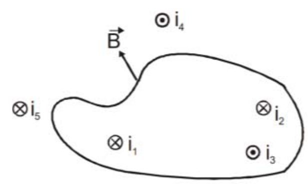
AMPERE'S CIRCUITAL LAW

Similar to Gauss's law in electrostatics, this principle offers a straightforward approach for determining magnetic fields in situations characterized by symmetry. Ampere's law, on the other hand, presents an alternative means of computing the magnetic field resulting from a specified current distribution.

Statement: The circulation $\oint \vec{B} \cdot d\vec{l}$ The magnetic field resulting from a closed circuit or an infinite wire with a steady current, along a closed path known as an Amperian path, is equivalent to μ_0 times the total current passing through the area enclosed by the closed curve. This holds true as long as the electric field within the loop remains constant.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$



In the diagram, the positive side is oriented into the plane, making i_1 and i_2 positive while i3 is negative. Consequently, the overall current passing through the area is $i_1 + i_2 - i_3$ any current outside the area is not included in writing the right - hand side of equation. The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the currents existing anywhere.

Ampere’s law may be derived from the Biot-Savart law may be derived from the Ampere’s law. Thus, the two are equivalent in scientific content. However, Ampere’s law is useful under contain symmetrical conditions.

Justification of Ampere’s law

Consider a long straight wire with an upward current I. Now, select a circular path with a radius symmetric to the wire. Let's divide this circular path of radius r into numerous small length vectors. $\Delta\vec{s} = \Delta s\hat{\phi}$, where $\hat{\phi}$ point along the Tangential direction with magnitude Ds (Figure).

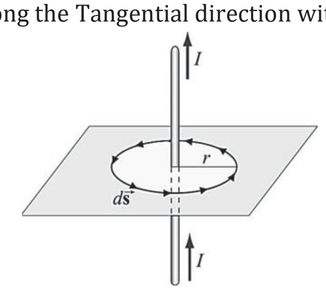


Figure: Amperian loop

In the limit $\Delta\vec{s} \to \vec{0}$, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \vec{B}\oint ds = \left(\frac{\mu_0 I}{2\pi r}\right) = \mu_0 I$$

The result above is obtained by choosing a closed path, or an “Amperian loop” that follows one particular magnetic field line. Let’s consider a slightly more complicated Amperian loop, as that shown in Figure.

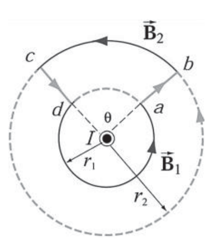


Figure: An Amperian loop involving two field lines

The line integral of the magnetic field around the contour abcda is

$$\oint_{abcda} \vec{B}\cdot d\vec{s} = \oint_{ab} \vec{B}\cdot d\vec{s} + \oint_{bc} \vec{B}\cdot d\vec{s} + \oint_{cd} \vec{B}\cdot d\vec{s} + \oint_{da} \vec{B}\cdot d\vec{s} = 0 + B_2(r_2\theta) + 0 + B_1[r_1(2\pi - \theta)]$$

Where the length of arc bc is $r_2\theta$ and $r_1(2\pi - \theta)$ for arc da. The first and the third integrals vanish since the magnetic field is perpendicular to the paths of integration. With

$$\oint_{abcda} \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi r^2}(r^2\theta) + \frac{\mu_0 I}{2\pi r_1}[r_1(2\pi - \theta)] = \frac{\mu_0 I}{2\pi}\theta + \frac{\mu_0 I}{2\pi}(2\pi - \theta) = \mu_0 I$$

We see that the same result is obtained whether the closed path involves one or two magnetic field lines. As shown above example, in polar coordinates (r,φ) with current flowing in the +z axis, the magnetic field is given by $\vec{B} = (\mu_0 I/2\pi r)\hat{\varphi}$ An arbitrary length element in the polar coordinates can be written as.

$$d\vec{s} = dr\hat{r} + rd\phi\hat{\phi}$$

Which implies

$$\oint_{\text{closed path}} \vec{B}\cdot d\vec{s} = \oint_{\text{closed path}} \left(\frac{\mu_0 I}{2\pi r^2}\right) rd\varphi = \frac{\mu_0 I}{2\pi}\oint_{\text{closed path}} d\varphi = \frac{\mu_0 I}{2\pi}(2\pi) = \mu_0 I$$

In other words, the line integral of $\oint \vec{B}\cdot d\vec{s}$ around any closed Amperian loop is proportional to I_{enc} , the current encircled by the loop.

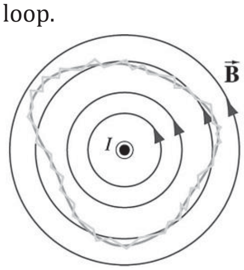


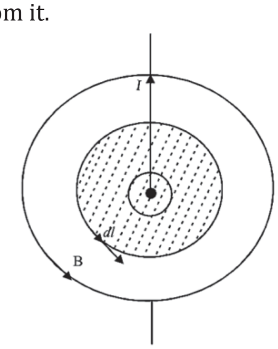
Figure: an Amperian loop of arbitrary shape

The extension to any closed loop of arbitrary shape, which encompasses numerous magnetic field lines (refer to, for instance, Figure), is recognized as Ampere's law.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$$

Calculation of magnetic field due to long straight wire

The illustration depicts a long straight current i. Our objective is to determine the magnetic field at a point P situated at a distance r from the wire. The figure presents the scenario in the plane perpendicular to the wire and passing through P. The current is oriented perpendicular to the diagram's plane, emanating from it.



Consider drawing a circle that intersects the given point, with the wire serving as its axis. Indicate the positive direction of the circle with an arrow. The circle's radius is denoted as r. The magnetic field resulting from the long, straight current at any point on the circle aligns with the tangent, as illustrated in the figure. This alignment is consistent with the direction of the length-element dI. Due to symmetry, all points on the circle are equivalent, implying that the magnetic field's magnitude should be uniform across these points. The circulation of the magnetic field along the circle is.

$$\oint \vec{B}\cdot\vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

The current crossing the area bounded by the circle is

$$\sum I_{en} = +I$$

Thus, from Ampere's law $$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

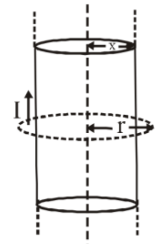
Magnetic field due to a current carrying thin long pipe

Case I : $r > R$ $$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

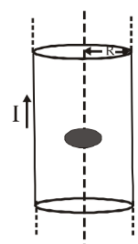


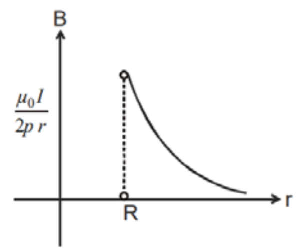
Case II : $r < R$ $$\oint \vec{B} \cdot \vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = 0$$

$$B(2\pi r) = \mu_0(0)$$

$$B = 0$$





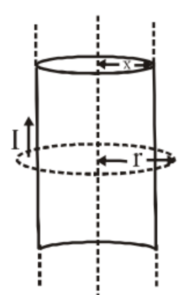
Magnetic field due to a current carrying rod having uniform current density

Case I : $r > R$ $$\oint \vec{B} \cdot \vec{dl} = \oint B \cdot dl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = +I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

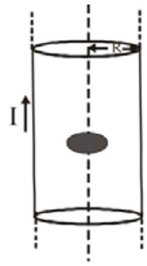


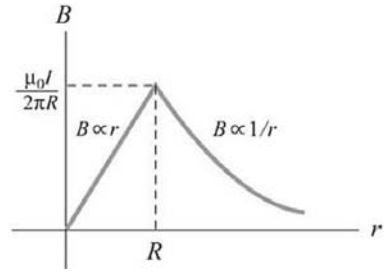
Case II : $r < R$ $$\oint \vec{B} \cdot \vec{dl} = \oint Bdl = B\oint dl = B(2\pi r)$$

$$\sum I_{en} = \frac{I}{R^2} r^2$$

$$B(2\pi r) = \mu_0 \frac{1}{R^2} r^2$$

$$B = \frac{\mu_0 I}{2\pi R^2} r$$





Magnetic Field Due to non uniform current density

Suppose that the current density in a wire of radius a varies with r according to $J = Kr^2$, where K is a constant and r is the distance from the axis of the wire. We have to find the magnetic field at a point distance r from the axis when (a) $r < a$ and $r > a$.

Choose a circular path centered on the axis of the conductor and apply Ampere's law.

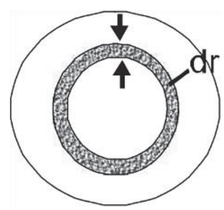
(a) To find the current passing through the area enclosed by the path integrate

$$dI = JdA = (Kr^2)(2\pi r dr)$$

$$I = \int dl = K\int_a^r 2\pi r^3 dr = \frac{K\pi r^4}{2}$$

$$\text{Since } \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$B2\pi r = \mu_0 \cdot \frac{\pi K r^4}{2} \Rightarrow B = \frac{\mu_0 K r^3}{4}$$

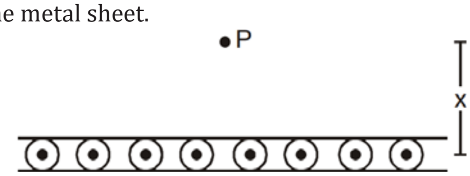


(b) If $r > a$, then net current through the Amperian loop is

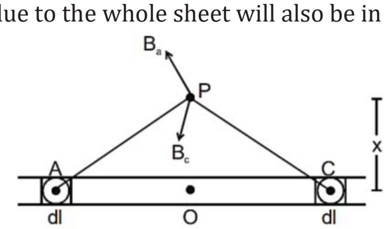
$$I = \int_a^r Kr^2 2\pi r dr = \frac{\pi K a^4}{2}$$

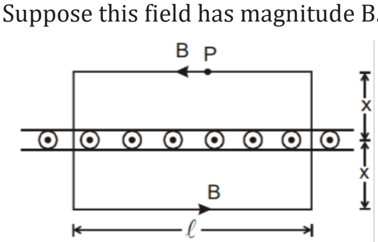
$$B = \frac{\mu_0 K a^4}{4r}$$

Ex. Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is λdl where λ is a constant. Find the magnetic field at a point P at a distance y from the metal sheet.



Sol. Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.





The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\ell = \mu_0\lambda\ell \text{ or, } B = \frac{1}{2}\mu_0\lambda.$$

Note that it is independent of y.

Ex. Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 4 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only

Sol. As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$ Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 2T.

Magnetic Field due to long Solenoid:

A solenoid is a helically wound long coil of wire, illustrating the magnetic field lines when carrying a constant current I. When the turns are closely packed and the length of the solenoid significantly exceeds its diameter, the resulting magnetic field inside becomes remarkably uniform. In the case of an "ideal" solenoid, characterized by infinite length and tightly packed turns, the magnetic field inside is both uniform and parallel to the axis, while it dissipates outside the solenoid.

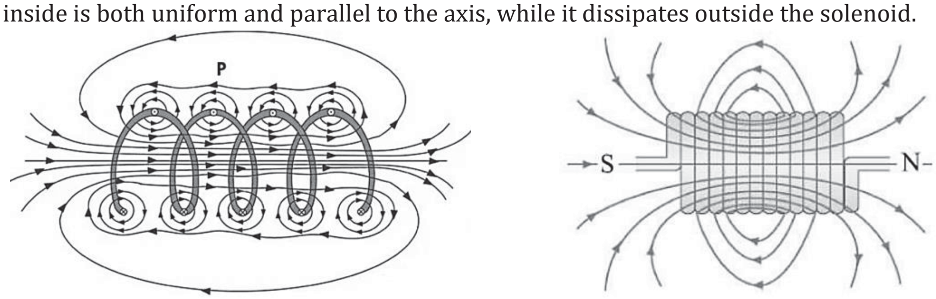


Figure: Magnetic field lines of a solenoid

Ampere's law can be employed for determining the magnetic field strength within a perfect solenoid. The diagram in the figure provides a cross-sectional representation of an ideal solenoid, serving as a basis for the computation. $\vec{B}$ We examine a rectangular route with dimensions length (l) and width (w), moving along the path in a counterclockwise direction. The line integral of $\vec{B}$ along this loop is.

$$\oint \vec{B}\cdot d\vec{s} = \oint_1 \vec{B}\cdot d\vec{s} + \oint_2 \vec{B}\cdot d\vec{s} + \oint_3 \vec{B}\cdot d\vec{s} + \oint_4 \vec{B}\cdot d\vec{s}$$
$$= 0 + 0 + B\ell + 0$$

In the aforementioned context, the contributions along sides 2 and 4 amount to zero due to $\vec{B}$ is perpendicular to $d\vec{s}$. In addition, $\vec{B} = \vec{0}$ alongside 1, as the magnetic field exists solely within the confines of the solenoid. Conversely, the aggregate current enclosed by the Amperian loop is $I_{enc} = n/I$, where n is the total number of turns per unit length. Applying Ampere's law yields.

$$\oint \vec{B}\cdot d\vec{s} = B\ell = \mu_0 n\ell I$$
$$B = \mu_0 nI$$

Magnetic Field due to Toroid

Examine a toroid comprising N turns, depicted in the figure, and determine the magnetic field at every point.

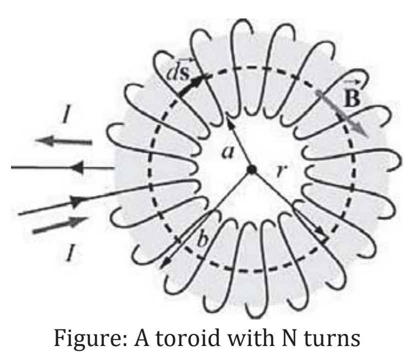


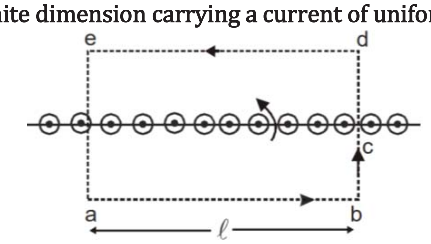
Figure: A toroid with N turns

A toroid can be conceptualized as a solenoid wrapped around with its ends connected. Consequently, the magnetic field is entirely contained within the toroid, with the field oriented in the azimuthal direction (clockwise, as determined by the direction of the current flow, as illustrated in the figure).
Applying Ampere's law, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \oint Bds = B\oint ds = B(2\pi r) = \mu_0 nI$$
$$B = \frac{\mu_0 NI}{2\pi r}$$

Here, r represents the distance measured from the center of the toroid. In contrast to the magnetic field of a solenoid, the magnetic field within the toroid is non-uniform and diminishes proportionally to 1/r.

Magnetic Field thin sheet of infinite dimension carrying a current of uniform linear current density i



$$\oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^e \vec{B} \cdot \vec{dl} + \int_e^f \vec{B} \cdot \vec{dl} + \int_f^g \vec{B} \cdot \vec{dl}$$
$$\int_a^b B \cdot \cos\theta \cdot dl + 0 + 0 + \int_d^e Bdl + 0 + 0 = \int_a^b Bdl + B\int_d^e dl = Bl + Bl = 2Bl$$
$$\sum I_{en} = il$$
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \sum I_{en}$$
$$2BI = \mu_0 il$$
$$B = \frac{\mu_0 I}{2}$$