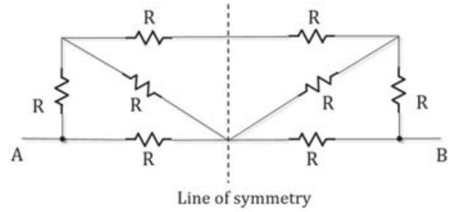
SYMMETRICAL CIRCUITS, MIRROR AND FOLDING SYMMETRY

Symmetrical Circuits, mirror and folding symmetry

When the distribution of resistances across a line in a circuit is symmetrical, it's referred to as mirror symmetry. In a circuit with line symmetry, the currents flowing through identical branches on either side of the line are equal. In mirror-symmetric circuits, certain junctions can be eliminated. After such removal, the circuit can be depicted as illustrated.

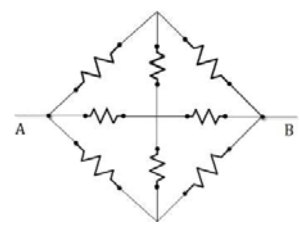


Equivalent resistance of the circuit is,

$$R_{eq} = \frac{6R}{5}\,\Omega$$

Ex. If each of the resistances in the circuit is R, then find equivalent resistance between terminals A and B.

Sol. Here, the resistances above and below line AB are similar. Thus, the values of potential are same at junction which can be connected after folding the circuit about line AB.



After folding, the resultant circuit becomes as shown below:

This is a balanced Wheatstone bridge.

Equivalent resistance of balanced Wheatstone bridge,

$$R_{AB} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

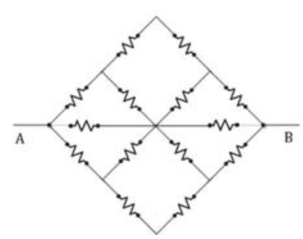
$$= \frac{(\frac{R}{2}+\frac{R}{2})\times(R+R)}{\frac{R}{2}+R+\frac{R}{2}+R}$$

$$= \frac{(R)\times(2R)}{3R} = \frac{2R}{3}$$

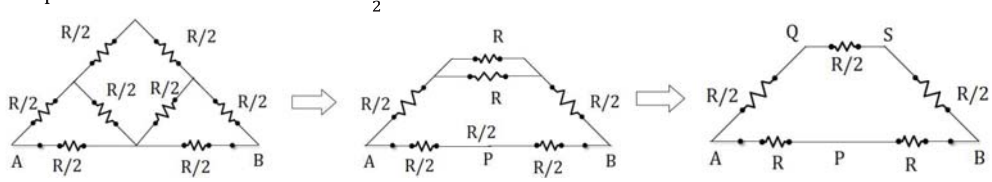
$$R_{eq} = \frac{2R}{3}\,\Omega$$

Ex. If each of the resistances in the circuit is R, then find equivalent resistance between terminals A and B.

Sol. The AB and CD lines possess folding and mirror symmetry. Using folding symmetry with AB as the folding axis, we get the following figure:



After folding the circuit w.r.t the folding axis AB, the resistances which get overlapped with each other are essentially parallel to each other. Since the value of each resistance is R, equivalent resistance of those parallel combination becomes $\frac{R}{2}$.



Applying mirror symmetry on the adjacent circuit w.r.t CP, we can say that current through AP and PB is same. Therefore, we can assume that there is no junction at point P and the circuit can be simplified as shown below:

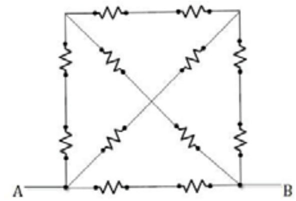
The resistance of the branch AQSB: $\frac{3R}{2} = 1.5R$

The resistance of the branch APB: 2R

Therefore, equivalent resistance: $R_{eq} = \frac{1.5R.2R}{3.5R} = \frac{6R}{7}$

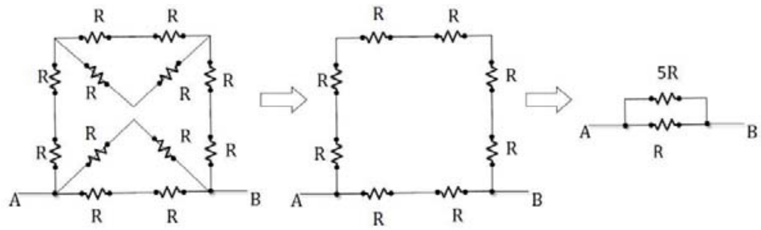
$$R_{eq} = \frac{6R}{7}\,\Omega$$

Ex. If each of the resistances in the circuit is R, then find equivalent resistance between terminals A and B.



Sol. The line AB doesn't act as the folding axis but the line CD possesses mirror symmetry.

Therefore, the junction P can be omitted as shown in the adjacent figure. Since the value of each resistance is R, the equivalent resistance of the encircled portion in the adjacent figure is R.

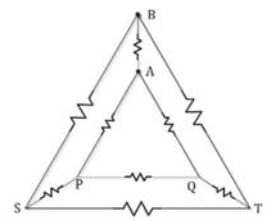


Therefore, equivalent resistance:

$$R_{eq} = \frac{5R.R}{6R} = \frac{5R}{6}$$

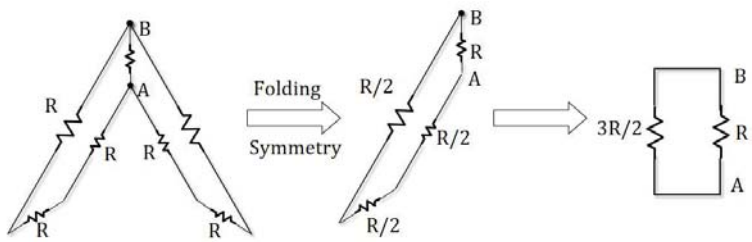
$$R_{eq} = \frac{5R}{6}$$

Ex. If each of the resistances in the circuit is R, then find equivalent resistance between terminals A and B.



Sol. No mirror symmetry exits in the problem but any line passing through AB acts as the folding axis. Therefore, the point P and Q will have same potential. The same is true for S and T.

Hence, no current will pass through the resistance connected between 'P and Q' and 'S and T'. So, those two resistances can be removed from the circuit, as shown below.



Therefore, equivalent resistance:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{2}{3R}$$

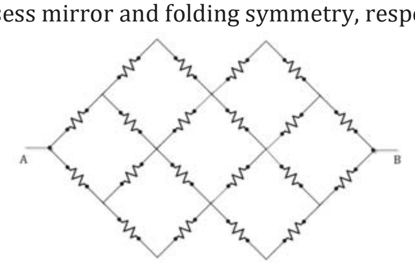
$$\frac{1}{R_{eq}} = \frac{3+2}{3R}$$

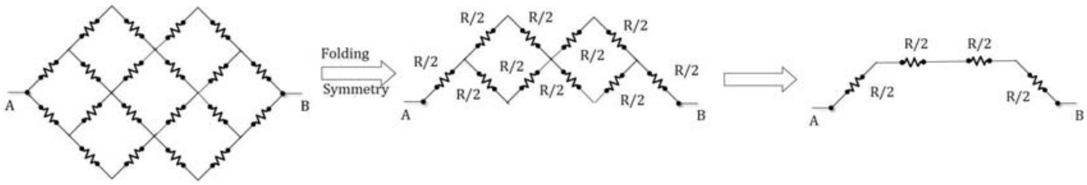
$$R_{eq} = \frac{3R}{S}$$

$$R_{eq} = \frac{3R}{5}\Omega$$

Ex. If each of the resistances in the circuit is R, then find equivalent resistance between terminals A and B.

Sol. The AB and CD lines possess mirror and folding symmetry, respectively.





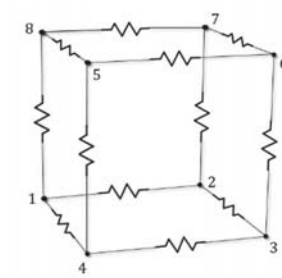
Therefore, equivalent resistance:

$$R_{eq} = 2R\Omega$$

Ex. Twelve resistors, each having a resistance R, are connected to form a cube. Find the equivalent resistance:

(a) Along body diagonal (b) Along an edge (c) Along face diagonal

Sol. There are four body diagonals in the given cube: $1 \leftrightarrow 6, 2 \leftrightarrow 5, 3 \leftrightarrow 8, 4 \leftrightarrow 7$



There are twelve face diagonals in the given cube: $1 \leftrightarrow 5, 4 \leftrightarrow 8, 4 \leftrightarrow 6, 3 \leftrightarrow 5, 3 \leftrightarrow 7, 2 \leftrightarrow 6, 2 \leftrightarrow 8, 1 \leftrightarrow 7, 8 \leftrightarrow 6, 5 \leftrightarrow 7, 1 \leftrightarrow 3, 2 \leftrightarrow 4$.

(a) Along body diagonal:

Consider the body diagonal: $1 \leftrightarrow 6$

If we go from 1 to 2, then our path is along the edge of the cube whereas if we go from 6 to 2, then our path is along the face diagonal of the cube.

Therefore, point 2 is symmetric w.r.t 1 and 6.

Edge Face Digonal

$1 \rightarrow 2 \leftarrow 6$

↳ 4 ↵

↳ 8 ↵

Similarly other such points are: Therefore, points 2, 4 and 8 are at same potential.

Now, If we go from 1 to 3, 8 then our path is along the face diagonal of the cube whereas if we go from 6 to 3, then our path is along the edge of the cube. Therefore, point 3 is also symmetric w.r.t 1 and 6.

Similarly other such points are:

Edge Face Digonal

$1 \rightarrow 3 \leftarrow 6$

↳ 5 ↵

↳ 7 ↵

Therefore, points 3, 5 and 7 are at same potential.

Here, three resistances connected between 1 and '2, 4, 8'. The same is true between 6 and '3, 5, 7'. Therefore, we have covered six resistances but in the cube, there are twelve resistances. Thus, the remaining six resistances should be connected between '2, 4, 8' and '3, 5, 7', as shown below.

Equivalent resistance between 1 and '2, 4, 8': $\frac{R}{3}$

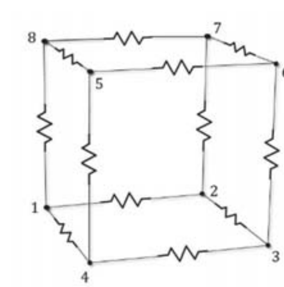
Equivalent resistance between 6 and ' 3,5,7 ': $\frac{R}{3}$

Equivalent resistance between '2, 4, 8' and ' 3,5,7 ': $\frac{R}{6}$

Therefore, the configuration between 1 and 6 becomes:

$$R_{eq} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{2R+R+2R}{6} = \frac{5R}{6}$$

$$R_{eq} = \frac{5R}{6}\Omega$$



(b) Along an edge

Consider the edge: $1 \leftrightarrow 2$

If we go from 1 to 8, then our path is along the edge of the cube whereas if we go from 2 to 8, then our path is along the face diagonal of the cube. Therefore, point 8 is symmetric w.r.t 1 and 2.

Similarly, other such points are:

Edge Face Digonal

1 → 8 ← 2

↳ 4 ↵

Therefore, points 4 and 8 are at same potential.

Now, If we go from 1 to 7, then our path is along the face diagonal of the cube whereas if we go from 2 to 7, then our path is along the edge of the cube. Therefore, point 7 is also symmetric w.r.t 1 and 2. Similarly other such points are:

Face Digonal Edge

1 → 7 ← 2

↳ 3 ↵

Therefore, points 3 and 7 are at same potential.

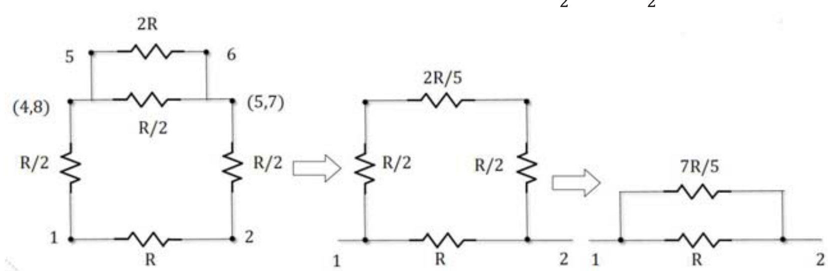
Here, three resistances are connected between 1 and '4, 8'. The same is true between 2 and '3, 7'. Therefore, we have covered four resistances but in the cube, there are twelve resistances. Thus, intuitively the remaining eight resistances can be connected as shown below.

Equivalent resistance between 1 and ' 4,8 ': $\frac{R}{2}$ Equivalent resistance between 2 and ' 3,7 ': $\frac{R}{2}$

Equivalent resistance between ' 3,7 ' and ' 4,8 ': $\frac{R}{2}$ Equivalent resistance between ' 3,7 ' and 6: $\frac{R}{2}$

Equivalent resistance between ' 4,8 ' and 5: $\frac{R}{2}$

Total resistance between '4, 8' and '3, 7' via points 5 and 6 is : $\frac{R}{2} + R + \frac{R}{2} = 2R$



The equivalent resistance between 1 and 2 is:

$$R_{eq} = \frac{\frac{7R}{5} \cdot R}{\frac{7R}{5} + R} = \frac{7R/5}{12/5}$$

$$R_{edge} = \frac{7R}{12}\Omega$$

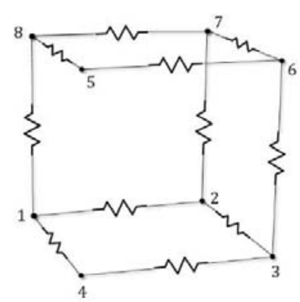
$$R_{eq} = \frac{7R}{12}\Omega$$

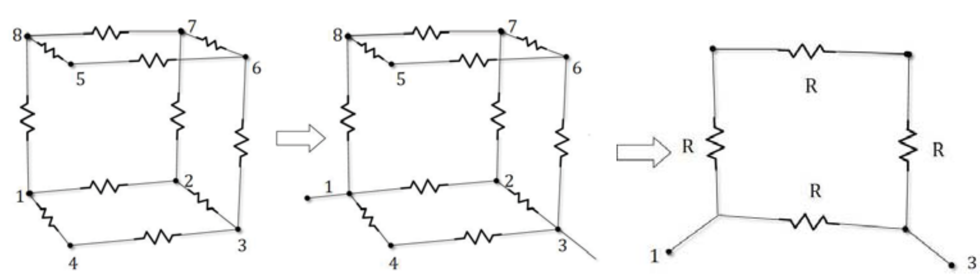
(c) Along face diagonal

Consider the face diagonal: 1 ↔ 6

If we imagine a plane containing 2, 4, 5 and 7 as shown in the figure, this plane generates the mirror symmetry in the cube. Therefore, the same current will flow from 1 to 4 and from 4 to 3 . The same is true for the current going from 1 to 3 via point 2.

Hence, no current will pass through the resistance connected between ' 4 and 5 ' and ' 2 and 7 ' . So, those two resistance can be removed from the circuit . Therefore, the circuit becomes as shown below



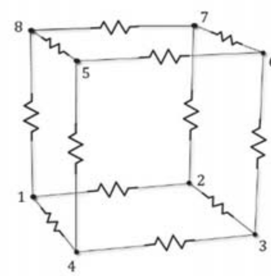


Equivalent resistance of the top and bottom part of the circuit: R

Therefore, the equivalent resistance between the points 1 and 3 is given by,

$$\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{R} \Rightarrow R_{eq} = \frac{3R}{4}$$

$$R_{eq} = \frac{3R}{4}\Omega$$



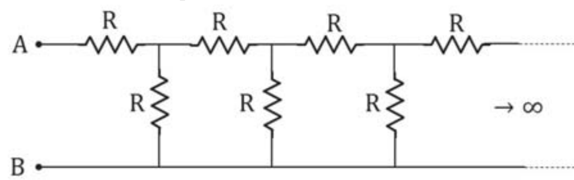
Summary Electric Formula for Box →

Edge	Face diagonal	Body diagonal
$\frac{7R}{12}$	$\frac{3R}{4}$	$\frac{5R}{6}$

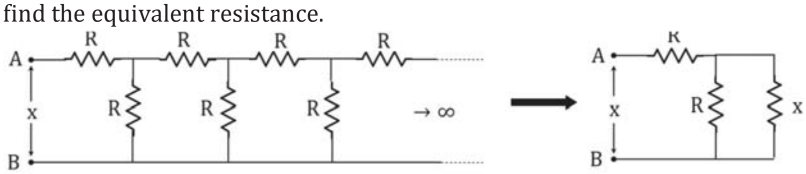
Ex. Find equivalent resistance between A and B in the given infinite network.

Sol. Let the equivalent resistance between A and B is x.

If we remove one repeating network from the given an infinite combination of resistances, then the equivalent resistance between point A and B remains same.



First identify the repeating pattern. Then, by keeping one such pattern, replace the rest of the network by an equivalent resistance x as shown. After that, by solving a quadratic equation of x, we can find the equivalent resistance.



The equivalent resistance between the points A and B is given by,

$$\frac{R \cdot x}{R + x} + R = x$$

$$R \cdot x + R^2 + R \cdot x = R \cdot x + x^2$$

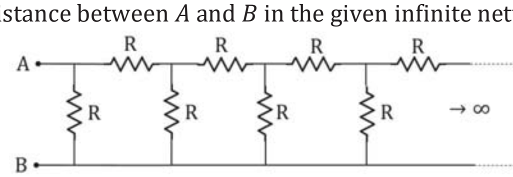
$$x^2 - R \cdot x - R^2 = 0$$

$$x = \frac{R \pm \sqrt{R^2 + 4R^2}}{2} = \frac{R \pm R\sqrt{5}}{2}$$

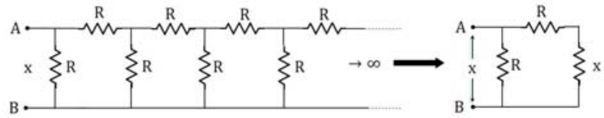
Since the resistance can’t be negative, the equivalent resistance will be given by,

$$x = \frac{R(1 + \sqrt{5})}{2}$$

Ex. Find equivalent resistance between A and B in the given infinite network.



Sol. Let the equivalent resistance between A and B is x. Keeping one repeating pattern, if we replace the rest of the network by the equivalent resistance, the configuration of the circuit becomes:



The equivalent resistance between the points A and B is given by,

$$\frac{(R+x)\cdot R}{2R+x} = x$$

$$R^2 + Rx = 2xR + x^2$$

$$x^2 + Rx - R^2 = 0$$

$$x = \frac{-R \pm \sqrt{R^2+4R^2}}{2} = \frac{R(\sqrt{5}-1)}{2}$$

$$x = \frac{R(\sqrt{5}-1)}{2}$$