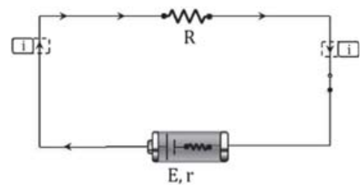
MAXIMUM POWER TRANSFER THEOREM AND MEASURING INSTRUMENT

Maximum Power Transfer Theorem



The battery possesses an electromotive force (EMF) of E, along with internal resistance r, while the circuit features resistance R.

The current in the circuit: $i = \frac{E}{R+r}$

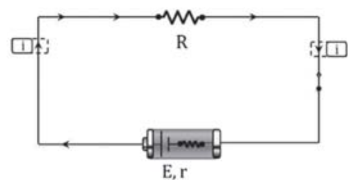
The input power from the battery, which is the power supplied by the battery to the resistors: $P_{input} = Ei$

The output power from the battery, namely the power dissipated across the external resistor: $P_{outmut} = i^2R$

The power dissipation resulting from the internal resistance of the battery: $P_{loss} = i^2r$

If the battery remains constant, with fixed values of E and r, the output power becomes entirely dependent on R, namely, $P_{output} = i^2R = \frac{E^2}{(R+r)^2}R = P(R)$ If we keep on changing the external resistance, then at $R \to \infty$, the power tends to zero because the denominator of the expression of power has R_2 term.

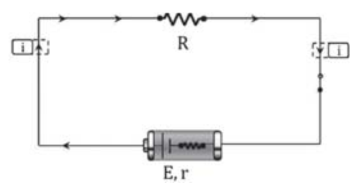
$$\frac{dP(R)}{dR} = 0 \Rightarrow \frac{E^2}{(R+r)^2} - \frac{2E^2 \cdot R}{(R+r)^3} = 0 \Rightarrow \frac{2R}{(R+r)} = 1 \Rightarrow 2R = R + r \Rightarrow R = r$$



Maximum power transfer occurs from the battery when the internal resistance equals the external resistance.

The value of the maximum power that a battery can provide will be $P_{max} = \frac{\varepsilon^2}{4r}$ (for R = r)

Maximum Power Transfer Theorem Efficiency



The battery's efficiency is defined as the ratio of the output power to the input power of the battery.

$$\text{Efficiency} = \frac{P_{output}}{P_{input}}$$

Among many batteries, if a battery produces a large output power w. r. t a constant input power, then it is said that the battery has higher efficiency than other batteries.

In general, $P_{output} = i^2R$ and $P_{input} = Ei$. Thus the efficiency (η) of the battery (In percentage) is defined as:

$$\eta = \frac{P_{output}}{P_{input}} = \frac{iR}{E} \times 100$$

R: Total external resistance of the battery

E: The EMF of the battery

For the given circuit,

$$i = \frac{E}{R+r}$$

$$P_{input} = Ei$$

$$P_{input} = \frac{E \cdot E}{(R+r)} = \frac{E^2}{(R+r)}.$$

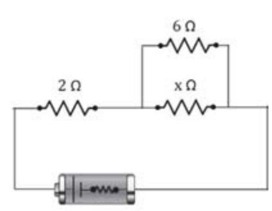
$$\text{Poutput} = i^2 - R$$

$$= \frac{E^2}{(R+r)^2} \cdot R$$

$$\eta = \frac{\text{Poutput}}{\text{Pinput}} \times 100$$

$$= \frac{E^2 \cdot R \cdot (R+r)}{(R+r)^2 \cdot E^2} \times 100$$

$$= (\frac{R}{R+r}) \times 100$$



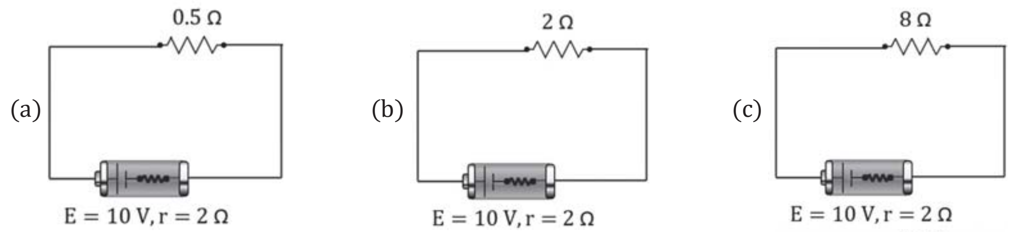
Ex. Three resistances 2 Ω, 6 Ω and x Ω are connected to a battery as shown. Find the value of x to drag maximum power from the battery.

Sol. To drag maximum power from the battery, the total (i.e., equivalent) external resistance of the circuit should be equal to the internal resistance of the circuit.

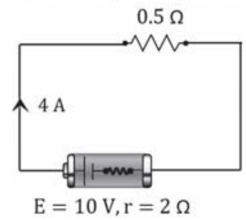
Therefore,

$$R_{eq} = 4\Omega.$$
$$2 + \frac{6\cdot x}{6+x} = 4$$
$$6 \cdot x = 2(6 + x)$$
$$6x = 12 + 2x$$
$$4x = 12$$
$$x = 3\Omega$$

Ex. Three resistances 0.5 Ω, 2 Ω and 8 Ω are connected in the three distinct circuits as shown. Find the value of output Power and efficiency.

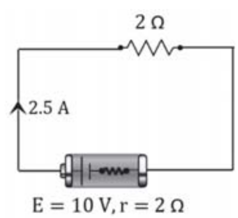


Sol. (a) Internal resistance: $r = 2\Omega$ External resistance: $R = 0.5\Omega$
EMF of the battery: $E = 10V$ Current: $i = 4$ A
Therefore, input power: $P_{input} = Ei = 40\omega$ att
Output power: $P_{output} = i^2R = 4^2 \times 0.5 = 8$ wott
Efficiency (in percentage) $= \frac{P_{output}}{P_{input}} \times 100$

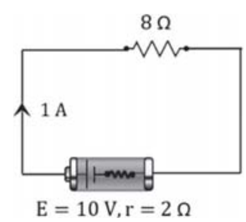


$$\eta = \frac{8}{40} \times 100 = 20\%$$
$$P_{output} = 8 \text{ W}$$
$$\eta = 20\%$$

(b) Internal resistance: $r = 2\Omega$ External resistance: $R = 2\Omega$
EMF of the battery: $E = 10$ V Current: $i = 2.5$ A
Therefore, input power: $P_{input} = Ei = 25$ watt
Output power: $P_{output} = i^2R = (2 \cdot 5)^2 \times 2. = 12.5$ watt
Efficiency (in percentage) $= \frac{P_{output}}{P_{input}} \times 100 = \frac{12.5}{25} \times 100 = 50\%$



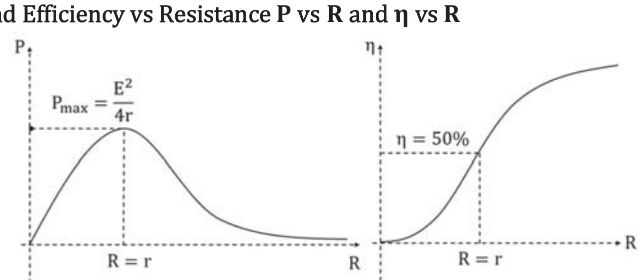
(c) Internal resistance: $r = 2\Omega$ External resistance: $R = 8\Omega$
EMF of the battery: $E = 10$ V Current: $i = 1$ A
Therefore, input power: $P_{input} = Ei = 10 \times 1 = 10$ watt .
Output power: $P_{output} = i^2R = 1^2 \times 8 = 8$ watt.
Efficiency (in percentage) $= \frac{P_{output}}{P_{input}} \times 100$

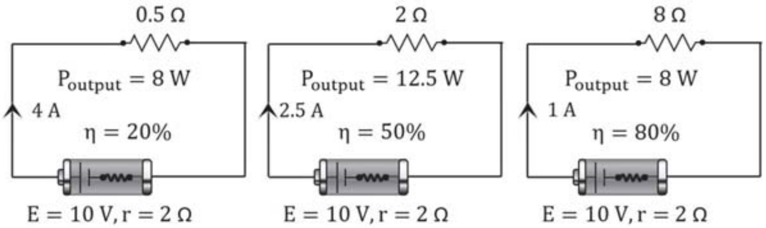


$$\eta = \frac{8}{10} \times 100 = 80\%$$

If $R \geq r$ and we keep on increasing R, the efficiency of the battery increases but the output power of the battery decreases.
When output power is maximum, the efficiency of the battery becomes 50 %.

Power vs Resistance and Efficiency vs Resistance P vs R and η vs R





For the given circuit, the output power of the battery is,

$$P = 10^2 \times 8 = 800 \text{ W}$$

If we incorporate a secondary battery with an electromotive force of 2 volts and an internal resistance of 1 ohm in series with the existing battery in the circuit, the resulting circuits current will be.

$$i = \frac{(100+2)}{(8+2+1)} = \frac{102}{11} = 9.3 \text{ A}$$

In this case, $(E - ir)$ across the small battery becomes $[2 - (9.3 \times 1)] = -7.3 (< 0)$. Since $(E - ir)$ across the small battery becomes negative , we can't be benefitted from the addition of small battery with the large battery.

Measuring instrument - Battery Eliminator, Resistance Box, Rheostat, Galvanometer, Ammeter and voltmeter

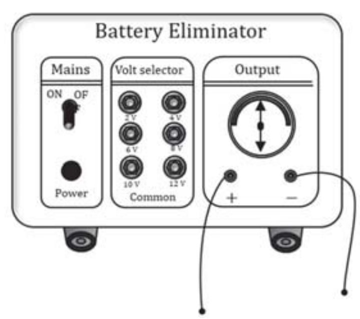
Experimental Voltage ≡ D.C. Voltage
(2 V − 10 V)

D.C. Voltage

Simple Cell | Battery Eliminator

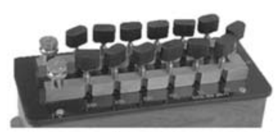
Measuring Instruments Battery Eliminator

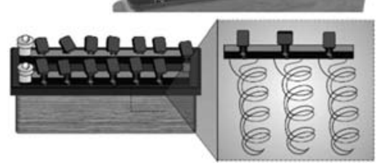
- If we incorporate a secondary battery with an electromotive force of 2 volts and an internal resistance of 1 ohm in series with the existing battery in the circuit, the resulting circuits current will be.
- The battery eliminator supplies a consistent DC voltage, converting 220 volts AC current into 2 to 10 volts DC current for experiments.
- A double headed electric wire is usually connected between the power and anyone of the volt selector to give that specific voltage.
- When the double-headed electric wire is linked between the power source and the 4-volt selector, the 'Battery eliminator' will produce an output of 4 volts.



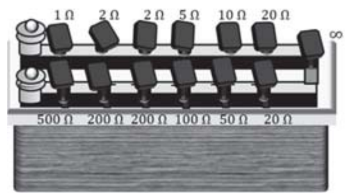
Measuring Instruments Resistance Box

- A device utilized for introducing predetermined resistances into the circuit.
- Typically, plugs are constructed from conductors with minimal resistance. Upon insertion into the resistance box, current flows through the metal strip (indicated by the yellow bar in the diagram), thus ideally resulting in no resistance being generated.



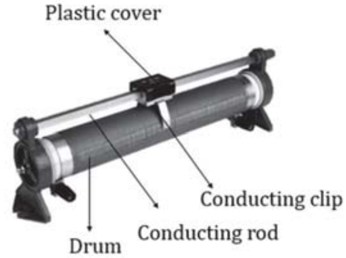


- When the plugs are removed from the resistance box, the current passes through the wires (the coiled wires in the diagram), resulting in resistance being generated.
- Any integer value of resistance can be generated depending upon the capacity of resistance box.
- The total resistance in the circuit provided by the resistance box equals the combined resistances of the positions where the plugs are removed.

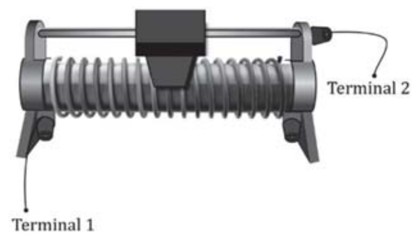


- If the neighboring resistance box is connected to a circuit through a connecting wire, and we remove the plugs at 5 Ω and 10 Ω, then the overall resistance imparted to the external circuit is. $(5+10)=15\Omega$
- For the resistance box shown in the figure, keeping the plug marked at ∞ attached in the box, total 1 Ω to 1110 Ω resistance can be put in the external circuit,
- When the plug labeled as ∞ is removed from the box, infinite resistance is introduced to the external circuit, resulting in an open circuit condition.

Measuring Instruments Rheostat

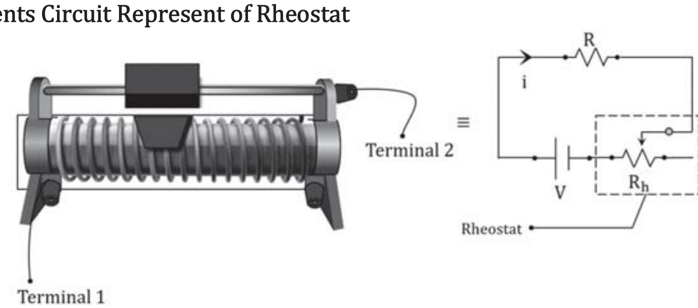


- A rheostat is an electrical device utilized for easily adjusting the resistance within a circuit.
- Terminal 1 and terminal 2 are connected with the external circuit.
- The wires tightly bound inside the drum produce resistance within the circuit.
- The current enters via terminal 1, traverses through the wire, and exits through terminal 2.
- The conducting clip having plastic cover on the head makes the connection between terminal 1 and terminal 2 via conducting clip.
- If the clip is moved within a certain distance from terminal 1, the rheostat produces resistance within that range due to the wire bindings.

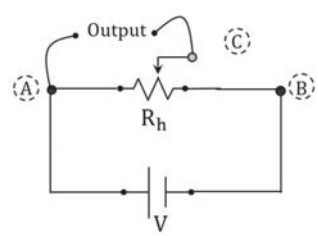


- Don't connect terminal 1 and terminal 3 with the external circuit. Otherwise, the circuit will pick up the whole resistance of the wire bindings in the drum.

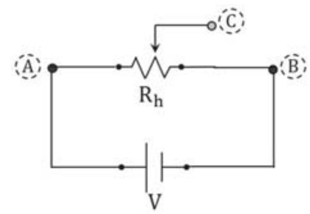
Measuring Instruments Circuit Represent of Rheostat



Measuring Instruments Rheostat

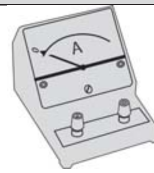
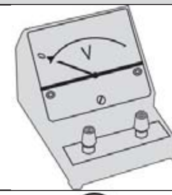


- The output voltage of the circuit can be adjusted by connecting the battery between points A and B and extracting the output from points A and C.
- The output voltage fluctuates with the movement of the rheostat arrow.
- The output voltage from the circuit will consistently be lower than the voltage of the battery.
- The rheostat is alternatively referred to as a potential divider.



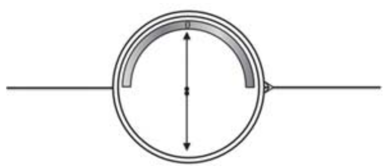
Ex. Connect a battery and take the output between two points, so that it works as a Potential Divider.

Sol. Battery is connected between A and B point, Output is taken from A and C point or, from A and C point

	Gravimeter	Ammeter	Voltmeter
Actual			
Symbol	G	A	V
Use	1. Detects extremely small current values. 2. The needle deflection indicates changes in the direction of current.	1. Measures higher current values. 2. Ammeter is connected in series circuit.	1. Measures voltage difference across any two points of the circuit. 2. Voltmeter is connected in parallel to the circuit.

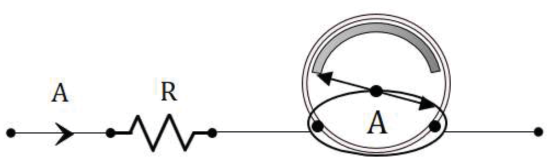
Measuring Instruments Gravimeter

A device that measures circuit current and indicates the direction of current flow.



Measuring Instruments Ammeter

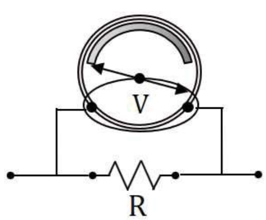
A device for measuring circuit current.



The ammeter has a higher capacity to measure current compared to the galvanometer.
The ammeter is wired in series within the circuit.

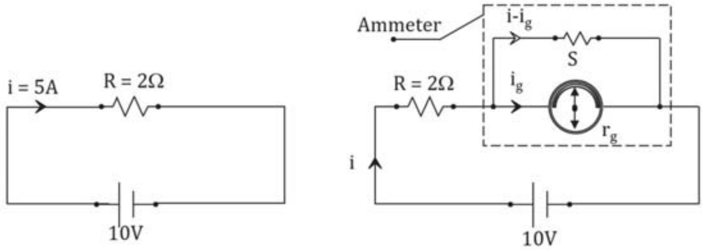
Measuring Instruments Ammeter

A device utilized for measuring the voltage across a circuit.

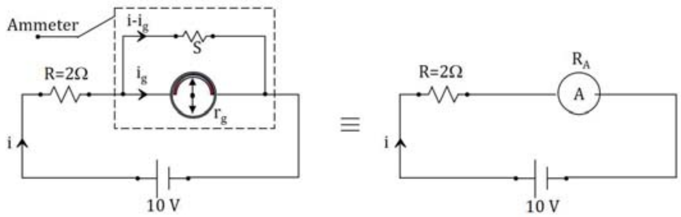


The voltmeter is linked in parallel within the circuit.

Measuring Instruments Conversion Of Galvanometer To Ammeter



A shunt resistance (S) is added to the circuit as an extra measure to prevent damage to the galvanometer.
The integration of a galvanometer with a shunt resistance is termed as an ammeter.



To uphold a consistent current in the circuit, it's imperative to keep the resistance of the ammeter (RA) as low as achievable.
Good Ammeter $\Rightarrow R_A \rightarrow 0$
Ideal Ammeter $\Rightarrow R_A = 0$
Ammeter resistance (R_A) is given by

$$\frac{1}{R_A} = \frac{1}{S} + \frac{1}{r_g}$$

Choose the value of S such that the value of R_A and r_g should be minimum.

$$\text{If } S \rightarrow 0 (i - i_g) \uparrow$$

$$i_g \rightarrow 0 R_A \rightarrow 0$$

Current in Ammeter (i) is given by

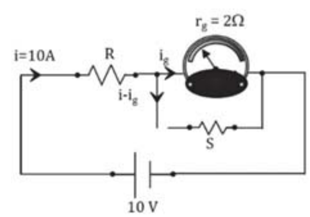
$$i_g r_g = (i - i_g) S$$

$$i_1 : i_g = \frac{1}{s} : \frac{1}{i_g} \Rightarrow i_1 : i_g = r_g : s \Rightarrow i_g = \frac{S}{r_g} . i$$

$$i = (1 + \frac{r_g}{S}) i_g$$

$\left(1 + \frac{r_g}{S}\right) \rightarrow$ Constant
$i_g \rightarrow$ Galvanometer current

Ex. A galvanometer has resistance 2 Ω with maximum current measuring capacity 0.1 A. Find the shunt resistance (S) of an ammeter which can measure current up to 10 A.



Sol. Given,

$$r_g = 2\Omega$$

$$i_g = 0.1 \text{ A}$$

$$i = 10 \text{ A}$$

We have,

$$i = \left(1 + \frac{r_g}{S}\right) i_g$$

$$10 = \left(1 + \frac{2}{S}\right) \times 0.1$$

$$1 + \frac{2}{S} = 100$$

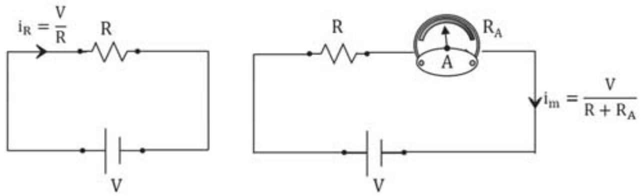
$$\frac{2}{S} = 99$$

$$S = \frac{2}{99\Omega}$$

Measuring Instruments Conversion Of Galvanometer To Ammeter

Percentage Error In Ammeter

Despite the minuscule resistance of the ammeter, it influences the overall resistance of the circuit, resulting in a slight deviation from the actual current value. The percentage deviation or error in the actual current value is therefore.



$$\% \text{ Error } = \frac{i_R - i_m}{i_R} \times 100$$

$i_R \rightarrow$ Real current

$i_m \rightarrow$ Measured current

$$= \frac{\frac{V}{R} - \frac{V}{R+R_A}}{\frac{V}{R}} \times 100$$

$$= \frac{R_A}{R+R_A} \times 100$$

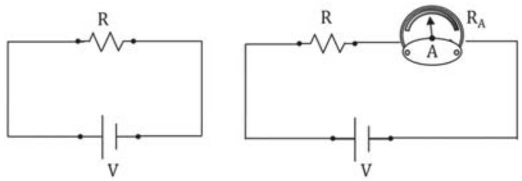
$$\% \text{ Error } = \frac{R_A}{R+R_A} \times 100$$

$R_A \rightarrow 0$

$\%_{Error} \rightarrow 0$

The percentage error also relies on the characteristics of the external circuit.

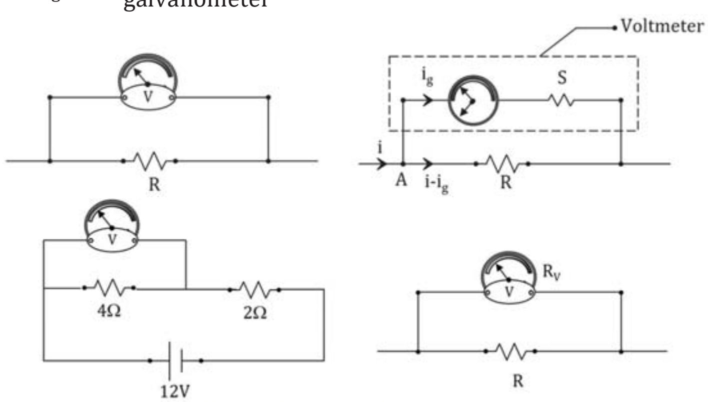
For good performance (least error), $R \gg R_A$



Measuring Instruments Conversion Of Galvanometer To voltmeter

To measure the potential drop across a circuit, a voltmeter is connected in parallel to it. Since a voltmeter is constructed using a galvanometer, the current flowing through it must be minimal. To achieve this, a high-resistance shunt is connected in series with the galvanometer, ensuring only a small current flows through it. Ideally, a voltmeter should possess infinite resistance.

If $S \uparrow$ $\quad R_V \uparrow i_g \downarrow, \rightarrow$ Saves the galvanometer

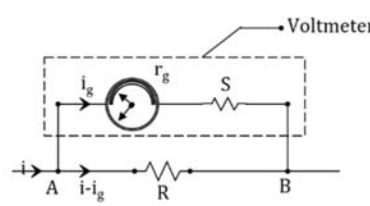


If we want that the circuit should not be affected by installing a voltmeter, then

Good Voltmeter $\Rightarrow R_V \rightarrow \infty$

Ideal Voltmeter $\Rightarrow R_V = \infty$

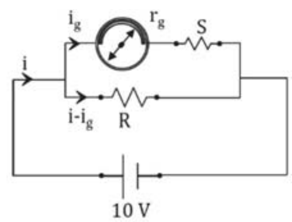
Potential across the voltmeter is given by,



$$(i - i_g)R = i_g(r_g + S)$$

$$V = i_g(r_g + S)$$

Ex. A galvanometer has resistance 2 Ω with maximum current measuring capacity 0.1 A. Find the shunt resistance (S) of a voltmeter which can measure voltage up-to 10 V.



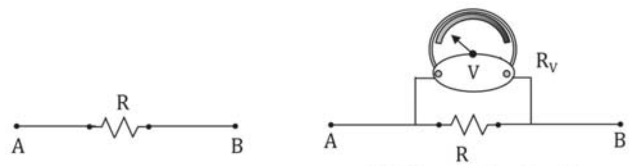
Sol. Given,

$$r_g = 2\Omega$$
$$i_g = 0.1\text{ A}$$
$$V = 10\text{ A}$$

We have,

$$V = i_g(r_g + S)$$
$$10 = 0.1(2 + s)$$
$$100 - 2 = s = 98\Omega$$
$$S = 98\Omega$$

Measuring Instruments Conversion Of Galvanometer To voltmeter
Percentage Error In voltmeter



V_R(Real voltage) V_m(Measured voltage)

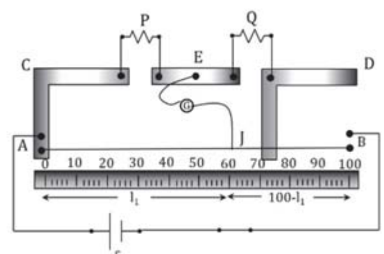
$$\%\text{ Error } = \frac{V_R - V_m}{V_R} \times 100$$

Measuring Instruments Mater Bridge

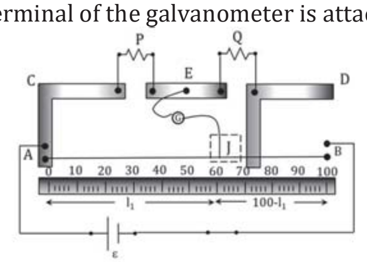
The meter bridge serves the purpose of determining the resistance of a wire whose value is initially unknown.

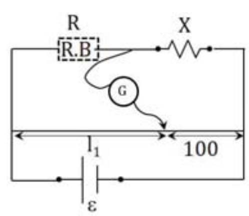
The design of the meter bridge draws upon the principles of the Wheatstone bridge.

A one-meter wire is linked between conducting plates positioned on a plank. Resistors are attached at points P and Q as illustrated in the diagram.

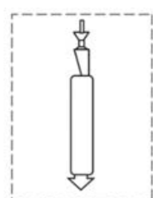


A resistance box is linked at point P, while the unknown resistance is connected at point Q. The galvanometer is connected between these resistances. The other terminal of the galvanometer is attached to a jockey, which can be moved along the plank.



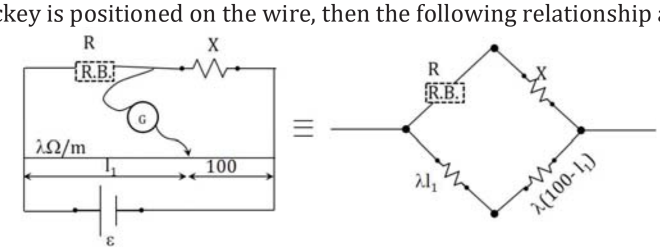


The jockey is positioned on the wire in such a way that the deflection in the galvanometer is zero, resulting in a balanced Wheatstone bridge configuration. The unknown resistance is determined by applying the condition of a balanced Wheatstone bridge.



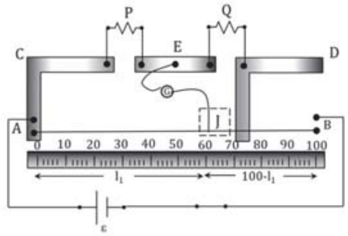
J →Jockey

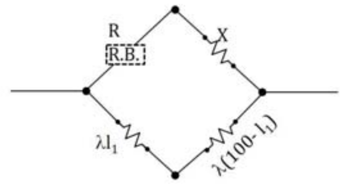
When the current reading on the galvanometer reaches zero, it indicates that the Wheatstone bridge is in a state of balance. assume λ denotes the resistance per unit length of the wire, and x represents the unknown resistance. If l_1 represents the distance from the left end to the point where the jockey is positioned on the wire, then the following relationship applies:



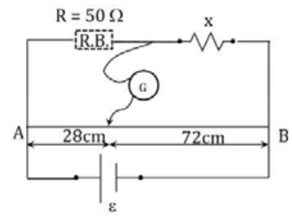
Unknown resistance (x) is given by,

$$x = \frac{(100-l_1)}{l_1} \times R$$





Ex. Find the unknown resistance (x), if the balance point for the jockey is at 28 cm from point A as shown.



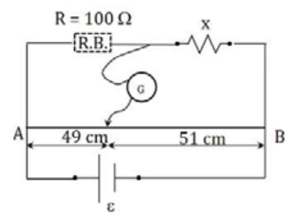
Sol. Given,

$$l_1 = 28 \text{ cm}$$
$$R = 50\Omega$$

We have,

$$x = \frac{(100-l_1)}{l_1} \times R$$
$$28 \times x = 72 \times 50$$
$$x = \frac{18\times50}{7} = 128.5\Omega$$
$$x = 128.5\Omega$$

Ex. Find the unknown resistance (x) if the balance point for the jockey is at 49 cm from point A as shown.



Sol. Given,

$$l_1 = 49 \text{ cm}$$
$$R = 100\Omega$$

We have,

$$x = \frac{(100-l_1)}{l_1} \times R$$
$$x \times 49 = 100 + 51$$
$$x = \frac{5100}{49.}$$
$$x = 104\Omega$$

Potentiometer

- Necessity of potentiometer practically voltammeter has a finite resistance. (Ideally it should be∞) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement, it draws no current from the circuit. It means its effective resistance is infinite.
- Working principle of potentiometer any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.
- Potentiometer wire made up of alloys of manganic, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature co–efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

Circuits of Potentiometer

Primary circuit

Contains constant source of voltage rheostat or Resistance Box

Secondary, Unknown or galvanometer circuit

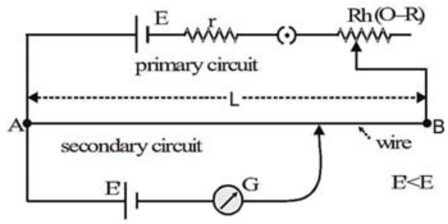
Let ρ = Resistance per unit length of potentiometer wire

Potential gradient (x) (V/m)

The fall of potential per unit length of potentiometer wire is called potential gradient

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resitance of potentiometer wire}}{\text{length of potentiometer wire}} = I(\frac{R}{L})$$

The potential gradient depends only on primary circuit and is independent of secondary circuit.



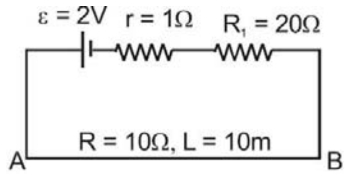
Ex. Primary circuit of potentiometer is shown in figure determine:

(a) Current in primary circuit

(b) Potential drop across potentiometer wire AB

(c) Potential gradient (means potential drop per unit length of potentiometer wire)

(d) Maximum potential which we can measure above potentiometer



Sol. (a) $i=\frac{\varepsilon}{r+R_1+R}=\frac{2}{1+20+10}\Rightarrow i=\frac{2}{31}A$

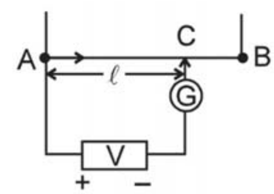
(b) $V_{AB}=iR=\frac{2}{31}\times 10\Rightarrow v_{AB}=\frac{20}{31}$ volt

(c) $x=\frac{V_{AB}}{L}=\frac{2}{31}$ volt/m

(d) Maximum potential which we can measure by it = potential drop across wire AB = $\frac{20}{31}$ volt

Ex. How to measure an unknown voltage using potentiometer.

Sol. The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition $V=x\ell$.



Application of Potentiometer

(a) To find emf of unknown cell and compare emf of two cells.

In case I,

In figure, (2) is joint to (1) then balance length $=\ell_1$

$$\varepsilon_1 = x\ell_1$$

In case II,

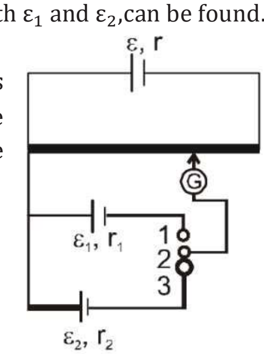
In figure, (3) is joint to (2) then balance length $=\ell_2$

$$\varepsilon_2 = x\ell_2$$

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{\ell_1}{\ell_2}$$

If any one ε_1 or $\varepsilon_2$is shown the other can be found. If x is known then both ε_1 and ε_2,can be found.

Ex. In an experiment to determine the emf of an unknown cell, its emf is compared with a standard cell of known emf ε_1 = 1.12 V. The balance point is obtained at 56cm withs t a n - dard cell and 80 cm with the unknown cell. Determine the emf of the unknown cell.



Sol. Here, $\varepsilon_1 = 1.12$ V $\ell_1 = 56$ cm; $\ell_2 = 80$ cm

Using equation

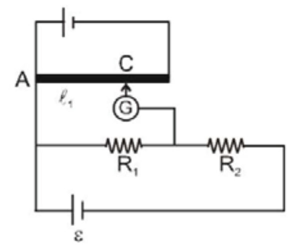
$$\varepsilon_1 = x\ell \quad \dots (1)$$

$$\varepsilon_2 = x\ell_2 \quad \dots (2)$$

We get $\frac{\varepsilon_1}{\varepsilon_2}=\frac{\ell_1}{\ell_2}\Rightarrow \varepsilon_2=\varepsilon_1(\frac{\ell_2}{\ell_1})$

$$\varepsilon_2 = 1.12(\frac{80}{56}) = 1.6\text{V}ans$$

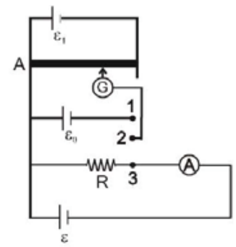
(b) To find current if resistance is known



$$V_A - V_C = x\ell_1$$
$$IR_1 = x\ell_1$$
$$I = \frac{x\ell_1}{R_1}$$

Similarly, we can find the value of R_2 also. Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.

Ex. A standard cell of emf $\varepsilon_0 = 1.11$ V is balanced against 72 cm length of a potentiometer. The same potentiometer is used to measure the potential difference across the standard resistance R = 120Ω. When the ammeter shows a current of 7.8 mA, a balanced length of 60 cm is obtained on the potentiometer.



1. Determine the current flowing through the resistor.
2. Estimate the error in measurement of the ammeter.

Sol. Here, $\ell_0 = 72$ cm; $\ell = 60$cm; R = 120Ω and $\varepsilon_0 = 1.11$ V

1. By using equation $\varepsilon_0 = x\ell_0$ ... (1)

$V = IR = x\ell$ ... (2)

From equation (1) and (2)

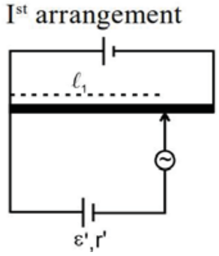
$$I = \frac{\varepsilon_0}{R}\left(\frac{\ell}{\ell_0}\right) \therefore I = \frac{1.11}{120}\left(\frac{60}{72}\right) = 7.7 \text{ mA}$$

2. Since the measured reading 7.8 mA (> 7.7 mA) therefore, the instrument has a positive error.

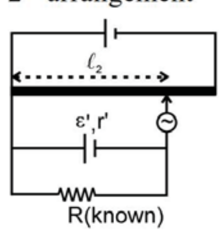
$$\Delta I = 7.8 - 7.7 = 0.1\text{mA},$$
$$\frac{\Delta I}{I} = \frac{0.1}{7.7} \times 100 = 1.3\%$$

(c) To find the internal resistance of cell.

I[st] arrangement



2[nd] arrangement



by first arrangement $\varepsilon' = x\ell_1$

by second arrangement $IR = x\ell_2$

$$I = \frac{X\ell_2}{R}$$

$$I = \frac{\varepsilon'}{r'+R} \Rightarrow \frac{\varepsilon'}{r'+R} = \frac{x\ell_2}{R} \Rightarrow \frac{x\ell_1}{r'+R} = \frac{x\ell_2}{R} \Rightarrow r' = \left[\frac{\ell_1 - \ell_2}{\ell_2}\right]R$$

Applications of Potentiometer

- To gauge the potential difference across a resistance.
- To determine the electromotive force (emf) of a cell.
- Comparison between two electromotive forces (EMFs), denoted as E_1/E_2.
- To determine the internal resistance of a primary cell.
- Comparison between two resistances.
- To determine an unknown resistance connected in series with the given resistance.
- To determine the current in a specified circuit.
- Calibrating an ammeter or verifying its readings (in amperes).
- Calibrating a voltmeter or verifying its readings (in volts).
- To measure the electromotive force (emf) of a thermocouple (in millivolts or mV).

Difference between potentiometer and voltmeter

Potentiomer	Voltmeter
It accurately measures the unknown electromotive force (emf) by comparing it with a known emf source. During the measurement of unknown potential difference, the resistance of the potentiometer becomes infinite. This method relies on the zero deflection principle and exhibits high sensitivity. The potentiometer finds applications in various scenarios, such as measuring the internal resistance of a cell, calibrating ammeters and voltmeters, evaluating thermos emf, and comparing electromotive forces.	It provides an approximate measurement of the unknown electromotive force (emf). During the emf measurement, it draws some current from the emf source. When measuring unknown potential differences, the voltmeter has a high but finite resistance. This method is based on the deflection principle and has relatively low sensitivity. Its sole application is the measurement of emf or unknown potential differences.