frac{3}{4}\frac{mv^2}{a}$$

Q is $$P_{at} \cdot Q = qE\hat{i} \cdot (-2v\hat{j}) = 0$$

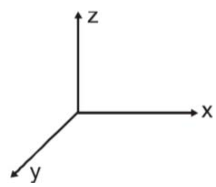
At Q, rate of work done by both the fields is zero.

Ex. Ex. A particle of mass 1×10^{-26} kg and charge 1.6×10^{-19} C travelling with a velocity 1.28×10^{6} ms^{-1} in the +x direction enters a region in which a uniform electric field E and uniform magnetic field of induction B are present such that $E_x = E_y = 0, E_x = -102.4 kVm^{-1}$ and $B_x = B_z = 0. B_y = 8 \times 10^{-2} Wbm^{-2}$ The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at t = 5×10^{-4} s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at t = 7.45×10^{-6} s ?

Sol. Let $\hat{i}, \hat{j}$ and $\hat{k}$ Be unit vector along the positive directions of x, y and z axes. Q = charge on the particle = 1.6×10^{-19} C $\vec{v} =$ velocity of the charged particle $= (1.28 \times 10^6) ms^{-1}\hat{i}$

$\vec{E} =$ electric field intensity : $= (-102.4 \times 10^3 Vm^{-1})\hat{k}$

$\vec{B} =$ magnetic induction of the magnetic field: $(8 \times 10^{-2} Wbm^{-2})\hat{j}$



$\vec{F}_e =$ electric force on the charge

$$q\vec{E} = [1.6 \times 10^{-19}(-102.4 \times 10^3)Nj\hat{k}$$

$$= 163.84 \times 10^{-16} N(-\hat{k})$$

$\vec{F}_m =$ magnetic force on the charge $= q\vec{v} \times \vec{B}$

$$[1.6 \times 10^{-19}(1.28 \times 10^6)(8 \times 10^{-2})N](\hat{i} \times \hat{j})$$

$$= (163.84 \times 10^{-16}N)(\hat{k})$$

The two forces $\vec{F}_e$ and $\vec{F}_m$ are along z-axis and equal, opposite and collinear.

The net force on the charge is zero and hence the particle does not get deflected and continues to travel along x–axis.

(a) At time t = 5×10^{-6} s

$$x = (5 \times 10^{-6})(1.28 \times 10^6) = 6.4 \text{ m}$$

Coordinates of the particle = (6.4 m, 0, 0)

(b) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x–z plane (i.e. the plane of velocity and magnetic force), anticlockwise as seen along +y axis.

Now, $\frac{mv^2}{r} = qvB$ where r is the radius of the circle.

$$r = \frac{mv}{qB} = \frac{(1\times10^{-26})(1.28\times10^6)}{(1.6\times10^{-19})(8\times10^{-2})}$$

The length of the arc traced by the particle in $[(7.45 - 5) \times 10^{-6}s]$

$(v)(t) = (1.28 \times 10^6)(2.45 \times 10^{-6})$

$$= 3.136 \text{ m} = \pi m = \frac{1}{2} \text{ circumference}$$

The particle has the coordinates $(6.4, 0, 2 \text{ m})$ as (x, y, z)