FORCE ON A MOVING CHARGE IN MAGNETIC FIELD WHEN ANGLE BETWEEN B AND V IS θ

Magnetic Field of a Moving Point Charge

Consider an infinitesimal current element shaped like a cylinder with a cross-sectional area A and length ds. This cylinder comprises n charge carriers per unit volume, all moving with a collective velocity $\vec{v}$ along the cylinder's axis. Let l denote the current in the element, defined as the quantity of charge passing through any cross-section of the cylinder per unit time. It is evident that the current I can be expressed as:

$$nAq|\vec{v}| = I$$

The overall number of charge carriers within the current element can be expressed as $dN = nAds$. Consequently, the magnetic field is affected by this. $d\vec{B}$ due to the dN charge carries is given by.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(nAq|\vec{v}|)d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(nAds)q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{(dN)q\vec{v}\times\hat{r}}{r^2}$$

Here, r represents the distance from the charge to the field measurement point P, and the unit vector is involved. $\hat{r} = \vec{r}/r$ Points extend from the field source (the charge) to P. For the differential length vector $d\vec{s}$ is defined to align parallel $\vec{v}$ In case of a single charge, $dN = 1$, the above equation becomes.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Magnetic Force on Moving Charge

When a charge q travels with a velocity $\vec{v}$ perpendicular to a magnetic field $\vec{B}$, the magnetic force acting on the moving charge is determined by the following formula:

$$\vec{F} = q(\vec{V}\times\vec{B}) \text{ Put q with sign.}$$

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point

Note

- $\vec{F}\perp\vec{V}$ and also $\vec{F}\perp\vec{B}$
- $\because \vec{F}\perp\vec{v}$ $\therefore$ The power arising from the magnetic force acting on a charged particle is zero. (Utilize the power formula) $P = \vec{F}\cdot\vec{v}$ for its proof).
- Since the $\vec{F}\perp\vec{B}$ Therefore, the work performed by the magnetic force is zero throughout the entire motion. The magnetic force does not have the capability to alter the speed (or kinetic energy) of a charged particle; instead, it can only modify the direction of velocity.
- A stationary charged particle experiences no magnetic force.
- If $\vec{V}\parallel\vec{B}$ Even in this scenario, the magnetic force on the charged particle is nonexistent. The particle moves along a straight line only under the influence of the magnetic field.

Ex. A particle with a charge and a mass of 5 mg $q = +2\mu C$ has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ Determine the magnetic force acting on the charged particle and its acceleration at this moment due to the magnetic field. $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Sol. $\vec{F} = q\vec{v}\times\vec{B} = 2\times10^{-6}(2\hat{i} - 3\hat{j}\times 4\hat{k})\times(3\hat{j} - 2\hat{k}) = 2\times10^{-6}[-6\hat{i} + 4\hat{j} + 6\hat{k}]N$

By Newton's Law $\vec{a} = \frac{\vec{F}}{m} = \frac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k}) = 0.8(-3\hat{i} + 2\hat{j} + 3\hat{k})m/s^2$

Motion of charged particles under the effect of magnetic force

Particle released if $v = 0$ then $f = 0$

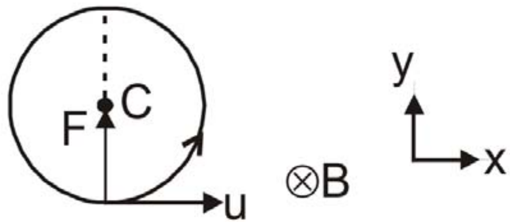
Particle will remain at rest

$$\vec{V}\parallel\vec{B} \text{ here } \theta = 0 \text{ or } \theta = 180°$$

$$F_m = 0 \therefore \vec{a} = 0 \therefore \vec{V} = \text{ const.}$$

Particle will move in a straight line with constant velocity

Initial velocity $\vec{U}\perp\vec{B}$ and $\vec{B} =$ uniform



In this instance, with Θ_B oriented in the z-direction, the magnetic force in the z-direction becomes negligible. ($\because \vec{F}_m \perp \vec{B}$). Now there is no initial velocity in z-direction.
Particle will always move in xy plane
velocity vector is always $\perp \vec{B} \therefore F_m = quB =$ constant

$$quB = \frac{mu^2}{R} \Rightarrow R = \frac{mu}{qB} = \text{ constant.}$$

The particle moves in a curved path whose radius of curvature is same everywhere, such curve in a plane is only a circle.
Path of the particle is circular.

$$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

Here p = linear momentum; k = kinetic energy

$$v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

Time period $T = 2\pi m/qB$
frequency $f = qB/2\pi m$
No τε: ω, f, T are independent of velocity.

Ex. A proton (p), an α-particle, and a deuteron (D) are traversing circular paths with identical kinetic energies within the same magnetic field. Determine the ratio of their radii and time periods. (Disregard interactions between particles).

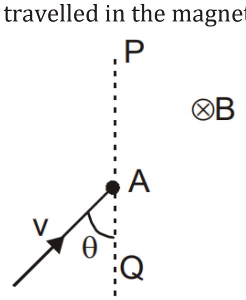
Sol.

$$R = \frac{\sqrt{2mK}}{qB}$$

$$R_p : R_a : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2\cdot 4mK}}{2qB} : \frac{\sqrt{2\cdot 2mK}}{qB} = 1 : 1 : \sqrt{2}$$

$$T = 2\pi m/qB = 1 : 2 : 2 \text{ Ans}$$

Ex. A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ. Find
1. Time spent, 2. Distance travelled in the magnetic field 3.Impulse of magnetic force.



Sol. The particle will move in the field as shown
Angle subtended by the arc at the centre = 2θ

1. Time spent by the charge in magnetic field

$$\omega t = \theta \Rightarrow \frac{qB}{m} t = \theta \Rightarrow t = \frac{m\theta}{qB}$$

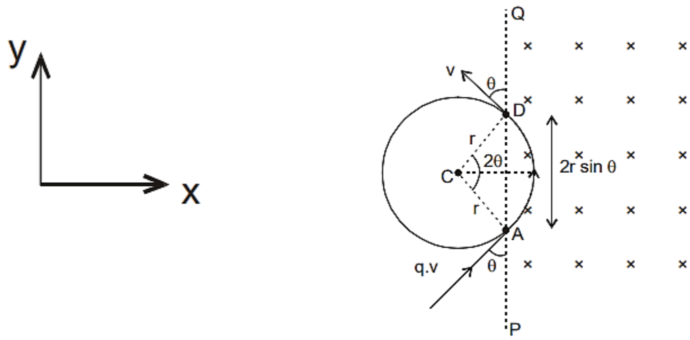
2. Distance travelled by the charge in magnetic field :

$$= r(2\theta) = \frac{mv}{qB} \cdot 2\theta$$

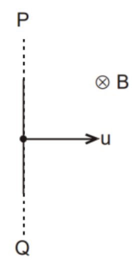
3. Impulse = change in momentum of the charge

$$= (-mv\sin\theta\hat{i} + mv\cos\theta\hat{j}) - (mv\sin\theta\hat{i} + mv\cos\theta\hat{j})$$

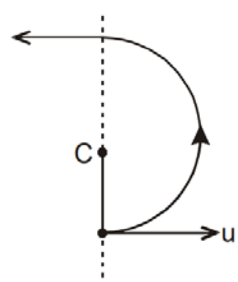
$$= -2mv\sin\theta\hat{j}$$



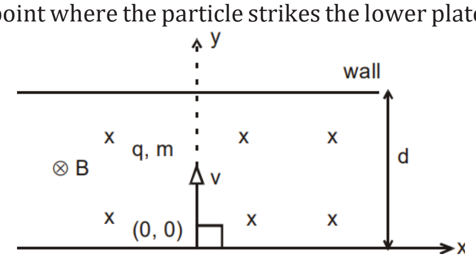
Ex. In the figure shown the magnetic field on the left on 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Sol. The path will be semicircular time spent $= T/2 = \pi m/qB$

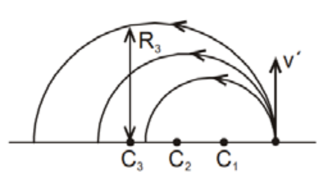


Ex. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



Sol. **1.** The path of the particle will be circular larger the velocity, larger will be the radius. For particle not to s. Strike $R < d$

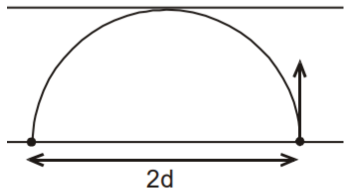
$$\frac{mv}{qB} < d \Rightarrow v < \frac{qBd}{m}$$



2. for limiting case $v = \frac{qBd}{m}$

$R = d$

Coordinate = (–2d, 0, 0)

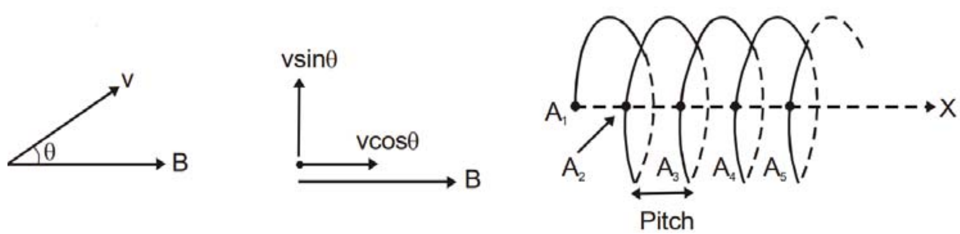


Helical path

When the charge's velocity is not perpendicular to the magnetic field, we have the option to decompose the velocity into two components. $-v_{\parallel}$, parallel to the field and $v_{\perp}$, , perpendicular to the field. The components v|| remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis

Suppose a particle possesses an initial velocity within the plane of the paper, and there is a consistent and uniform magnetic field also within the plane of the paper.



The particle starts from point A_1

It completes its one revolution at A_2 and 2^{nd} revolution at A_3 and so on. X – axis is the tangent to the h e l i x points

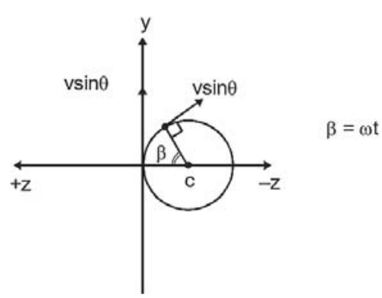
$A_1, A_0, A_2, \ldots..$ all are on the x-axis. distance $A_1A_2 = A_3A_4 = \cdots.. = v\cos\theta.T =$ pitch

Where T = Time period

Let the initial position of the particle be (0, 0, 0) and v sin θ in +y direction. Then

In x: $F_x = 0, a_x = 0, v_x =$ constant $= v\cos\theta, x = (v\cos\theta)t$

In y-z plane



From figure it is clear that

$$y = R\sin\beta, v_y = v\sin\theta\cos\beta$$

$$z = -(R - R\cos\beta)$$

$$v_z = v\sin\theta\sin\beta$$

acceleration towards centre $= (v\sin\theta)^2/R = \omega^2 R$

$$a_y = -\omega^2 R\sin\beta, a_z = -\omega^2 R\cos\beta$$

At any time : the position vector of the particle (or its displacement w.r.t. initial position)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, x, y, z \text{ already found}$$

$$\text{velocity } \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}, v_x, v_y, v_z \text{ already found}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}, a_x, a_y, a_z \text{ already found}$$

Radius $$q(v\sin\theta)B = \frac{m(v\sin\theta)^2}{R} \Rightarrow R = \frac{mv\sin\theta}{qB}$$

$$\omega = \frac{v\sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f.$$

Ex. A charged particle P leaves the origin with speed $v = v_e$, at some inclination with the x-axis. There is uniform magnetic field B along the x-axis. P strikes a fixed target T on the x-axis for a minimum value of $B = B_0$. Find the condition so that P will also strike if you change magnetic field and speed.

Sol. Let d = distance of the tangent T from the point of projection. P will strike T if d an integral multiple of the pitch. Pitch

$$(2\pi\frac{m}{qB_0})v_0\cos\theta = N(2\pi\frac{m}{qB})v\cos\theta$$

Here N is a natural number