TEMPERATURE DEPENDENT RESISTANCE AND SIMPLE CIRCUIT THEORY

Temperature dependent resistivity for conductors and semi-conductors

Conductors	Semiconductors
As $T\uparrow, \tau\downarrow$ $m, n, e =$ constant	As $T\uparrow, \tau\downarrow, n\uparrow$ $m, e =$ constant
When the temperature of a conductor rises, the thermal energy of the electrons within it increases, leading to greater randomness in their movement. Consequently, the average time interval between two consecutive collisions, denoted as τ, decreases.	With the increase in temperature of a semiconductor, there is a rise in the quantity of free electrons present within it, consequently elevating the free electron density (n). As a result of this temperature increase, the average time interval between two consecutive collisions, denoted as τ, decreases.
$T\uparrow \Rightarrow \tau\downarrow \Rightarrow \rho\uparrow \Rightarrow R\uparrow$	$T\uparrow \Rightarrow n\uparrow$ but $\tau\downarrow \Rightarrow$ But increase of n dominates $\Downarrow$ $P\downarrow R\downarrow$

Temperature dependence for conductors

The original resistivity of the conductor: ρ_i

The starting temperature of the conductor: T_i

Consider increasing the temperature of the conductor by ΔT.

Final resistivity of the conductor: ρ_f

Final temperature of the conductor: $T_f = T_i + \Delta T$

The change in the resistivity ($d\rho = \rho_f - \rho_i$) the variation in the conductor's temperature relies on the subsequent factors:

$$d\rho \propto dT$$
$$d\rho \propto \rho_i$$

Therefore $$d\rho = \alpha\rho_i dT \qquad \ldots (1)$$

$\alpha =$ Coefficient of thermal resistance

At any given moment, if the resistivity of the conductor is denoted as ρ and the change in temperature is represented by dt, then according to relation (1), we obtain:

$$d\rho = \alpha\rho dT \Rightarrow \int_{\rho_i}^{\rho_f} \frac{d\rho}{\rho} = \int_{T_i}^{T_f} \alpha dT \Rightarrow \ln\left(\frac{\rho_f}{\rho_i}\right) = \alpha(T_f - T_i)$$

$$\ln\left(\frac{\rho_f}{\rho_i}\right) = \alpha\Delta T \Rightarrow \rho_f = \rho_i e^{\alpha\Delta T}$$

$$\rho_f = \rho_i e^{\alpha\Delta T}$$

This applies to a notable alteration in temperature, with α remaining constant.

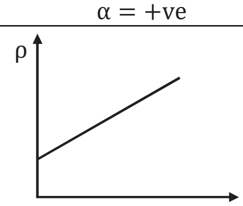
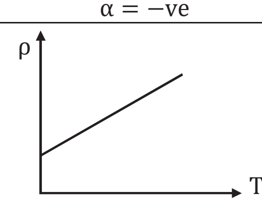
Considering the typical range of α is 10^{-2} to 10^{-3} and in all practical cases $\Delta T < 100°C$, thus, αdT is small quantity.

$$\rho_f = \rho_i e^{\alpha\Delta T} \xrightarrow{\alpha dT \text{ is a small}} \rho_f = \rho_i[1 + \alpha\Delta T]$$

$\rho_i =$ Resistivity at temperature T_i,

$\rho_f =$ Resistivity at temperature T_f,

$\alpha =$ Temperature coefficient of resistivity

Conductors	Semiconductors
$\alpha = +ve$	$\alpha = -ve$
	
Resistivity increases with increase in temperature.	Resistivity decreases with increase in temperature.

If α is constant:
$\rho_f = \rho_i e^{\alpha \Delta T}$

If α is constant and αdT is small:
$\rho_f = \rho_i[1 + \alpha\Delta T]$

If α varies with temperature:

$$\int_{\rho_i}^{\rho_f} \frac{d\rho}{\rho} = \int_{T_i}^{T_f} \alpha(T)dT \Rightarrow \ln\left(\frac{\rho_f}{\rho_i}\right) = \int_{T_i}^{T_f} \alpha(T)dT \Rightarrow \rho_f = \rho_i e^{\int_{T_i}^{T_f} \alpha(T)dT}$$

If α varies with temperature and $\int_{T_j}^{T_f} \alpha(T)dT$ is small:

$$\rho_f = \rho_i[1 + \int_{T_i}^{T_f} \alpha(T)dT]$$

Resistivity of a conductor at temperature T

$$\rho_f = \rho_i[1 + \alpha\Delta T]$$

Resistance of a conductor at temperature T

$$R_f = R_i[1 + \alpha\Delta T]$$

Change in length and area is negligible effect as compound to ρ
We disregard the influence of length and area due to the coefficient of thermal expansion being smaller than the coefficient of thermal resistance.
For semiconductors formulae are same but,

$$\alpha = -ve$$

Ex. A copper rod has a resistance of 10 Ω at 10°C. What will be its resistance at 50°C, given $\alpha = 2 \times 10^{-3}\ {}^\circ C^{-1}$

Sol. Given:
Initial temperature of the conductor: $T_i = 10^\circ C$
Final temperature of the conductor: $T_f = 50^\circ C$
Initial resistance of the conductor: $R_i = 10\Omega$
Coefficient of thermal resistance: $\alpha = 2 \times 10^{-3}\ {}^\circ C^{-1}$
The resistance of the copper rod at 50°C will be,

$$R_f = R_i[1 + \alpha\Delta T]$$
$$R_f = 10(1 + 2 \times 10^{-3} \times 40)$$
$$[\text{ Since } \Delta T = T_f - T_i = 40^\circ C]$$
$$= 10(1 + 0.08)$$
$$= 10 \times 1.08 = 10.8\Omega$$
$$R_f = 10.8\Omega$$

Ex. A semiconductor rod has a resistance of 10 Ω at 10°C. What will be its resistance at 50°C, given $\alpha = -2 \times 10^{-3}\ {}^\circ C^{-1}$.

Sol. Given:
Initial temperature of the conductor: $T_i = 10^\circ C$
Final temperature of the conductor: $T_f = 50^\circ C$
Initial resistance of the conductor: $R_i = 10\Omega$
Coefficient of thermal resistance: $\alpha = -2 \times 10^{-3}\ {}^\circ C^{-1}$
The resistance of the copper rod at 50°C will be,

$$R_f = R_i[1 + \alpha\Delta T]$$
$$R_f = 10(1 - 2 \times 10^{-3} \times 40)$$
$$[\text{ Since } \Delta T = T_f - T_i = 40^\circ C]$$
$$= 10(1 - 0.08)$$
$$= 9.2\Omega$$
$$R_f = 9.2\Omega$$

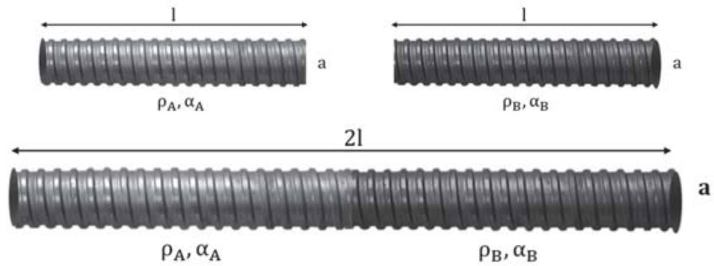
Ex. For a material α varies with temperature as $\alpha = AT^2 + BT^3$. What will be its resistance at T°C, if at 0°C its resistance is R_0.

Sol. Given:
Initial temperature of the conductor: $T_i = 0^\circ C$
Final temperature of the conductor: $T_f = T^\circ C$
Initial resistance of the conductor: $R_i = R_o$
Coefficient of thermal resistance: $\alpha = AT^2 + BT^3$ (Temp. dependent)
Here, α varies with temperature and assume $\int_{T_i}^{T_f} \alpha(T)dT$ is small. Therefore,

$$R_f = R_0(1 + \int_t^{T_t} \alpha(T) \cdot dT).$$
$$= R_0[1 + \int_{T_i}^{T_i}(AT^2 + BT^3) \cdot dt -$$
$$= R_0[1 + \frac{AT^3}{3} + \frac{BT^4}{4}].$$
$$R_f = R_o[1 + \frac{AT^3}{3} + \frac{BT^4}{4}]$$

Ex. Two rods A, B of different materials having same length l and cross-sectional area a are joined such that the resistance of resulting material doesn't vary with temperature. Find the relation between (ρ_A, α_A) and (ρ_B, α_B)

Sol.



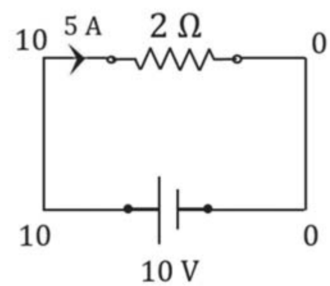
The rods are joined in such a way that the resistance of resulting material i.e., the equivalent resistance of the configuration doesn't vary with temperature. Therefore, the equivalent resistance of the rods before and after an increase of temperature should be same.

$$\frac{\rho_A l}{Al} + \frac{\rho_B l}{A} = \frac{\rho_A l}{A}(1 + \alpha_A \Delta T) + \frac{P_B l}{Al}(1 + \alpha_B \Delta T)$$

[Since the connection of the rods are same as the series connection of the resistances, the equivalent resistance will be, $R_{eq} = R_A + R_B$]

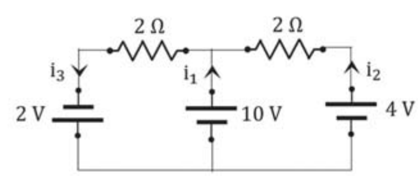
$$\rho_{/A} + p_B = \rho_A/\rho_A \alpha_A \Delta T + \rho_B + \rho_B \alpha_B \Delta T$$
$$\rho_A \alpha_A + \rho_B \alpha_B = 0$$

Simple Circuit Theory



There is no voltage drop across the connecting wires.
Choose a single point as the reference point.
Set zero potential at the negative terminal of the battery. If the battery's potential difference is x V, then the potential at the positive terminal of the battery will be (0 + x) = xV
There will be no alteration in current as it flows through the battery.

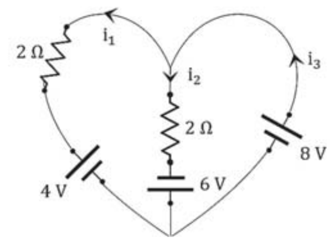
Ex. Find the current in each branch of the circuit.



Sol. Consider the zero potential at the lower conducting wire AFE.
The potential at point B: (0 − 2) = −2 V
The potential at point C: (0 + 10) = +10 V
The potential at point D: (0 + 4) = +4 V
The potential difference across the 2 Ω resistance connected between point B and C is, [10 − (−2)] = 12 V Therefore, the current through it: $\frac{12}{2} = 6A$
The potential difference across the 2 Ω resistance connected between point C and D is, [10 − 4] = 6 V Therefore, the current through it $\frac{6}{2} = 3A$
Therefore,
The current from B to A is: $i_3 = 6A$
The current from D to E is: $i_2 = 3A$
Since the current obeys algebraic addition, the current from F to C will be: $i_1 = i_2 + i_3 = 9A$

$$i_1 = 9A, i_2 = 3\,A, i_3 = 6\,A$$

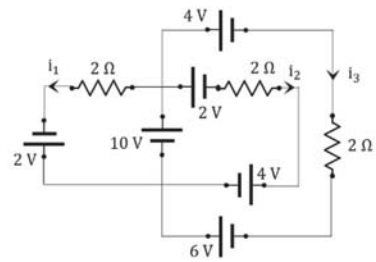
Ex. Find the current in each branch of the circuit.



Sol. Consider the zero potential at point A. Thus,
The potential at point B: $(0+4)=4$ V
The potential at point E: $(0-6)=-6$ V
The potential at point D: $(0+8)=+8$ V
Since there is no resistance between point C and D, these two points will have same potential. Thus, the potential at point C: +8 V
The potential difference across the 2 Ω resistance connected between point B and C is, $[8-4]=4$ V Therefore, the current through it from C to B: $i_1=\frac{4}{2}=2A$
The potential difference across the 2 Ω resistance connected between point C and E is, $[8-(-6)]=14$ V Therefore, the current through it from C to E: $i_2=\frac{14}{2}=7$ A
Since the current obeys algebraic addition, the total current at point A flowing towards D will be:

$$i_3=i_1+i_2=(2+7)=9A$$
$$i_1=2A, i_2=7A, i_3=9A$$

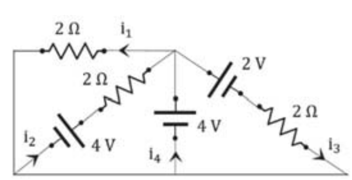
Ex. Find the current in each branch of the circuit.



Sol. Consider the zero potential at point A. Since there is no resistance between point A and B, these two points will have same potential. Thus, the potential at point B: 0 V.
If we go from positive terminal of the battery to the negative terminal of the battery, then potential at the negative terminal = (potential of the positive terminal − potential difference of the battery)
If we go from negative terminal of the battery to the positive terminal of the battery, then potential at the positive terminal = (potential of the negative terminal + potential difference of the battery)
Therefore,
The potential at point C: $(0-2)=-2$ V
For the battery having potential difference 10 V, the potential at point A is 0 V. Thus, potential at point D: $(0+10)=10$ V
Since there is no resistance between point D and E, these two points will have same potential. Thus, the potential at point E: 10 V.
Thus, the potential at point F: $(10-4)=6$ V
Since the potential at point D is 10 V, the potential at point H$(10-2)=8$ V
Since the potential at point A is 0 V, the potential at point K$(0+4)=4$ V
Since there is no resistance between point K and L, these two points will have same potential.
Thus, the potential at point L :4 V
Since the potential at point A is 0 V, the potential at point G :$(0-6)=-6$ V
The potential difference across the 2 Ω resistance connected between point D and C is, $[10-(-2)]=12$V Therefore, the current through it from D to C: $i_1=\frac{12}{2}=6$ A
The potential difference across the 2 Ω resistance connected between point H and L is, $[8-4]=4$V Therefore, the current through it from H to : $i_2=\frac{4}{2}=2$ A
The potential difference across the 2 Ω resistance connected between point F and G is $[6-(-6)]=12$V therefore, the current through it from H to L $i_3=\frac{12}{2}=6$ A

$$i_1=6\text{ A}, i_2=2\text{ A} i_3=6\text{ A}$$

Ex. Find the current in each branch of the circuit.

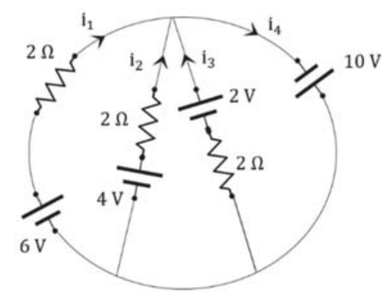


Sol. Consider the zero potential at point A. Since there is no resistance between point A, B, F and E, these points will have same potential. Thus, the potential at point B, F a nd E: 0 V Now,

we have the potential at one end of each battery. So, it's time to find out the potential at the other end of the battery.
Potential at the negative terminal = (potential of the positive terminal − potential difference of the battery)
Potential at the positive terminal = (potential of the negative terminal + potential difference of the battery)
Therefore
The potential at point G: $(0 + 4) = 4$ V
The potential at point C: $(0 + 4) = 4$V
Since the potential at point C is 4V, the potential at point D: $(4 - 2) = 2$ V
The potential difference across the 2Ω resistance connected between point B and C is $[4 - 0] = 4$V
Therefore, the current through it from C to B: $i_1 = \frac{4}{2} = 2$A
[Since C has higher potential than B, the current will flow from C to B]
The potential difference across the 2Ω resistance connected between point G and C is $[4 - 4] = 0$ V
Therefore, the current through it : $i_2 = \frac{0}{2} = 0$ A
The potential difference across the 2Ω resistance connected between point D and E is, $[2 - 0] = 2$V. therefore the current through is from D to E $i_3 = \frac{2}{2} = 1$A [Since D has higher potential than E the current will flow from D to E.]
Since $i_1 = 2$A and $i_2 = 0$ the current in the branch AF is 2A; and since $i_3 = 1$A the current in the branch EF is 1 A. thus total current at point F is $i_4 = (2 + 1) = 3$ A

$$i_1 = 2A, i_2 = 0, i_3 = 1A, i_4 = 3A$$

Ex. Find the current in each branch of the circuit.

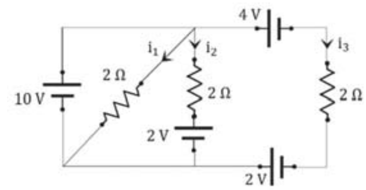


Sol. Consider the zero potential at point A. Since there is no resistance between point A, B and F, these points will have same potential. Thus, the potential at point B and F: 0 V.
Since the potential at point B is V the potential at point D: $(0 + 4) = 4$ V and the potential at point C: $(0 + 6) = 6$ V
Since the potential at point F is 0 V to go at point G from point F we have to go from positive terminal to negative terminal of the 10 V battery the potential at point G : $(0 - 10) = -10$V since the potential at point G is –10 V the potential at point E :$(-10 - 2) = -12$ V
The potential difference across the 2 Ω resistance connected between point C and G is, $[6 - (-10)] = 16$V Therefore, the current through it from C to G : $i_1 = \frac{16}{2} = 8$A [Since *C* has higher potential than G, the current will flow from C to G]
Similarly, the potential difference across the 2 Ω resistance connected between point D and G is, $[4 - (-10)] = 14$ V
Therefore, the current through it from D to G : $i_2 = \frac{14}{2} = 7$A
The potential difference across the 2 Ω resistance connected between point H and E is, $[0 - (-12)] = 12$V Therefore, the current through it from H to E : $i_3 = \frac{12}{2} = 6$A [Since A (or, H) has higher potential than E, the current will flow from H to E]
Since all the currents i_1, i_2 and i_3 are towards point G, the total current at point F is, $i_4 = (8 + 7 + 6) = 21$ A

$$i_1 = 8A, i_2 = 7A, i_3 = 6A, i_4 = 21A$$

Ex. Find the current in each branch of the circuit.

Sol. Consider the zero potential at point A (Most of the time, try to set zero potential at negative terminal of a battery). Since there is no resistance between point A and F, the potential at point F: 0 V.



Since the potential at point A is 0 V, the potential at point B: $(0 + 10) = 10V$

Since there is no resistance between point B and C, the potential at point C: 10 V

Since the potential at point C is 10 V, the potential at point D: $10 - 4 = 6$ V

Since the potential at point F is 0 V, the potential at point G: $0 + 2 = 2$ V and the potential at point E: $0 - 2 = -2$ V

The potential difference across the 2 Ω resistance connected between point A and C is, $10 - 0 = 10$ V. Therefore, the current through the resistance from C to A : $i_1 = \frac{10}{2} = 5$ A

The potential difference across the 2 Ω resistance connected between point C and G is, $10 - 2 = 8$ V. Therefore, the current through the resistance from C to G: $i_2 = \frac{8}{2} = 4A$

The potential difference across the 2 Ω resistance connected between point D and E is, $[6 - (-2)] = 8V$ Therefore, the current through the resistance from D to E : $i_3 = \frac{8}{2} = 4$ A

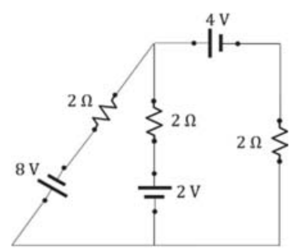
Since the currents i_2 and i_3 are towards point F, the total current at point F is, $(4 + 4) = 8A$

Since this 8 A current and the current i_1 are towards point A, the total current at point A is, $(8 + 5) = 13A$

$$i_1 = 5A, i_2 = 4A, i_3 = 4A$$

Ex. Find the electric current in the circuit shown.

Sol. Consider the zero potential at point A. Since there is no resistance between point A, F and E, the potential at point F and E: 0 V



Since the potential at point A is 0 V, the potential at point B: $(0 + 8) = 8V$

Since the potential at point F is 0 V, the potential at point G: $(0 + 2) = 2$ V

Now, to go at point C from any point, we have to cross at least one resistance. Therefore, we can't find the potential at C with knowledge that we have till now. We must know the 'Kirchhoff's current law' to find the potential at point C.

We will be able to solve this problem without using 'Kirchhoff's current law' if there is only one battery in ant one branch of the given circuit.