KIRCHHOFF’S LAW AND SIMPLE CIRCUIT THEORY

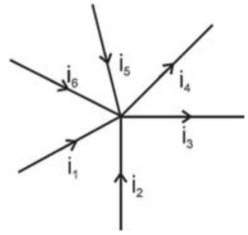
1. **Kirchhoff’s Current Law (Junction law)**

This principle derives from the law of conservation of charge, asserting that "The algebraic sum of currents converging at a point in the circuit is zero" or, in other words, the total current entering a junction is equal to the total current leaving the junction.

$$\Sigma I_{in} = \Sigma I_{oupt}.$$

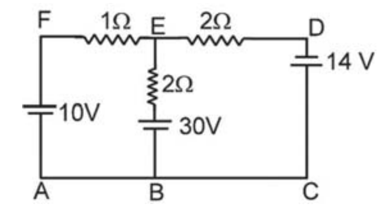
It is also known as KCL (Kirchhoff's current law).

Ex. Find relation in between current i_1, i_3, i_3, i_4, i_s and i_6.



Sol. $i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$

Ex. Find the current in each wire.



Sol. Let potential at point B = 0. Then potential at other points are mentioned.

Potential at E is not known numerically. Let potential at E = x

Now applying Kirchhoff’s current law at junction E. (This can be applied at any other junction also).

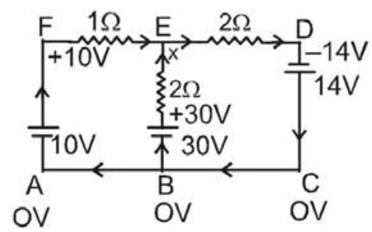
$$\frac{x-10}{1} + \frac{x-30}{2} + \frac{x+14}{2} = 0$$

$$4x = 36 \Rightarrow x = 9$$

Current in EF $= \frac{10-9}{1} = 1$ A From F to E

Current in BE $= \frac{30-9}{2} = 10.5$ A from B to E

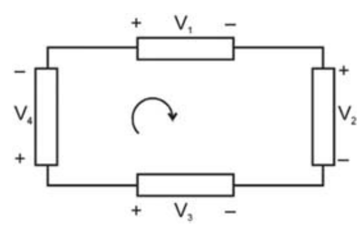
Current in DE $= \frac{9-(-1\ )}{2} = 11.5$ A From E to D



2. **Kirchhoff’s Voltage L aw (LOOP LAW)**

The sum of all potential differences along a closed loop is zero when considered algebraically. IR + ΣEMF = 0"

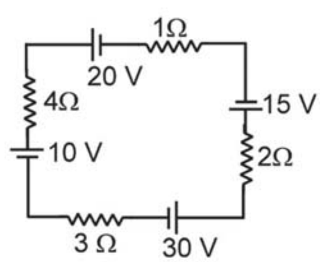
The closed loop can be traveled in any direction. When moving through the loop, assign a positive sign to the expression if the potential increases, and assign a negative sign if the potential decreases. (Assume the provided sign convention.)



$-V_1 - V_2 + V_3 - V_4 = 0.$

Boxes may contain resistor or battery or any other element (linear or nonlinear). It is also known as KVL.

Ex. Find current in the circuit



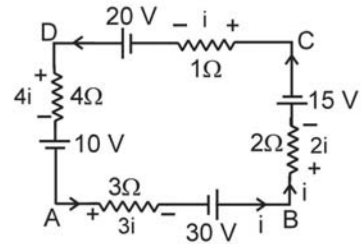
Sol. All the elements are connected in series current is all of them will be same

let current = i

Applying Kirchhoff voltage law in ABCDA loop

$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10i = 25 \Rightarrow i = 2.5A$$



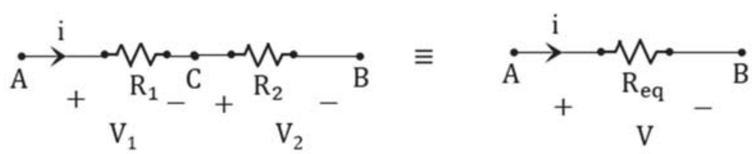
Note:

- If a wire is elongated to n times its original length, the resulting resistance will be proportional to n^2.
- If a wire is stretched such that it's radius is reduced to $\frac{1}{n}$th of it's original values, then resistance will increases n4 times similarly resistance will decrease n^4 time if radius is increased n times by contraction.
- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual
- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual
- To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- Ohm's law is not a fundamental law of nature. As it is possible that for an element :–
  1. V depends on I non-linearly (e.g. vacuum tubes)
  2. Relation between V and I depends on the sign of V for the same value [Forward and reverse Bias in diode]
  3. The relation between V and I is non-unique. That is for the same I there is more than one value of V
- In general
  1. Resistivity of alloys is greater than their metals.
  2. Temperature coefficient of alloys is lower than pure metals.
  3. Resistance of most of nonmetals decreases with increase in temperature. (e.g. Carbon)
  4. The resistivity of an insulator (e.g. amber) is greater than the metal by a factor of 1022
- Temperature coefficient (α) of semiconductor including carbon (graphite), insulator and electrolytes is negative.

Parallel And Series Combinations Of Resistor

Resistors Series combination

The series connection of resistors is formed when identical electrical currents pass through each resistor.



Total potential difference between point A and B is,

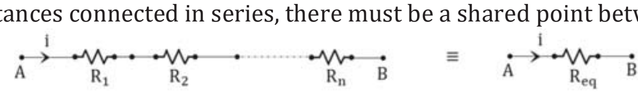
$$V = v_1 + v_2$$
$$v = iR_1 + iR_2$$

If the group of resistances is replaced by a single resistance R_{eq} such that R_{eq} draws the same current from the battery as the group of resistances have drawn, then R_{eq} will be known as the 'Equivalent resistance'. The potential difference between point *A* and *B* is, $V = iR_{eq}$

Combining these two obtained equations, we get,

$$iR_{eq} = iR_1 + iR_2$$
$$R_{eq} = R_1 + R_2$$

For two resistances connected in series, there must be a shared point between them.

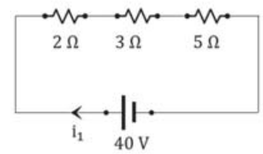


If there are n resistors connected in series, then the resultant resistance will be,

$$R_{eq} = R_1 + R_2 + \cdots \ldots + R_n$$

Ex. Find the potential drop across 3 Ω resistors in the circuit shown below.

Sol. In this case, given three resistances are in series. Therefore, the equivalent resistance of the circuit is, $R_{eq} = 2 + 3 + 5 = 10\Omega$



Therefore, the current in the circuit will be,

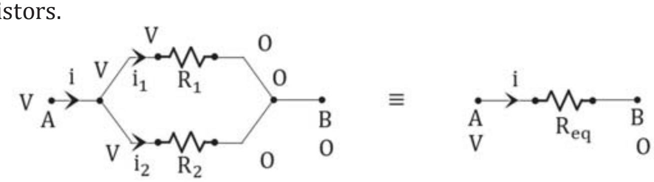
$$i = \frac{40}{10} = 4\text{ A}$$

The potential difference across the 3 Ω resistor will be given by,

$$V = 3 \times 4 = 12\text{volt}$$

Resistors Parallel combination

The parallel connection of resistors is formed when an identical potential difference is generated across all resistors.



The potential difference across each resistances are same. The total current is given by,

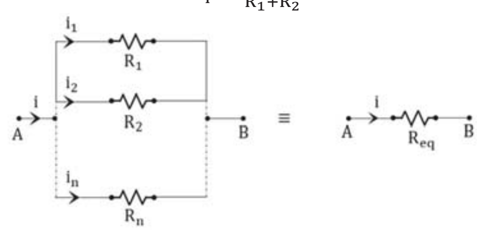
$$i = i_1 + i_2$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

The potential difference between point A and B is, $V = iR_{eq}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

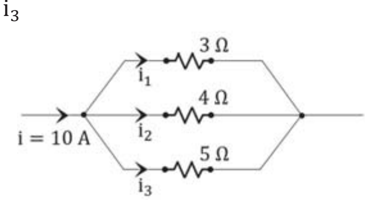


If there are n resistors arranged in parallel, the resulting resistance will be equivalent.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \ldots + \frac{1}{R_n}$$

Ex. In the electrical circuit shown below, find the values of i_1, i_2 and i_3

Sol. If R_{eq} is the equivalent resistance of the given circuit, then,



$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$\frac{1}{R_{eq}} = \frac{20+15+12}{60}$$

$$R_{eq} = \frac{60}{47}$$

Therefore, the potential difference across each resistance is given by,

$$V = iR_{eq}$$

$$V = 10 \times \frac{60}{47} = \frac{600}{47}$$

Hence, the current through each resistance is given by,

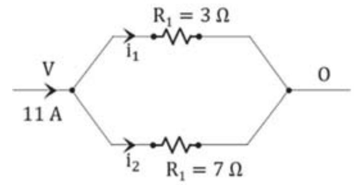
$$i_1 = \frac{600}{47\times3} = \frac{200}{47}$$

$$i_2 = \frac{660}{47\times4} = \frac{150}{47} \qquad [\text{ Since } i = \frac{V}{R}\ ]$$

$$i_3 = \frac{120}{47}$$

$$i_1 = \frac{200}{47}\text{A}, i_2 = \frac{150}{47}\text{A}, i_3 = \frac{120}{47}\text{A}$$

Short Trick To Find The Current In Parallel Combination



The circuit has a total current of 11 A, and the potential difference across each resistor is V.

Therefore, $i_1 = \frac{V}{3}$ and $i_2 = \frac{V}{7}$

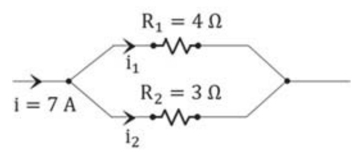
The ratio of these two currents is given by,

$i_1 : i_2 = \frac{v}{3} : \frac{v}{7} \Rightarrow i_1 : i_2 = \frac{1}{3} : \frac{1}{7} \Rightarrow i_1 : i_2 = 7 : 3$

Therefore,
$$\frac{i_1}{i_2} = \frac{7}{3}$$
$$\frac{i_1}{i_1 + i_2} = \frac{7}{10}$$
$$\frac{i_1}{11} = \frac{7}{10}$$
$$i_1 = \frac{77}{10}$$

Thus,
$$i_2 = \frac{33}{10}$$
$$i_1 : i_2 = \frac{1}{3} : \frac{1}{7}$$
$$\Downarrow$$
$$i_1 : i_2 = \frac{1}{R_1} : \frac{1}{R_2} = x : y$$
$$\Downarrow$$
$$i_1 = \frac{x}{(x+y)}(\text{ Total current }) \quad i_2 = \frac{y}{(x+y)}(\text{ Total current })$$

Ex. In the electrical circuit shown below, find the values of i_1 and i_2.

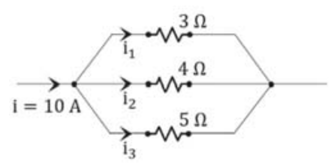


Sol. We have: $R_1 = 4\Omega$ and $R_2 = 3\Omega$

Using short trick, we can write,
$$i_1 : i_2 = \frac{1}{R_1} : \frac{1}{R_2} = \frac{1}{4} : \frac{1}{3} = 3 : 4 = x : y$$

Therefore,
$$i_1 = \frac{x}{(x+y)}(\text{ Total current }) = \frac{3}{7} \times 7 = 3\text{ A}$$
$$i_2 = \frac{y}{(x+y)}(\text{ Total current }) = \frac{4}{7} \times 7 = 4\text{ A}$$
$$i_1 = 3\text{A}, i_2 = 4\text{A}$$

Ex. In the electrical circuit shown below, find the values of i_1, i_2 and i_3.



Sol. We have: $R_1 = 3\Omega, R_2 = 4\Omega$ and $R_3 = 5\Omega$

Using short trick, we can write,

$i_1 : i_2 : i_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} = \frac{1}{3} : \frac{1}{4} : \frac{1}{5} = 20 : 15 : 12 = x : y : z$

Therefore,
$$i_1 = \frac{x}{(x+y+z)}(\text{ Total current }) = \frac{20}{47} \times 10 = \frac{200}{47}\text{A}$$
$$i_2 = \frac{y}{(x+y+z)}(\text{ Total current }) = \frac{15}{47} \times 10 = \frac{150}{47}\text{A}$$
$$i_3 = \frac{z}{(x+y+z)}(\text{ Total current }) = \frac{12}{47} \times 10 = \frac{120}{47}\text{A}$$
$$i_1 = \frac{200}{47}\text{A}, i_2 = \frac{150}{47}\text{A}, i_3 = \frac{120}{47}\text{A}$$