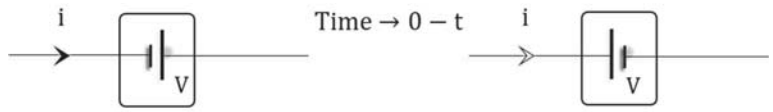
ELECTRICAL ENERGY AND POWER

Power in a battery and resistor

Power in battery



Energy supplied by cell (E) $= q \times V$

Power, $P = \frac{dE}{dt} = V \times \frac{dq}{dt} = Vi$

Power delivered by cell (P) $= Vi$

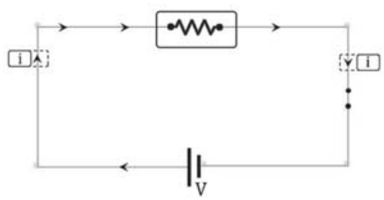
Energy stored by cell (E) $= -q \times V$

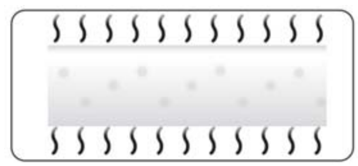
Power consumed by cell (P) $= -Vi$

Power In Resistor

The power dissipated across the resistance in the provided circuit is.

$$P_R = Vi$$
$$k = (iR)(i) = i^2R$$
$$P = i^2R = Vi = \frac{V^2}{R}$$





Power is consistently dissipated from the resistor, manifesting as heat and light.

A resistor is considered a passive component.

S.I. unit of power is joule /second or watt

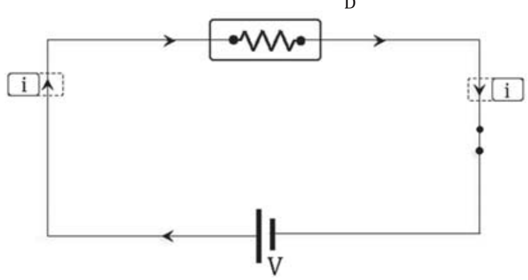
Power dissipated by resistor = Power delivered by cell

Energy / Heat

Energy realized in time (t) = Heat dissipated in time (t)

An electrical resistor transforms electrical energy into non-electrical forms, typically thermal energy or heat. Consequently, when a circuit is linked to an external battery, the resistance within the circuit generates heat. Therefore,

$$P = i^2R = Vi = \frac{V^2}{D}$$



Rate of generation of thermal energy

Therefore, $P = \frac{dE}{dt} = i^2R \Rightarrow \text{Heat (H)} = \int_0^t i^2R(dt)$

If the current (i) and the resistance (R) throughout the circuit remain constant, then the heat generated in the circuit over time (t) will be.

$$H = i^2Rt = Vit = \frac{V^2t}{R}$$

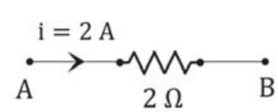
Ex. Current $i = 2$ A is flowing through a resistance R = 2 Ω as shown. Find the heat dissipated across the resistor in 3 s.

Sol. Given,

Current: i = 2 A

Resistance: R = 2Ω

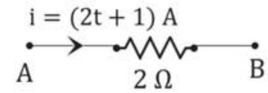
Time interval: 3 s



Given that the current (i) and the resistance (R) across the circuit are consistent, the heat generated within the circuit over a period of time t will be.

$$H = i^2Rt = 2^2 \times 2 \times 3 = 24J$$
$$H = 24J$$

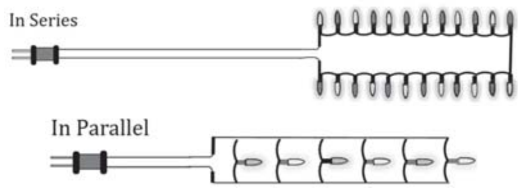
Ex. Current $i = (2t + 1)\ A$ is flowing through a resistance $R = 2\ \Omega$ as shown. Find the heat dissipated across the resistor in 5 s.



Sol. Current: $i = (2t + 1)A$ Resistance: $R = 2\Omega$ Time interval: $t = 5$ s
Since the current (i) is time dependent, heat produced in the circuit in time t will be,

$$H = \int_0^t i^2Rdt$$
$$H = 2\int_0^5 (2t + 1)^2dt = 2\int_0^5(4t^2 + 4t + 1)\,dt = 2[\frac{4t^3}{3} + \frac{4t^2}{2} + t]_0^5$$
$$H = 2[\frac{4t^3}{3} + 2t^2 + t]_0^5 = 2[\frac{4\times125}{3} + 50 + 5] = 2[\frac{500}{3} + 55] = 2 \times \frac{665}{3} = 443.33 \approx 443\Omega$$
$$H = 443J$$

Mini Light



In Indian households, all appliances are typically linked in parallel to ensure that each appliance receives an equal voltage supply from the power source.

Electricity Bill

In Indian households, the voltage supply is 220 volts AC with a frequency of 50 Hz.
All appliances are linked in parallel.
We receive electricity bills based on energy usage.

$$1\text{ unit } = 1kWh$$
$$[kWh = \text{ KiloWatt Hour }]$$
$$\Downarrow$$
1000 W appliance used for 1 hr consumes the 1 unit energy.
$$1\text{ unit } = 1kWh = 1KW \times 1hr = 1000\ W \times 3600sec = 36 \times 10^5J$$

Ex. In an Indian household, 2 fans each of 100 W run for 3 hours, 2 lights each of 50 W is switched on for 8 hours and 1 air-conditioner of 1000 W run for 2 hours. If energy unit cost ₹ 10/ unit, what will be the electric bill for one day?

Sol.

Appliance	Rating (W)	Time (T)	Energy Consumed
Fan	100	3	$2 \times 100 \times 3 = 600Wh$
Light	50	8	$2 \times 50 \times 8 = 800Wh$
Air-conditioner	1000	2	$1 \times 1000 \times 2 = 2000Wh$

Total energy consumed in the house in one day:
$$(600 + 800 + 2000) = 3400Wh = 3.4kWh = 3.4\text{ unit}$$
Therefore, the electric bill:
$$(3.4 \times 10) = 34₹$$
₹34.00

Ex. An electric kettle consists of two resistors R_1 and R_2. If only R_1 is used, it takes 3 min to make tea and if R_2 is used, it takes 5 min to make the tea. How much time it will take to make the tea, if R_1 and R_2 are used by connecting them in
(a) Parallel
(b) Series ?

Sol. 1. Since the electric kettle is connected in the household power supply, the potential difference (V) across it remains same all the time.

2. To make tea, a particular amount of heat is required. Therefore, heat required (H) to make tea is constant.

We know that when voltage is constant the power generated across the resistance is

$$\frac{V^2}{R} \times t = H$$

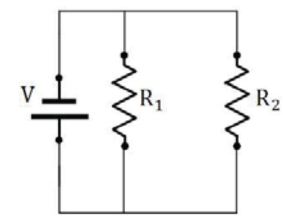
When only R_1 is used: $$\frac{V^2}{R_1} \times 3 = H \quad \dots (1)$$

When only R_2 is used: $$\frac{V^2}{R_2} \times 5 = H \quad \dots (2)$$

R_1 and R_2 are connected in Parallel

Let the time required to make the tea is t.

Therefore, the same heat H will be given by,



$$\left(\frac{V^2}{R_1} \times t\right) + \left(\frac{V^2}{R_2} \times t\right) = H$$

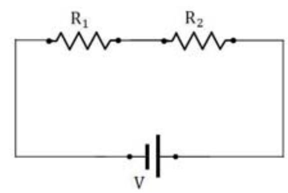
$(\frac{H}{3} \times t) + (\frac{H}{5} \times t) = H$[From equation (1) and (2)]

$$\frac{t}{3} + \frac{t}{5} = 1 \Rightarrow t = \frac{15}{8} \text{ min}$$

$$t = \frac{15}{8} \text{min}$$

R_1 and R_2 are connected in Series

In this case, the same heat H will be given by,



$$\frac{V^2}{(R_1+R_2)} \times t = H$$

$\frac{V^2}{\left[\frac{3V^2}{H} + \frac{5V^2}{H}\right]} \times t = H$[From equation (1) and (2)]

$$\frac{H}{(3+5)} \times t = H$$

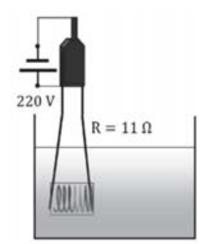
$$\frac{t}{8} = 1 \Rightarrow t = 8 \text{ min}$$

[From equation (1) and (2)]

$$t = 8 \text{ min}$$

Ex. In a container carrying 2 Kg water (S = 1 Cal/gm° C) at temperature T = 5 °C, an immersion rod of resistance R = 11 Ω is placed and connected to a voltage source of 220 V. If the rod is only 80 % efficient in transferring heat dissipated to the water, find the time taken for the water to reach 45 °C.

Sol. Let the time taken for the water to reach 45°C is t.



Mass of the water: m = 2 Kg

Initial temperature of the water: $T_i = 5$ °C

Final temperature of the water: $T_f = 45$°C

Specific heat capacity: $S = \frac{\frac{1 Cal}{gm°C} 4200 J}{kg°C}$

Heat required for the water: $mS\Delta T = mS(T_f - T_i)$

The immersion rod is connected with a power supply having voltage:

V = 220 Volts

Resistance of the immersion rod : R = 11Ω

Therefore, the heat generated by the immersion rod is: $(\frac{V^2}{R} \times t)$

It is given that the rod is only 80 % efficient in transferring heat dissipated to the water.

Therefore,

$$mS(T_f - T_i) = 0.8\left(\frac{V^2}{R} \times t\right)$$

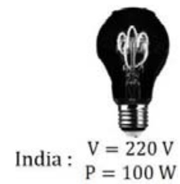
$$2 \times 4200 \times (45 - 5) = 0.8\left(\frac{220^2}{11} \times t\right)$$

$$2 \times 4200 \times 40 = 0.8 \times 220 \times 20 \times t$$

$$t = \frac{2100}{22} \approx 95.45 \text{sec}$$

$$t = 95.45 \text{sec}$$

Ex. A bulb rated 100 W and 220 V is made to be used in India. If it is used in USA where the voltage is 110 V, then find the power consumed by it.



Sol. Let the resistance of the bulb is R

Therefore, in India, we have,

$$P = \frac{V^2}{R} \Rightarrow 100 = \frac{220^2}{R} \Rightarrow R = \frac{220^2}{100}$$

In USA, the voltage is, $V = 110$ volt and the resistance of the bulb remains same.

Thus, the power consumed by the bulb in USA is,

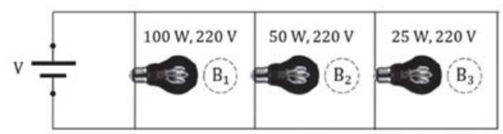
$$P = \frac{V^2}{R} \Rightarrow P = \frac{110^2}{(220^2/100)} \Rightarrow P = \frac{110^2 \times 100}{220^2} = 25\text{ W}$$

$$P = 25\text{W}$$

Ex. Three 220 V bulbs B_1, B_2 and B_3 rated at 100 W, 50 W and 25 W, respectively are connected in parallel . Arrange them in decreasing order of their brightness.

Sol.

$$\left.\begin{aligned}&\text{The resistance of } B_1: R_1 = \frac{220^2}{100}\Omega\\&\text{The resistance of } B_2: R_2 = \frac{220^2}{50}\Omega\\&\text{The resistance of } B_3: R_3 = \frac{220^2}{25}\Omega\end{aligned}\right\} R_3 > R_2 > R_1$$

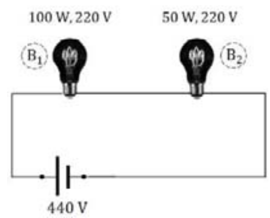


Since the bulbs are in parallel, the potential drop across each of them is same. Thus, their power should be determined by using, $P = \frac{V^2}{R}$. Since V is constant, $P \propto \frac{1}{R}$. Thus, $P_3 < P_2 < P_1$ and hence, the decreasing order of the brightness of the bulbs are: $B_1 > B_2 > B_3$.

Ex. Two 220 V bulbs B_1 and B_2 rated at 100 W and 50 W respectively are connected in series with a 440 V voltage source as shown, then:

(a) B_1 will burst first (b) B_2 will burst first

(c) Both will burst (d) None of them will burst



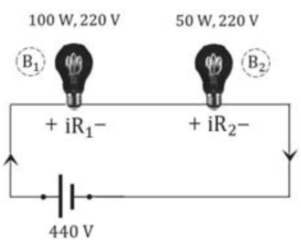
Sol. When the voltage across a bulb exceeds its rated voltage, then the bulb will burst.

The rating of electric bulb B_1: 100 W and 200 V

The rating of electric bulb B_2: 50 W and 200 V

The resistance of B_1: $R_1 = \frac{220^2}{100}\Omega = R$ (Say)

The resistance of B_2: $R_2 = \frac{220^2}{50}\Omega = 2R$



Since the bulbs are in series, the equivalent resistance of the connection is, $R_{eq} = R + 2R = 3R$

The voltage drop across B_1: $V_1 = \frac{440}{3R} \times R = \frac{440}{3} \approx 146.67$ V

The voltage drop across B_2: $V_2 = \frac{440}{3R} \times 2R = \frac{880}{3} \approx 293.33$ V

Since $V_1 < 220$ V but $V_2 > 220$ V, the bulb B_2 will burst.

Thus, option (b) is correct answer.