DRIFT SPEED, CURRENT DENSITY AND RESISTANCE OF MATERIAL

Mobility

The smoothness of charge carrier movement within the medium.

The measure of drift velocity relative to the applied electric field strength.

$$\mu = \frac{|v_d|}{E}$$

SI unit of mobility: $m^2V^{-1}s^{-1}$

$$\mu = \frac{1}{2}\left(\frac{e\tau}{m}\right)$$

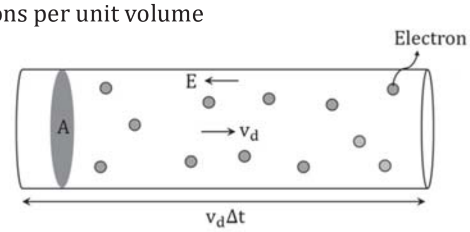
$$v_d = \frac{1}{2}\left(\frac{eE}{m}\right)\tau$$

Drift Speed And Current Density

Charge crossing through A in time Δt

$$\Delta Q = nAv_d\Delta te$$

n = No. of free electrons per unit volume



The current passing through a unit area is termed as current density.

$$j = \frac{\Delta Q}{A\times\Delta t} = \frac{nAv_d\Delta t}{A\Delta t}$$

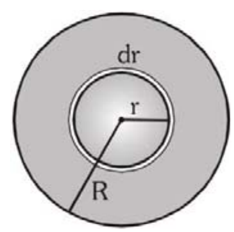
$$j = nev_d$$

Ex. The current density at a point is $\vec{J} = (2\times10^4\hat{j})Jm^{-2}$.

Find the rate of charge flow through a cross sectional area $\vec{S} = (2\hat{i} + 3\hat{j})cm^2$

Sol. The rate of flow of charges = current $= I = \int\vec{J}\cdot d\vec{S} \Rightarrow I = \vec{J}\vec{S} = (2\times10^4)[\hat{i}\cdot(2\hat{i} + 3\hat{j})]\times 10^{-4}$ A = 6 A

Ex. When a potential difference is applied across the ends of a wire composed of an alloy, it induces a current. The current density changes according to J = 3 + 2r, where r is the distance of the point from the axis. If R is the radius of the wire, the total current through any cross-section of the wire can be determined.



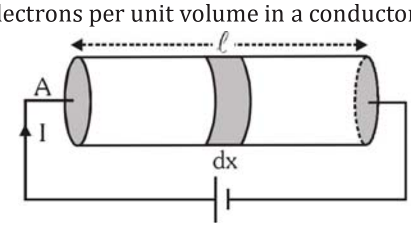
Sol. Take into account a circular strip with a radius r and thickness dr.

$$dI = \vec{J}\cdot d\vec{S} = (3+2r)(2\pi r dr)\cos 0^\circ = 2\pi(3r + 2r^2)dr$$

$$I = \int_0^R 2\pi(3r+2r^2)dr = 2\pi\left(\frac{3r^2}{2} + \frac{2}{3}r^3\right)^R = 2\pi\left(\frac{3R^2}{2} + \frac{2R^3}{3}\right) \text{ units}$$

Relation between Current Density, Conductivity and Electric Field

Let the number of free electrons per unit volume in a conductor = n



Total number of electrons in dx distance = n (Adx)

Total charge dQ = n(Adx)e

Current $I = \frac{dQ}{dt} = nAe\frac{dx}{dt} = neAv_d$, Current density $J = \frac{I}{A} = nev_d$

$= ne(\frac{eE}{m})\tau \because v_d = (\frac{eE}{m})\tau \Rightarrow J = (\frac{ne^2\tau}{m})E \Rightarrow J = \sigma E$, where conductivity $\sigma = \frac{ne^2\tau}{m}$

σ Depends only on the material of the conductor and its temperature.

In vector form $\vec{J} = \sigma\vec{E}$ Ohm's law (at microscopic level)

Ohm's Law and its validity

Ohm's Law

$$j = nev_d = \frac{ne^2\tau}{2m}E$$

$$\vec{j} = \sigma\vec{E}$$

σ = electrical conductivity of material

E = electric field applied

The voltage variance between two points correlates directly with both the current traversing through them and the resistance offered by the circuit between those points.

$$V = iR$$

$$E \cdot l = j \cdot A \cdot \frac{P_l}{l}$$

$$E = P_j$$

$$\vec{i} = \sigma\vec{E} \qquad (\because \frac{1}{p} = \sigma)$$

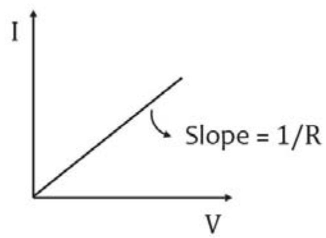
$$j = nev_d$$

Validity of Ohm's law

Ohm's law holds true solely for conductors exhibiting Ohmic behavior.

Ohmic conductors

Constant resistance



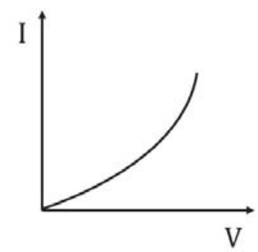
Metal at law v and I

(Provided temperature and other physical conditions remain constant.)

For example, Nichrome.

Non-Ohmic conductors

Variable resistance



Semiconductors, Alloys

V = RI

Resistance and resistivity

Resistance

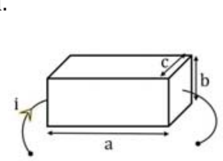
$$R = \frac{\rho l}{A}$$

l = Length along the direction of current

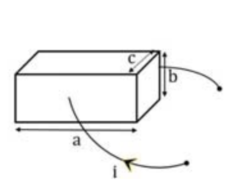
A = Area perpendicular to current direction

The resistance of the conductor varies based on the direction of current flow.

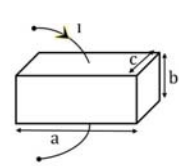
Here are examples where the resistance of a conductor changes when the direction of current flow is altered.



$R = \frac{\rho a}{bc}$

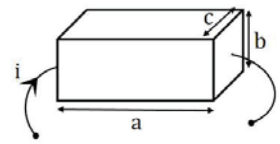


$R = \frac{\rho c}{ba}$



$R = \frac{\rho b}{ac}$

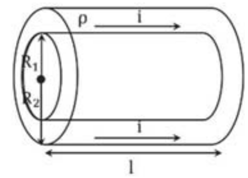
The resistance of a conductor is determined by



Material (as resistivity depends on material and temperature)
Dimensions
Direction of current

$$R = \frac{\rho l}{A}$$

l = Length along the direction of current
A = Area perpendicular to current direction



Ex. Find the resistance of the given conductor.

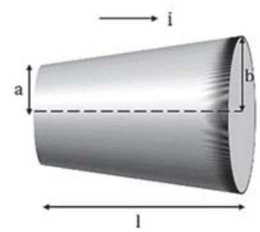
Sol. Cross section area of the conductor,

$$A = \pi[R_2^2 - R_1^2]$$

If ρ is the resistivity, resistance of the conductor is,

$$R = \frac{\rho l}{\pi[R_2^2 - R_1^2]}$$

Ex. A conductor of length *l* has a circular cross section as shown. The radius of cross section varies from a to b. The resistivity of the material is ρ. If $b - a \ll l$ then find the resistance of the conductor.

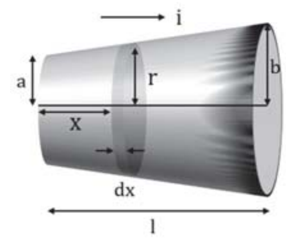


Sol. Consider a small disc of frustum as shown. The complete frustum is formed by arranging multiple such small discs with increasing radius in series.
As the radius of disc is not constant, assumption of disc causes some error. This error will be minimum when $b - a \ll l$ and thus, can be neglected.

Resistance of the elemental disc, $dR = \frac{\rho \cdot dx}{\pi r^2}$ ... (1)

From the figure, $r = \frac{(b-a)}{l}x + a$

Differentiating, $dr = \frac{(b-a)}{l} \cdot dx \Rightarrow dx = \frac{l}{(b-a)} \cdot dr$

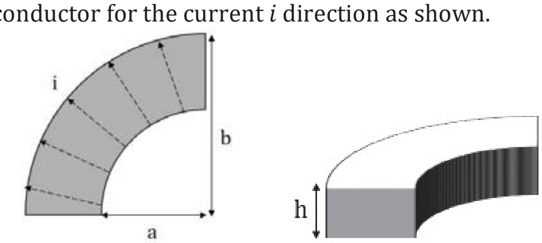


Putting in equation (1) and integrating for the complete frustum,

$$R_t^n = \int_a^b \frac{p \cdot l}{(b-a)\pi r^2} dr = \frac{Pl}{\pi(b-a)}\left[-\frac{1}{r}\right]_a^b = \frac{Pl}{\pi(b-a)} \cdot \left[\frac{1}{a} - \frac{1}{b}\right] = \frac{Pl}{\pi ab}$$

$$R = \frac{\rho l}{\pi ab}$$

Ex. A quarter part of a ring is given with an inner radius *a*, outer radius *b* and a height *h*. Find the resistance of the conductor for the current *i* direction as shown.

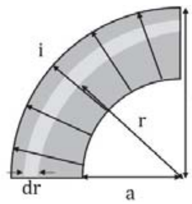


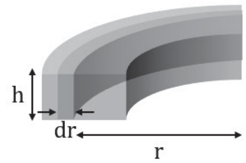
Sol. Here, the current is flowing radially outwards. Consider a small strip of quarter ring as shown. The complete quarter ring is formed by arranging multiple such small strips with increasing radial distance in series.

Resistance of the elemental strip, $dR = \frac{\rho \cdot dx}{\frac{\pi x h}{2}}$

Integrating for the complete quarter ring, $R^n = \frac{2\rho}{\pi h}\int_a^b \frac{dx}{x} = \frac{2p}{\pi h} \text{In}\left(\frac{b}{a}\right)$

$$R = \frac{2\rho}{\pi h}\ln\left(\frac{b}{a}\right)$$



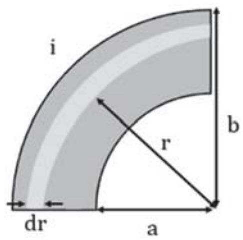


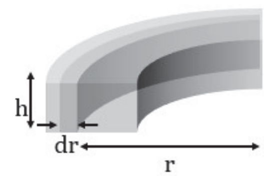
Ex. A quarter part of a ring is given with an inner radius *a*, outer radius *b* and a height *h*. Find the resistance of the conductor for the current *i* direction as shown.

Sol. Here, the current is flowing through a fix cross section. As the cross section is constant, we can directly write the resistance.

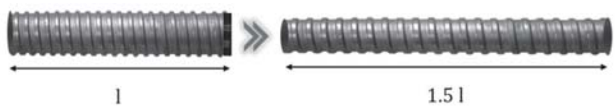
Given,
$$l = h$$
$$\text{area} = \frac{\pi(b^2 - a^2)}{4}$$

Resistance, $R = \frac{4\rho}{\pi(b^2 - a^2)}$





Ex. Given rod has a resistance R. This rod was beaten such that its length becomes 1.5 times of its initial value. What will be the new resistance of the rod ?



Sol. When the length of rod becomes 1.5 times of its initial value, let the area changes from A to A' . Assume the resistivity and density of material remains constant. Thus, volume remains the same after deformation.

As the volume remains same,
$$l \cdot A = 1.5l \cdot A'$$
$$A' = \frac{2A}{3}$$

Resistance before deformation, $R = \frac{\rho l}{A}$

The resistance after deformation,
$$R' = \frac{\rho \cdot 1.5l}{A'} = \frac{\rho \cdot 1.5l}{2Al_3} = \frac{\rho l}{A} \cdot \left(\frac{3}{2}\right)^2 = (1.5)^2 \cdot R = \frac{9}{4}R$$
$$R' = \frac{9}{4}R$$

Resistivity ($\boldsymbol{\rho}$)

Resistance, $R = \frac{\rho l}{A}$

Resistivity, $\rho = \frac{2m}{ne^2\tau}$

The electrical resistivity (or conductivity) of a material is contingent upon

- Types of material
- Temperature

Types of material

<table>
<tr><th>Material</th><th>Example</th><th>Resistivity</th><th></th></tr>
<tr><td>Conductors</td><td></td><td>$\rho \approx 10^{-12} - 10^{-6}\Omega \cdot m$</td><td rowspan="3">$\rho = \frac{2m}{ne^2\tau}$</td></tr>
<tr><td>Semiconductors</td><td></td><td>$\rho \approx 10^{-5} - 10^{5}\Omega \cdot m$</td></tr>
<tr><td>Insulators</td><td></td><td>$\rho \approx 10^{5} - 10^{16}\Omega \cdot m$</td></tr>
</table>

Relation between conductivity (σ) and resistivity(ρ):

$$\rho \propto \frac{1}{\sigma}$$

The greater the conductivity, the lower the resistivity.
Conductivity ranking in ascending order:
Insulator < Semiconductor < Conductor
Order of increasing resistivity:
Conductor < Semiconductor < Insulator