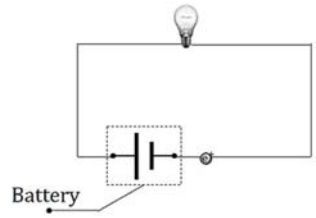
ELECTRIC CELL AND COMBINATIONS OF CELLS

Electric Cell



A device that sustains a potential difference across its terminals.

Transforms chemical energy into electrical energy.

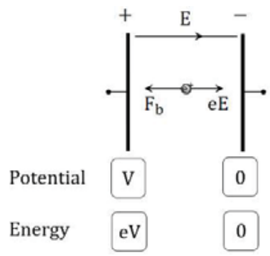
A battery serves as an energy source that provides power to the charge.

The charge acts as an energy carrier, transferring its energy to the resistance/load.

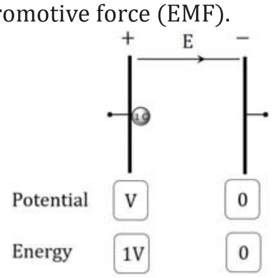
EMF and potential across battery

Electromotive Force

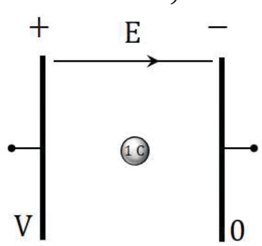
When we move a charge of 1 C from a low potential to a high potential, the internal force within the battery must perform work on it.



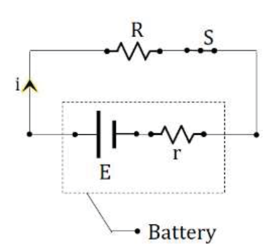
The work performed when moving a unit positive charge from a lower potential to a higher potential is referred to as electromotive force (EMF).



The energy expended to transport a unit positive charge from a lower potential to a higher potential is termed as EMF (Electromotive Force).



$$\text{Work done } = q \times \Delta V$$
$$q \times (V - 0)$$
$$q = 1C$$



Work done = 1 V

SI unit of EMF is volt (V)

Assume that a 1C charge battery has to do 12 J work to carry the lower potential to the higher potential. But the energy of 1 C increases only by 10 J, meaning 2 J Energy goes into internal loss.

We saw that 12 J will be called the EMF of the battery, 10 J will be called the potential drop in the battery. And we will represent 2 J as an internal resistance.

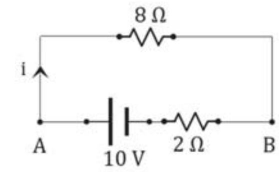
Ex. In following circuit, calculate:

(a) Potential difference across the terminals of the battery. (b) Current in the circuit.

Sol. Current in the circuit:

$$i = \frac{E}{R+r} = \frac{10}{8+2} = 1A$$

Potential difference across terminals of the battery:



$$V_A - 10 + 2 = V_B$$
$$V_A - V_B = 10 - 2$$
$$= 8\text{Volt}$$
$$V_B - V_A = 8\text{ V}$$

Potential difference across the battery depends on the external resistance. $V = ir$

Here,

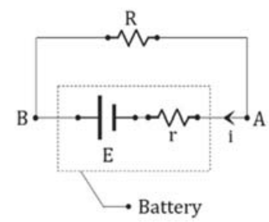
V = Depends on external circuit and battery both

E = Depends on battery i = Depends on external circuit

$$i = 1A$$

Potential Across the battery

$$V_B - E + ir = V_A$$
$$\text{P.D.} = V_B - V_A = V_{BA} = E - ir$$



r → Internal resistance of a cell

i → Current flowing through the circuit

Where i represents the circuit's current, contingent upon the external resistance.

The output voltage drawn from the battery is also influenced by the external resistance.

The potential difference is contingent upon the external resistance, whereas EMF remains unaffected by external resistance.

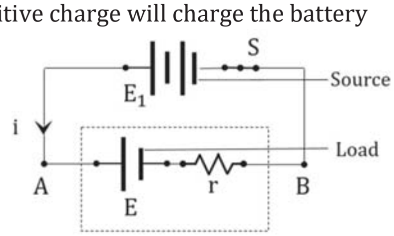
Charging and discharging of battery

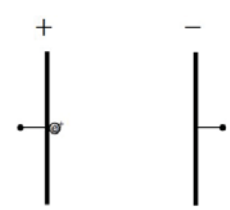
Charging of the battery

A bigger battery will recharge the smaller one as illustrated.

The larger battery compels positive charges to move within the smaller battery, from a higher potential to a lower potential.

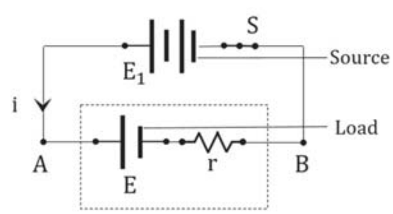
This positive charge will charge the battery

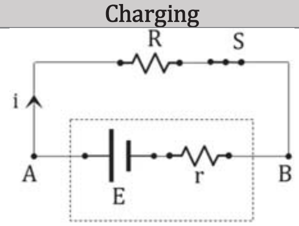
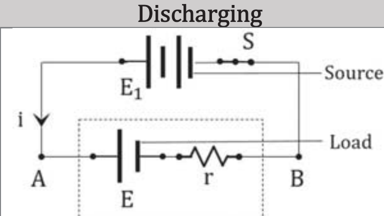




Potential across the battery is given by.

$$V_A - V_B = E + ir$$

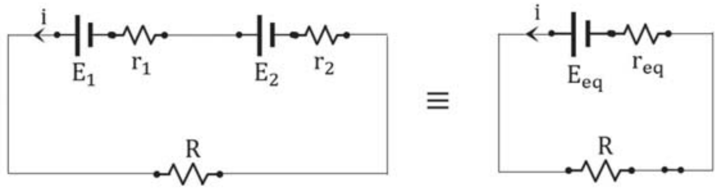


Charging	Discharging
	
$V_{AB} = E - ir$ $V_{AB} < E$	$V_{AB} = E + ir$ $V_{AB} > E$

$$\text{If } r = 0 \text{ then } V_{AB} = E$$

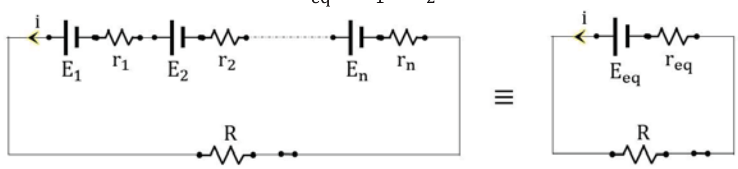
Series and parallel combination of cells

Series combination



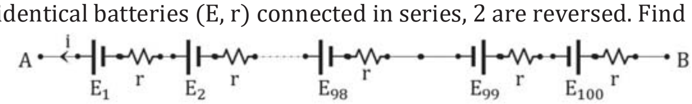
The total EMF of batteries arranged in series is the combined sum of the individual EMFs of all batteries. The EMFs of the batteries should be taken into account with their respective signs.
The total internal resistance of batteries arranged in series is the cumulative sum of the individual internal resistances, regardless of their orientation.

$$E_{eq} = E_1 + E_2$$
$$r_{eq} = r_1 + r_2$$



$$E_{eq} = \sum E_i$$
$$r_{eq} = \sum r_i$$

Ex. Out of 100 identical batteries (E, r) connected in series, 2 are reversed. Find out new E_{eq}, r_{eq}



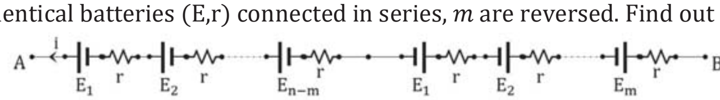
Sol. We know that $E_{eq} = \sum E_i$

$$E_{eq} = (98 - 2)E = 96E$$

(Because 98 batteries are arranged in same direction and 2 are opposite to them.)

$$E_{eq} = 100r$$
$$E_{eq} = 96E, r_{eq} = 100r$$

Ex. Out of *n* identical batteries (E,r) connected in series, *m* are reversed. Find out new E_{eq}, r_{eq}



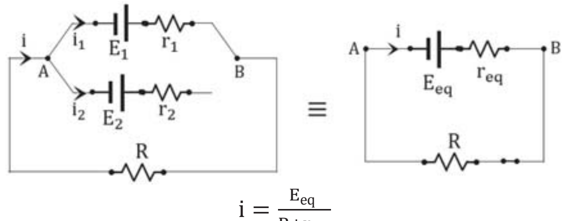
Sol. We know that $E_{eq} = \sum E_i$

$$E_{eq} = (n - m)E - mE = (n - 2m)E$$

(Because $(n - m)$ batteries are arranged in same direction and m are opposite to them.)

$$E_{eq} = nr$$
$$E_{eq} = (n - 2m)E, r_{eq} = nr$$

Parallel Combination



$$i = \frac{E_{eq}}{R + r_{eq}}$$

For n cells arranged in parallel combination, we can write following *n* equations for different loop

$$E_1 - i_1 r_1 - iR = 0 \quad \dots (1)$$
$$E_2 - i_2 r_2 - iR = 0 \quad \dots (2)$$
$$E_2 - i_3 r_3 - iR = 0 \quad \dots (3)$$

Dividing Equation by r_1 and r_2 and … r_n respectively

$$\frac{E_1}{r_1} = i_1 + i\frac{R}{r_1}$$

$$\frac{E_2}{r_2} = i_2 + i\frac{R}{r_2}$$

$$\sum\frac{E_i}{r_i} = i + iR\sum\frac{1}{r_1}$$

As stated earlier, $$i = \frac{E_{eq}}{R + r_{eq}}$$

$$i = \frac{\sum\frac{E_i}{r_i}}{1 + R\sum\frac{1}{\frac{1}{r_i}}}$$

Dividing numerator and denominator by $\sum\frac{1}{r_i}$

$$i = \frac{\sum\frac{E_i}{r_i}/\sum\frac{1}{r_i}}{R + \frac{1}{\sum\frac{1}{r_j}}}$$

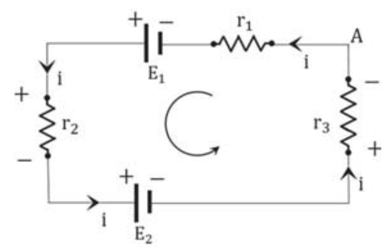
$$i = \frac{\frac{\sum\frac{E_i}{r_i}}{\sum\frac{1}{r_i}} \text{.................} E_{eq}}{R + \frac{1}{\sum\frac{1}{r_i}} \text{.................} r_{eq}}$$

If several batteries are connected in parallel combination, they can be replaced by one battery having EMF (E_{eq}) and internal resistance (r_{eq})

Equivalent voltage, $E_{eq} = \sum\frac{E_i}{r_i}/\sum\frac{1}{r_i}$

Equivalent resistance, $r_{eq} = \frac{1}{\sum\frac{1}{r_i}}$

Kirchhoff's Voltage Law (KVL)



Within a closed loop, the total algebraic sum of all potential differences equals zero.

Kirchhoff's Voltage Law (KVL) operates on the principle of energy conservation.

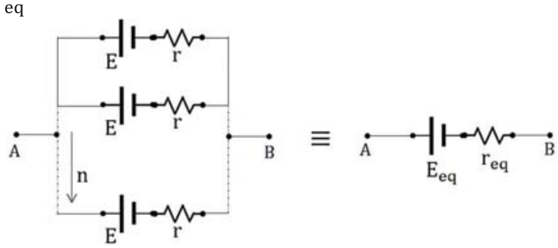
Applying KVL in the loop,

$$V_A - ir_1 + E_1 - ir_2 - E_2 - ir_3 = V_A$$

$$E_1 - ir_1 - E_2 - ir_2 - ir_3 = 0$$

Ex. If n identical batteries each of emf E and internal resistance r are connected in parallel combination as shown, then find E_{eq}, r_{eq}

Sol.



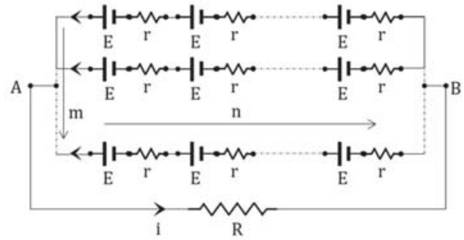
Equivalent EMF of the combination of cells,

$$E_{eq} = \frac{\sum\frac{E_i}{r_i}}{\sum\frac{1}{r_i}} = \frac{nE/r}{n/r} = E$$

Equivalent internal resistance of the combination of cells,

$$r_{eq} = \frac{1}{\sum\frac{1}{r_i}} = \frac{1}{n/r} = \frac{r}{n}$$

Ex. Consider N = nm identical cells, each of emf E and internal resistance r. Suppose n cells are joined in series and m such lines are connected in parallel. The combination drives a current in an external resistanceR. Find E_{eq} and r_{eq}.



Sol. Here, n cells are connected in series. Equivalent EMF of n such cells in series is

$$E_{n,\, series} = nE$$

Equivalent internal resistance of *n* cells in series is,

$$r_{n,\, series} = nr$$

We know that when same cells are connected in parallel combination, EMF of the equivalent combination remains same as EMF of individual EMF.

Here, we can assume m cells having EMF nE are connected in parallel. Thus, equivalent EMF of the circuit is,

$$E_{eq} = nE$$

When same cells are connected in parallel combination, internal resistance of the equivalent combination is,

Here, we can assume m cells having internal resistance *nr* are connected in parallel. Thus, equivalent internal resistance of the circuit is,

$$r_{eq} = \frac{1}{\sum \frac{1}{r_i}} = \frac{1}{m/nr} = \frac{nr}{m}$$

$$E_{eq} = nE, r_{eq} = \frac{nr}{m}$$