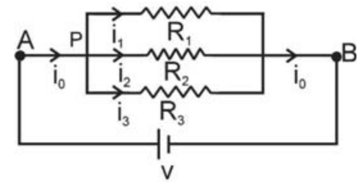
EQUIVALENT RESISTANCE AND CURRENT DISTRIBUTION IN CIRCUIT

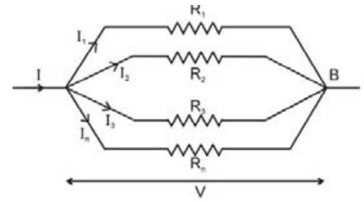


Equivalent resistance

Applying Kirchhoff's junction law at point P

$$i_0 = i_1 + i_2 + i_3$$
$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \ldots \ldots . + \frac{1}{R_n}$$



Conclusions: (about parallel combination)

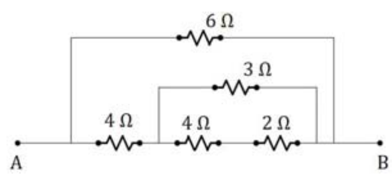
1. Potential difference across each resistor is same.
2. $I = I_1 + I_2 + I_3 + \cdots \ldots \ldots . I_n$.
3. Effective resistance (R) then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \ldots \ldots + \frac{1}{R_n}$. (R is less than each resistor).
4. Current in different resistors is inversely proportional to the resistance.

$$I_1 : I_2 : \ldots \ldots \ldots . I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \ldots \ldots . : \frac{1}{R_n}.$$
$$I_1 = \frac{G_1}{G_1 + G_2 + \cdots \ldots\ldots + G_n} 1, I_2 = \frac{G_2}{G_1 + G_2 + \cdots \ldots\ldots + G_n} 1, \text{etc.}$$

where $G = \frac{1}{R} =$ Conductance of a resistor. [Its unit is Ω^{-1} or ℧(mho)]

Ex. In the circuit shown below, find the equivalent resistance between A and B.

Sol. Consider the loop enclosed in blue box first. Resistances 4 Ω and 2 Ω are connected in series and 3 Ω is connected parallel to it. Thus, equivalent resistance of this loop is,



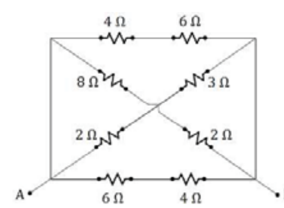
$$\frac{6 \cdot 3}{6+3} = \frac{6 \cdot 3}{9} = 2\Omega$$

Now, replacing this loop with a 2 Ω resistance new circuit will look the adjacent figure. Resistances 4 Ω and 2 Ω are connected in series and 6 Ω is connected parallel to it. Thus, equivalent resistance of this loop is,

$$R_{eq} = \frac{6 \times 6}{6+6} = 3\Omega$$
$$R_{eq} = 3\Omega$$

Ex. In the circuit shown below, find the equivalent resistance between A and B.

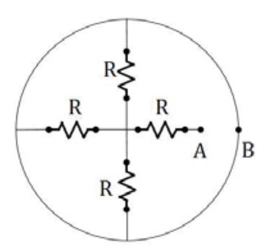
Sol. If we observe carefully, we can find that there are four branches, each having two resistances in series between point A and B. After adding the resistances in series in the branches, given circuit can be represented as shown in figure.



The equivalent resistance of the simplified circuit is

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} + \frac{1}{5} + \frac{1}{10} \Rightarrow \frac{1}{R_{eq}} = \frac{3}{10} + \frac{1}{5}$$
$$R_{eq} = \frac{10}{3+2} = 2\Omega$$
$$R_{eq} = 2\Omega$$

The process to determine the equivalent resistance in a complex circuit involves the following steps.



1. Suppose that there is a potential difference V across the terminals of the provided circuit, with one terminal at a potential of 0.
2. Consider two other unknown potentials, denoted as x and y, within the given circuit.
3. Designate V and 0 as the endpoints, and place x between them, on the given scale.
4. Position all the resistances between the designated points and proceed to solve the circuit.

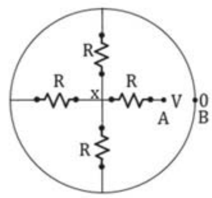
Ex. In the circuit shown below, find the equivalent resistance between A and B.

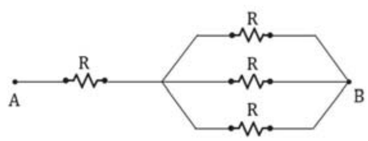
Sol. Here, three resistances R are connected in parallel through circular loop between point x to point B. Resistance R is connected in series with above combination between point A to x. The circuit can be represented as shown in the figure.

Thus, equivalent resistance is,

$$R_{eq} = R + \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = R + \frac{R}{3} = \frac{4R}{3}$$

$$R_{eq} = \frac{4R}{3}$$

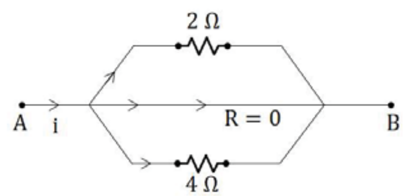




Ex. In the circuit shown below, find the equivalent resistance between A and B.

Sol. In an 'ideal' short circuit, there is no resistance and thus no voltage drop across the connection. The whole current will flow through the wire having no resistance.

Here, one of the three branches between point A to point B is short circuited. Thus, all current will flow through this branch and equivalent resistance of the circuit will be zero. This can also be calculated from formula as follows.



$$\frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{0} \Rightarrow R_{AB} = 0$$

$$R_{eq} = 0\Omega$$

Ex. n the circuit shown below, find the equivalent resistance between A and B.

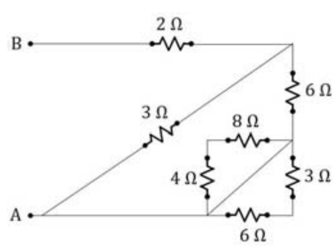
Sol. Here, one of the three branches is short circuited in the part of circuit enclosed in yellow circle. Thus, all current will flow short circuited conductor and equivalent resistance of the part will be zero. The equivalent circuit can be represented as shown in figure.

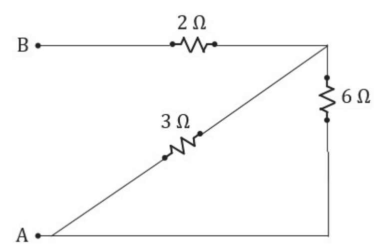
Equivalent resistance of the circuit is,

$$R_{eq} = 2 + \frac{1}{\frac{1}{3}+\frac{1}{6}}$$

$$= 2 + \frac{6}{3} = 4\Omega$$

$$R_{eq} = 4\Omega$$



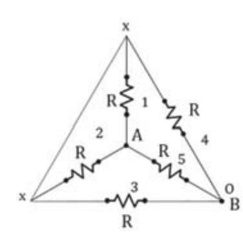


Ex. In the circuit shown below, find the equivalent resistance between A and B.

Sol. Number the resistances as shown in figure.

Resistance 1 and 2 are connected in parallel combination between point A to x. Similarly, resistance 3 and 4 are connected in parallel combination between point x to B. The circuit can be represented as shown in the figure.

Equivalent resistance of the circuit is,



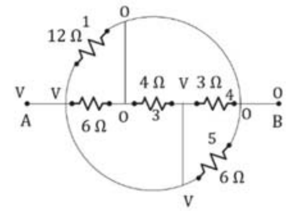
$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{\frac{R\times R}{R+R}+\frac{R\times R}{R+R}} = \frac{1}{R} + \frac{1}{\frac{R}{2}+\frac{R}{2}} = \frac{1}{R} + \frac{1}{R}$$

$$R_{eq} = \frac{R}{2}$$

$$R_{eq} = \frac{R}{2}$$

Ex. In the circuit shown below, find the equivalent resistance between A and B.

Sol. Number the resistances as shown in figure.



From figure, all resistances are connected in parallel. The circuit can be represented as shown in the figure.

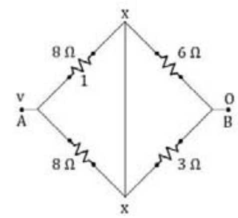
Equivalent resistance of the circuit is,

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6}$$

$$= \frac{4+2+1+3+2}{12} = 1\Omega$$

$$R_{eq} = 1\Omega$$

Ex. In the circuit shown below, find the equivalent resistance between A and B.



Sol. The circuit can be represented as shown in the figure.

Equivalent resistance of the circuit is

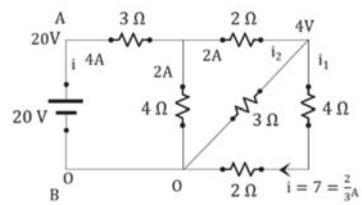
$$R_{eq} = \frac{8\times8}{8+8} + \frac{3\times6}{3+6}$$

$$= 4 + 2 = 6\Omega$$

$$R_{eq} = 6\Omega$$

Ex. In the electrical circuit shown below, find the current in 2 Ω resistance.

Sol. **Method 1: Using current distribution**



Equivalent resistance of the circuit, $R_{AB} = 5\Omega$

Current through battery, $i' = \frac{20}{5} = 4A$

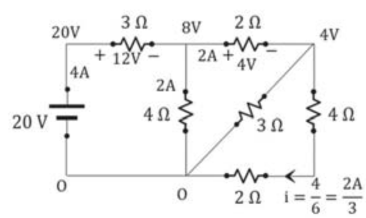
The 4A current divides into two branches. The 2A current further divides in two branches. Let's assume these currents be i_1 and i_2 as shown in figure.

Ratio of currents, $i_1 : i_2 = \frac{1}{6} : \frac{1}{3} = 1 : 2$

Current through 2Ω resistance, $i_1 = \frac{1}{3} \times 2 = \frac{2}{3}$

$$i = \frac{2}{3}A$$

Method 2: Using potential difference across resistor



Equivalent resistance of the circuit, $R_{AB} = S\Omega$

Current through battery, $i' = \frac{20}{5} = 4A$

The voltage drop across the resistance is found by multiplying current with resistance. The voltage at all points is shown in the figure.

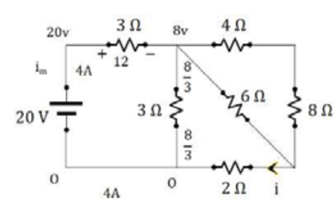
The voltage drops from 4V to 0 through 4Ω and 2Ω resistance.

Thus, current through 2Ω resistance, $i = \frac{4-0}{4+2} = \frac{4}{6} = \frac{2}{3}A$

$$i = \frac{2}{3}A$$

Ex. In the electrical circuit shown below, find the current in 2 Ω resistance.

Sol. Equivalent resistance of the circuit, $R_{AB} = 5\Omega$



Current through battery, $i' = \frac{20}{5} = 4\ A$

Current through 3Ω resistance, $i_{3\Omega} = \frac{8-0}{3} = \frac{8}{3}A$

Let the current through 2Ω resistance be i''.

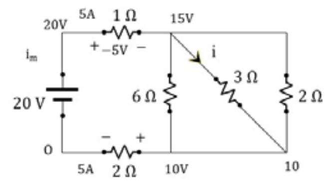
Applying Kirchhoff's current law, $i'' + \frac{8}{3} = 4$

$$i'' = 4 - \frac{8}{3} = \frac{4}{3}$$

$$i = \frac{4}{3}A$$

Ex. In the electrical circuit shown below, find the current in 3 Ω resistance

Sol. Equivalent resistance of the circuit, $R_{eq} = 4\Omega$



Current through battery, $i_m = \frac{20}{4} = 5A$

Let the current through branches from A to B be i_1, i_2 and i_3.

Using short trick, we can write, $i_1 : i_2 : i_3 = \frac{1}{6} : \frac{1}{3} : \frac{1}{2}$

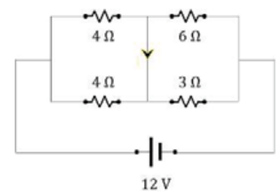
$= 1 : 2 : 3$

Therefore, $i_2 = \frac{y}{(x+y+z)}(i_m) = \frac{2}{1+2+3} \times 5 = \frac{5}{3}A$

$i = \frac{5}{3}A$

Ex. In the electrical circuit shown below, find the value of *i*.

Sol. Equivalent resistance of the circuit, $R_{eq} = 4\Omega$



Current through battery, $i_m = \frac{12}{4} = 3\text{ A}$

$i_m = 3A$

Simplified circuit:

Here point x represents point C and B on original diagram.

Potential at point C and B: $V_B = V_C = 12 - 3 \times 2 = 6V$

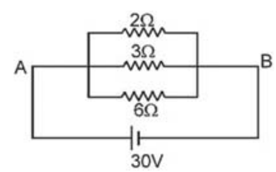
Current in branch AB: $i_{AB} = \frac{12-6}{4} = 1.5\text{ A}$

Current in branch BO: $i_{BO} = \frac{6-0}{6} = 1A$

Applying KCL at point B: $1.5 = i + 1 = i = 0.5A$

Ex. Find current passing through the battery and each resistor.

Sol. Method (I) It is easy to see that potential difference across each resistor is 30 V.



Current is each resistors are $\frac{30}{2} = 15\text{ A}, \frac{30}{3} = 10\text{ A And } \frac{30}{6} = 5\text{ A}$

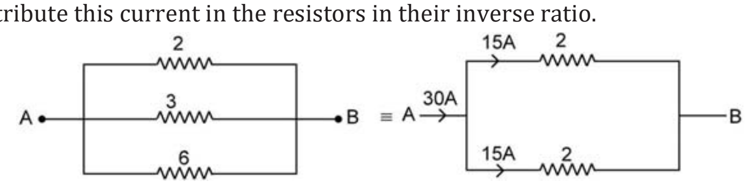
Current through battery is $= 15 + 10 + 5 = 30\text{ A}$.

Method (II)

By ohm's law $i = \frac{V}{R_{eq}} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1\Omega$

$R_{eq} = 1\Omega \Rightarrow i = \frac{30}{1} = 30A$

Now distribute this current in the resistors in their inverse ratio.

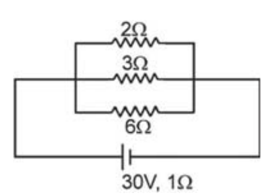


Current total in 3 Ω and 6 Ω is 15 A it will be divided as 10 A and 5 A.

The method (I) is better. But you will not find such an easy case everywhere

Ex. Find current which is passing through battery.

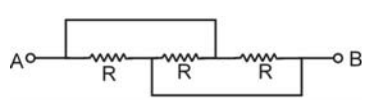
Sol. Here potential difference across each resistor is not 30 V Battery has internal resistance. Here the concept of combination of resistors is useful.



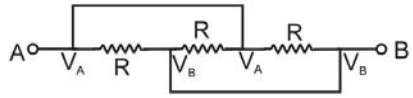
$R_{eq} = 1 + 1 = 2\Omega$

$i = \frac{30}{2} = 15A$

Ex. Find equivalent Resistance

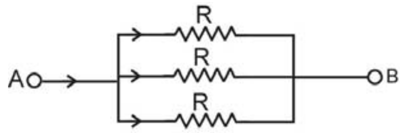


Sol.

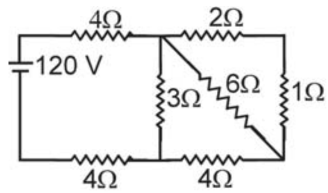


Here all the Resistance are connected between the terminals A and B Modified circuit is

$$R_{eq} = \frac{R}{3}$$



Ex. Find the current in 2Ω resistance



Sol. $2\Omega, 1\Omega$ in series $= 3\Omega$

$$3\Omega, 6\Omega \text{ in parallel } = \frac{18}{9} = 2\Omega$$

$$2\Omega, 4\Omega \text{ in series } = 6\Omega$$

$$6\Omega, 3\Omega \text{ in parallel } = 2\Omega$$

$$R_{eq} = 4 + 4 + 2 = 10\Omega$$

$$i = \frac{120}{10} = 12 \text{ A}$$

So current in 2Ω Resistance $= \frac{8}{3}$ A