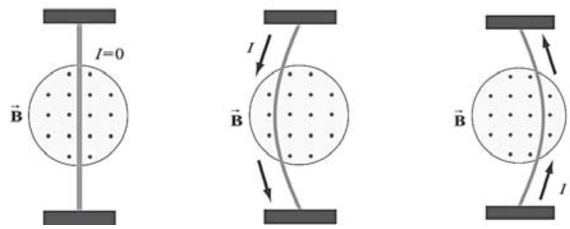
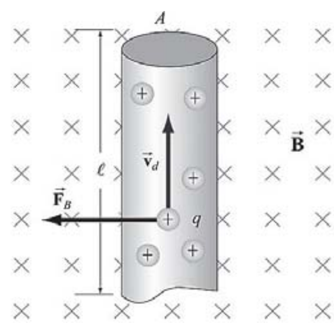
FORCE ON CURRENT CARRYING WIRE DUE TO MAGNETIC FIELD

Magnetic Force on a Current-Carrying Wire

We've just observed that a charged particle, when in motion through a magnetic field, undergoes a magnetic force. $\vec{F}_B$ Considering that an electric current is essentially a group of charged particles in motion, when placed within a magnetic field, a wire conducting a current will also encounter a magnetic force. Contemplate a lengthy, straight wire positioned in the space between two magnetic poles. The magnetic field extends out of the page and is symbolized by dots (.). It can be easily illustrated that when a downward current flows through the wire, it is deflected to the left. Conversely, when the current is directed upward, the deflection occurs to the right, as depicted in the figure.



To compute the force applied to the wire, examine a section of the wire with a length of ℓ and a cross-sectional area A, as illustrated in the figure. The magnetic field is directed into the page and is denoted by crosses (X).



The charges move at an average drift velocity $\vec{v}_d$ Since the total amount of charge in this segment is $Q_{tot} = q(nA\ell)$ where n is the number of charges per unit volume, the total magnetic force on the segment is.

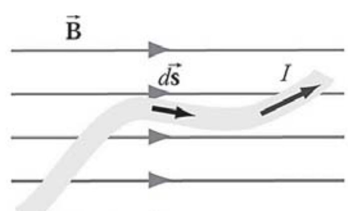
$$\vec{F}_B = Q_{tot}\,\vec{v}_d \times \vec{B} = qnA\ell(\vec{v}_d \times \vec{B}) = I(\vec{\ell} \times \vec{B})$$

Where $I - nqv_dA$ and $\vec{\ell}$ is a length vector with a magnitude ℓ and directed along the direction of the electric current.

Special Case - 1:

Wire of arbitrary shape placed in uniform magnetic field

For a wire of any shape, the magnetic force can be determined by adding up the forces exerted on the small segments constituting the wire. Let the differential segment be represented as $d\vec{s}$ (Figure)

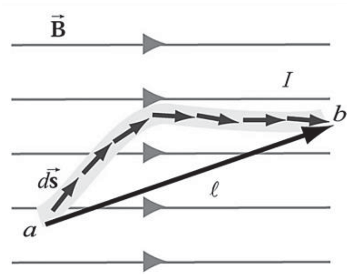


The magnetic force acting on the segment is : $dF_B = Ids \times B$

Thus, the total force is : $\vec{F}_B = I\int_a^b d\vec{s} \times B$

Where a and b represent the endpoints of the wire.

As an example, consider a curved wire carrying a current I in a uniform magnetic field $\vec{B}$ as show in figure.



Using the magnetic force on the wire is given by

$$\vec{F}_B = (I\int_a^b d\vec{s}) \times \vec{B} = I\vec{\ell} \times \vec{B}$$

Where $\vec{\ell}$ is the length vector directed from a to b. However, if the wire forms a closed loop of arbitrary shape (Figure), then the force on the loop becomes.

$$\vec{F}_B = I(\oint d\vec{s}) \times \vec{B}$$

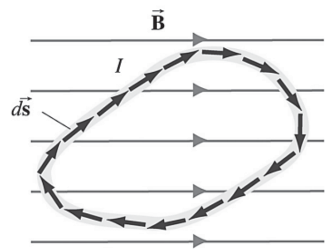
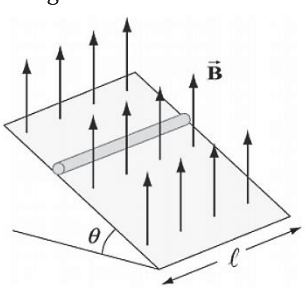


Figure: A closed loop carrying a current I in a uniform magnetic field

Since the det of differential length elements $d\vec{s}$ from a closed polygon, and their vector sum is zero, i.e. $\oint d\vec{s} = 0$. The net magnetic force on a closed loop is $\vec{F}_B = \vec{0}$

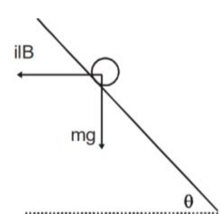
Ex. A conducting bar of length is placed on a frictionless inclined plane which is titled at an angle q from the horizontal, as shown in figure.



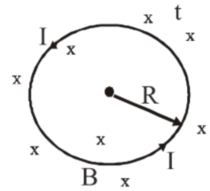
A uniform magnetic field is directed vertically. To impede the bar from descending, a voltage source is linked to the ends of the bar, allowing a current to flow through it. Ascertain the magnitude and direction of the current required for the bar to remain stationary.

Sol. For equilibrium $I\ell B\cos\theta = mg\sin\theta$

$$I = \frac{mg\sin\theta}{\ell B\cos\theta}$$

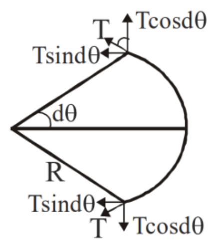


Ex. A current (I) carrying circular wire of radius R is placed in a magnetic field B perpendicular to its plane. Find the tension T along the circumference of the wire.

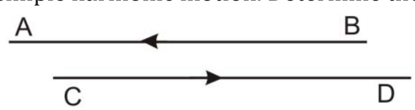


Sol. Sor small element portion

$$2T = d\theta IB$$
$$2Td\theta = 2RIBd\theta$$
$$T = IRB$$



Ex. A lengthy horizontal wire AB, capable of vertical movement, carries a constant current of 20 A and is in a stable position 0.01 m above another parallel wire CD. Wire CD is fixed in a horizontal plane and bears a constant current of 30 A, as depicted in the figure. Demonstrate that when AB is gently depressed, it undergoes simple harmonic motion. Determine the period of oscillation.



Sol. Let m be the mass per unit length of wire AB. At a height x about the wire CD, magnetic force per unit length on wire AB will be given by

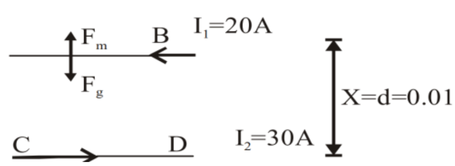
$$F_m = \frac{\mu_0 i_1 i_2}{2\pi x} \text{ (upwards)} \quad \ldots (1)$$

Wt. per unit of wire AB is

$F_g = mg$ (downwards)

At $x = d$, wire is in equilibrium

$$F_m = F_g \Rightarrow \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$
$$\frac{\mu_0 i_1 i_2}{2\pi d^2} = \frac{mg}{d} \quad \ldots (2)$$



When AB is depressed, x decreases therefore, F_m will increase, while F_g remains the same. Let AB is displaced by dx downwards. Differentiating equation (i) w.r.t. x, we get.

$$dF_m = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2} \cdot dx \quad \ldots (3)$$

i.e., restoring force, $F = d$ $F_m \propto -dx$

Hence the motion of wire is simple harmonic. From equation (2) and (3), we can write.

$$dF_m = -\left(\frac{mg}{d}\right) \cdot dx (x = d)$$

Acceleration of wire, $a = -\left(\frac{g}{d}\right) \cdot dx$

Hence period of oscillations

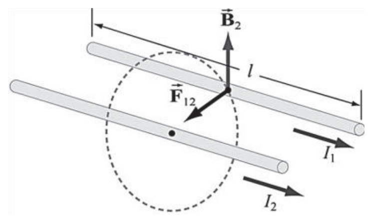
$$T = 2\pi\sqrt{\frac{dx}{a}} = 2\pi\sqrt{\left|\frac{disp.}{acc.}\right|}$$

$$T = 2\pi\sqrt{d/g} = 2\pi\sqrt{\frac{0.01}{9.8}}$$

$$T = 0.2\ s$$

Force between Two Parallel Wires:

As we've observed, a wire conducting an electric current generates a magnetic field. Moreover, when positioned within a magnetic field, a current-carrying wire undergoes a resultant force. Consequently, it is anticipated that two wires carrying currents will exert forces on each other. Contemplate a pair of parallel wires separated by a distance α and carrying currents I_1^2 and I_2^2 in the $+\xi$-direction, as shown in Figure.



The magnetic force $\vec{F}_{12}$, exerted by wire 2 on wire 1 may be computed as follows. Using the result from the previous example, the magnetic field lines due to I_2^2 going in the $+\xi$ direction are circle concentric with wire 2, with the field $\vec{B}_2$ pointing in the tangential direction. Thus, at an arbitrary point Π on wire 1, we have $\vec{B}_2 = -(\mu_0 I_2/2\pi a)\hat{j}$ which points in the direction perpendicular to wire 1, as depicted in Figure. Therefore,

$$\vec{F}_{12} = I_1\vec{\ell} \times \vec{B}_2 = (\ell\hat{i}) \times \left(-\frac{\mu_0 I_2}{2\pi a}\hat{j}\right) = -\frac{\mu_0 I_1 I_2 \ell}{2\pi a}\hat{k}$$

Clearly $\vec{F}_{12}$ The direction of the magnetic force between the wires is toward wire 2. The inference drawn from this straightforward calculation is that two parallel wires carrying currents in the same direction will mutually attract. Conversely, if the currents flow in opposite directions, the resultant force will be repulsive.

Definition of ampere

Consider two parallel wires separated by 1 m and carrying a current of 1A each. Then $i_1 = i_2 = 1A$ and d =1m, so that from equation

$$\frac{dF}{dl} = 2 \times 10^{-7} Nm$$

This is employed as the formal definition of the unit 'ampere' for electric current. When two parallel, extended wires, maintained 1 m apart in a vacuum, convey identical currents in the same direction, and there exists an attractive force of 2×10^{-7} newton per meter for each wire, the current in each wire is designated as 1 ampere.