Chapter 3

Current

CONDUCTANCE AND RESISTANCE AND OHM'S LAW

In the realm of electrostatics, our exploration of electric phenomena has primarily centered around stationary charges. In the preceding chapter, we delved into the concept of electric potential, denoted in volts. Now, we will uncover that this voltage serves as an "electrical pressure" capable of inducing a flow of charge or current, quantified in amperes (or simply amps, abbreviated as A). The resistance impeding this flow is measured in ohms (Ω). Electric currents manifest in various professions.

Meteorologists focus on lightning and the more gradual movement of charge through the atmosphere. Professionals in biology, physiology, and medical technology are engaged with nerve currents regulating muscles, particularly their restoration post-spinal cord injuries.

Electrical engineers address diverse electrical systems like power systems, lightning protection systems, information storage systems, and music systems.

Space engineers scrutinize the flow of charged particles from the Sun, as it can disrupt telecommunication systems in orbit and even power transmission systems on the ground.

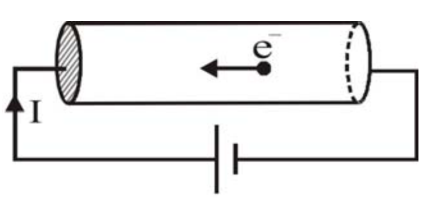
This chapter explores the fundamental physics of electric currents, elucidating why they can be established in certain materials but not in others.

We commence with an examination of the concept of electric current.

Electron Flow, Electric Current And Voltage

Electric Current

An electric current is formed when electric charges are in motion. A substance with electric charges that are essentially free to move is considered a conductor of electricity. The flow of electric charge occurs from a state of higher potential energy to a state of lower potential energy.



- Balanced and unbalanced Wheatstone Bridge
  - Balanced Wheatstone Bridge
  - Unbalanced Wheatstone Bridge
  - Symmetrical Circuits, Mirror and folding Symmetry
- Symmetrical Circuits, mirror and folding symmetry
  - Symmetrical Circuits, mirror and folding symmetry
  - Equivalent resistance of infinite resistant network
- Electric cell and combinations of cells
  - Electric cell
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  - Charging and discharging of battery
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  - KVL
  - Power in a battery
- Electrical Energy and power
  - Power in a battery and resistor
  - Energy / Heat
- Maximum Power transfer theorem and Measuring Instrument
  - Maximum power transfer theorem
  - Power vs Resistance and Efficiency vs Resistance
  - Measuring instrument - Battery Eliminator, Resistance Box, Rheostat, Galvanometer, Ammeter and voltmeter

Positive charge moves from regions of higher potential to lower potential, while negative charge moves in the opposite direction, from lower to higher potential. Metals like gold, silver, copper, and aluminum exhibit excellent conductivity. In a conductor, when electric charge flows from one location to another, the flow rate of charge is referred to as electric current (denoted as I). When charge is transferred within a conductor from one point to another, we identify the presence of electric current in that region. For positive moving charges, the current aligns with the direction of motion, while for negative charges, the current moves in the opposite direction. The average electric current through an area, when a charge ΔQ crosses it in a time interval Δt, is defined as.

Average current $I_{av} = \frac{\Delta Q}{\Delta t}$

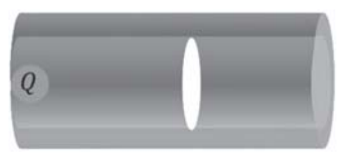
Instantaneous current $I = \text{Lim}_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

Unit

SI unit of electric current is *ampere*

$$1\text{ampere}(A) = \frac{1C}{1S}$$

One coulomb of charge passing through an area in one second.



$\Delta t = 1s$

Ex. For a given charge $q = 2t^2 + 1$ find the current at $t = 2s$.

Sol. Given$q = 2t^2 + 1$. Therefore, the current at $t = 2s$ is.

$$i = \frac{dv}{dt} = 4t$$

$$i = 8\text{ A}$$

This is the instantaneous current that we have found.

Suppose it is said to find the average current till 2 s.

Initial scenario: $t = 0q_i = 1$

Final scenario: $t = 2\sec q_\tau = 2 \times 2^2 + 1 = 9$

Therefore, the average current till 2 s will be,

$$i_{average} = \frac{q_f - q_i}{\Delta t}$$

$$i_{average} = \frac{9-1}{2}$$

$$i_{avg} = 4A$$

Ex. The current is given as $i = 2t^2$ find the charge flow till $t = 2s$.

Sol. We know, $i = \frac{dQ}{dt}$

$$\int dq = \int i \cdot dt$$

$$q = \int i(t) \cdot dt$$

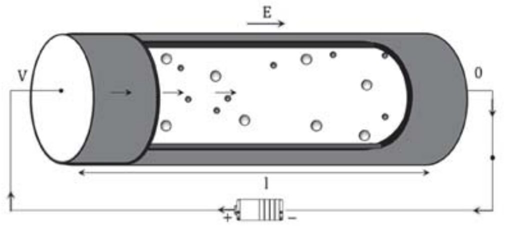
It is given that $i = 2t^2$. Therefore, the charge flow till $t = 2s$ is.

$$\int dq = \int_0^2 2t^2 \cdot dt$$
$$q_T = 2 \cdot [\frac{t^3}{3}]_0^2 = \frac{2}{3}[2^3 - 0]$$
$$q = \frac{16}{3}C$$

Electric Current

Cell:

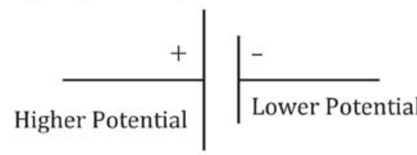
It supplies a potential difference to a conductor, which in turn delivers energy to the conductor.



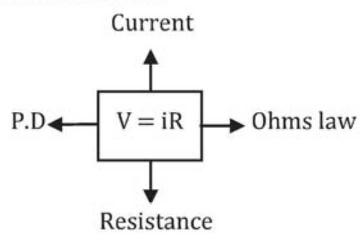
Notation of cell:

The extended bar indicates the side of the battery with higher potential, while the shorter bar signifies its lower potential side.

The cell generates energy, allowing free electrons to move within a conductor, thus forming an electric current in the conductor.



We know that the Ohm's law is defined as:



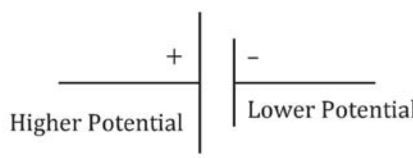
Ex. Find current i?

Sol. Applying Ohm's law, we get: $P \cdot d = 8 - 2 = i \times 2$

$i = 3A$

Notation of electric cell or battery:

The extended bar indicates the battery's side with higher potential, while the shorter bar represents its side with lower potential.

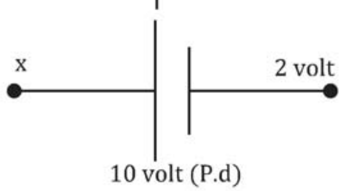


Let's say the potential difference of a battery is 10 volts, with the potential on the lower side being 2 volts.

Therefore, if the potential of the battery at the higher potential side is represented by x, then,

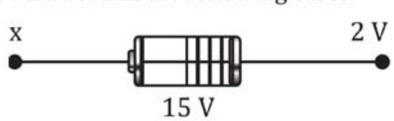
Higher potential – Lower potential = 10

$$x - 2 = 10$$
$$x = 12\text{Volt}$$

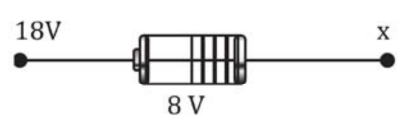


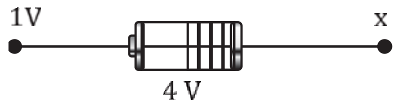
Ex. Find the value of x in the following cases.

(a)



(b)



(c) 

Sol. (a) $x - 2 = 15$
$x = 17$ Volt
(b) $18 - x = 8V$
$x = 10$ Volt
(c) $x - 1 = 4$
$x = 5$ Volt

(a) 17 V (b) 10 V (c) 5 V

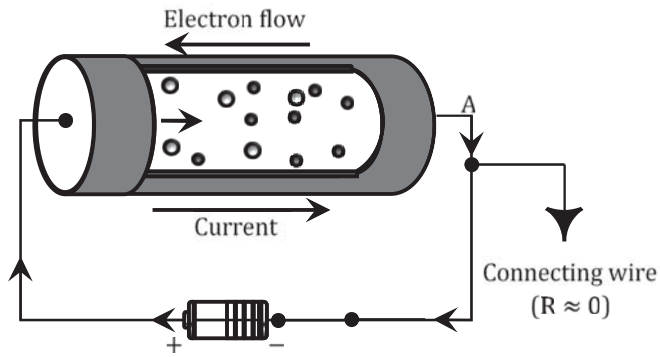
Connecting Wires

The potential difference across two points on the ideal connecting wire is consistently zero.

Hence, $V_A = V_B$

If a 2 Ω resistance is linked with a 10-volt battery, then the resulting current passing through the resistance will be,

$$i = \frac{10}{2} = 5A$$



Suppose the current in the circuit is created by the motion of positive charge carriers, represented by e^+ or positrons.

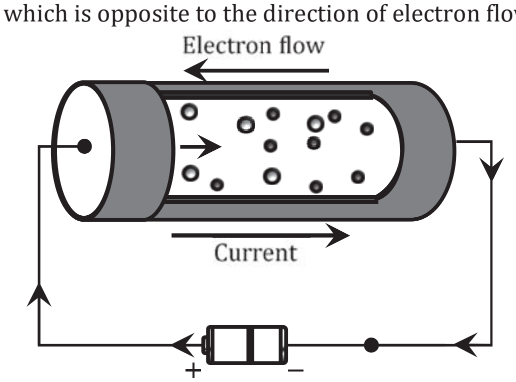
When the positive charge carrier (e^+ or positron) is positioned at the negative terminal of the battery, it possesses no energy. Nevertheless, once the same positron traverses through the battery and reaches the positive terminal, it acquires energy, denoted as $U = 10$ eV.

As the positron moves through the connecting wire, it maintains its energy without any loss. However, when it traverses through the resistance, it dissipates all of its energy. Therefore, the conclusion is as follows:

- The battery supplies energy to the charge carriers, typically electrons in reality.
- The resistance absorbs the energy carried by the charge carriers.
- The charge carrier conveys the energy.

Direction

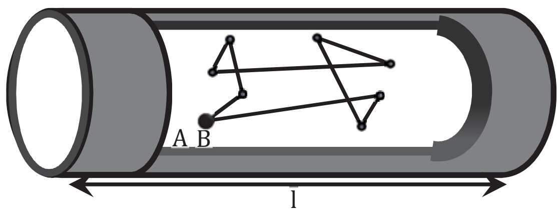
The electric current's direction is considered from positive (high potential) to negative (low potential) terminal, which is opposite to the direction of electron flow.



Drift Velocity and current

Drift Velocity

When a conductor or any material is linked to a battery, electrons typically undergo random motion, referred to as "Brownian motion," as they gain thermal energy from the surroundings of the conductor.



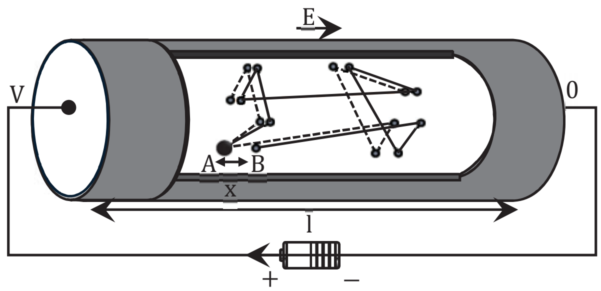
As a consequence of the free electrons' random motion within the conductor, the following outcomes are observed:

Net displacement = 0

Speed ≠ 0

Average velocity = 0 [Since net displacement is zero]

When a conductor is linked to a battery, it generates an electric field E within the conductor. Due to this electric field, the charge carriers (assuming e^+) undergo a force and are displaced by a distance x. Consequently, the charge carriers' effective displacement becomes x.



With battery connected, the motion of the charge carriers will remain random but there will be some effective displacement. Thus, the following results are observed:

Net displacement ≠ 0

Speed ≠ 0

Avg. velocity ≠ 0

$$\text{Speed} = \frac{\text{Total distance (from A to B)}}{\text{Time}}$$

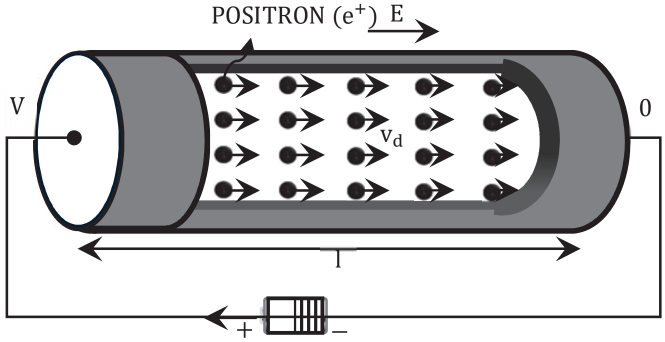
$$\text{Drift velocity } (v_d) = \frac{\text{Total displacement } (x)}{\text{Time}}$$

$$\text{Speed} \ggg \text{Drift velocity } (v_d)$$

Drift Velocity vd:

Net displacement per unit time.

Speed >> |Average Velocity|



Assumption All positrons are arranged in alignment and move with a constant drift velocity without undergoing collisions with one another.

Consider a cross-sectional area A, as depicted in the figure.
Considering the electric field, we understand that the average velocity of the electrons is the drift velocity vd. Consequently, within the time interval dt, the electrons travel a distance. $\Delta x = v_d dt$
Assuming n represents the number of electrons per unit volume (i.e., mobile electron density), then the charge passing through the cross-sectional area A within time dt is:

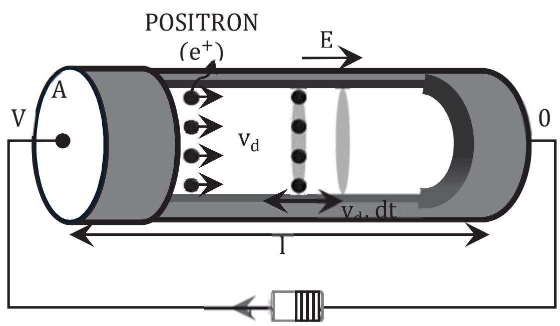
$$N = ndV = nAv_d dt$$

Hence, the total charge crossing through A in time dt is given by,

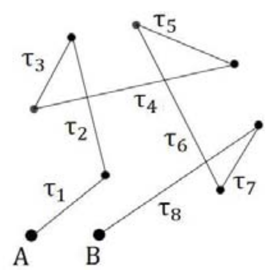
$$dq = Ne = (nAv_d \Delta t)e$$

[e= Charge of one positron or, electron]
Now, since the current is defined as; $i = \frac{dQ}{dt}$, therefore,

$$\frac{dq}{dt} = i = neAv_d$$



Mean Free Path and Relaxation Time



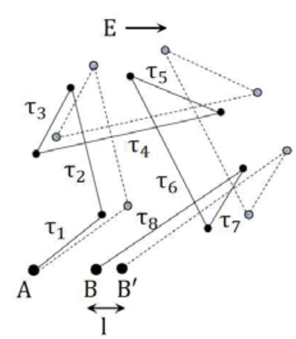
$\lambda =$ Mean free path
(Distance b/w two consecutive collisions)
$\tau =$ Relaxation time
If there are N successive collisions, the average time between each collision is

$$\tau = \frac{\tau_1 + \tau_2 + \tau_3 \ldots\ldots + \tau_N}{N}$$

Similarly, the average distance between successive collisions,

$$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 \ldots\ldots + \lambda_N}{N}$$

Drift Speed



Let's suppose the positron undergoes a shift to point B' due to the force exerted on it by the external electric field. Consequently, it appears as though the positron drifts from point B to B' due to the external electric field, and the duration of this process is denoted as the average relaxation time (τ).
Assuming the mass of a positron to be m, we know that its acceleration due to the electric field will be, $|a| = \frac{eE}{m}$
If we consider that the electron initiates motion from a stationary position but experiences acceleration a, then by applying the law of motion in one dimension, we obtain

$$l = ut + \frac{1}{2}at^2 \Rightarrow \lambda = \frac{1}{2}(\frac{eE}{m})\tau^2$$

The drift speed of positrons (which are actually electrons) refers to the average velocity they achieve within a conductor under the influence of an electric field. Mathematically, it is defined as follows:

$$v_d = \frac{\lambda}{\tau} \Rightarrow v_d = \frac{1}{2}(\frac{eE}{m})\tau$$

$$v_d = \frac{1}{2}(\frac{eE}{m})\tau$$

Ohm's Law

We have: $i = neAv_d$ and $v_d = \frac{1}{2}(\frac{eE}{m})\tau$

Combining these two relations, we get,

$$i = neAVd$$

$$i = neA \cdot \frac{eEC}{2m} = \frac{ne^2A\tau}{2m} \cdot \frac{V}{l}$$

For a conductor this is constant

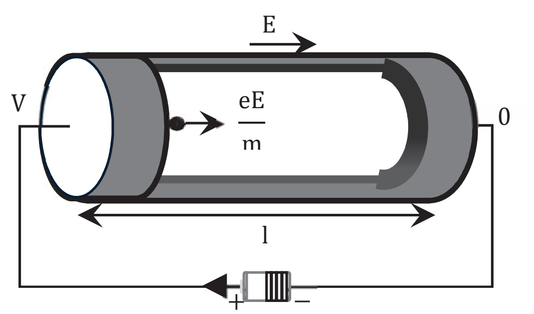
$$\frac{V}{i} = \frac{2m}{ne^2c} \cdot \frac{l}{A}$$

$$\frac{V}{i} = \text{ Constant]} \rightarrow \text{ Ohm's Law.}$$

Now, the constant represents the resistance of the conductor, defined as follows:

$$R = \frac{2m}{nc^2\tau}\frac{l}{A}$$

S.I Unit of resistance: (OhmΩ)



$$V = iR > R = \frac{2ml}{ne^2\tau A}$$

Upon close examination of the expression for R, it becomes evident that the resistance is contingent upon both the conducting material's properties (n,τ) and its dimensions (l, A).

The term $\frac{2m}{ne^2\tau}$ is known as the resistivity (ρ) of the conducting material and this term purely depends on the property of the material.

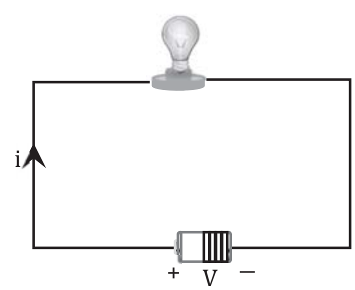
Hence, the expression for R can be formulated as:$R = \frac{\rho l}{A}$

Ohms Law Statement

The voltage drop across a conductor increases directly with the current passing through it, provided that the temperature remains constant.

$$V = iR$$

Where, $R = \frac{\rho l}{A}$



Resistivity and conductivity

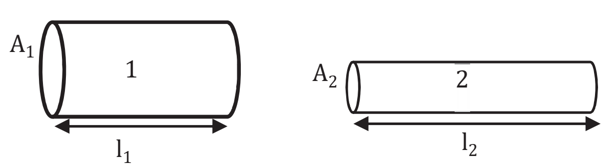
Ohms Law Resistivity

$\rho = \frac{2m}{ne^2\tau}$ S.I Unit: Ω. m

If we consider two cut pieces of the same conducting material but with different dimensions, their resistivity will be the same, but not their resistance.

Resistance of cut-piece 1: $R_1 = \rho\frac{l_1}{A_1}$

Resistance of cut-piece 2: $R_2 = \rho\frac{l_2}{A_2}$



Ohms Law Conductance And Conductivity

$$R = \frac{\rho l}{A} \text{ S.I Unit: } \Omega$$

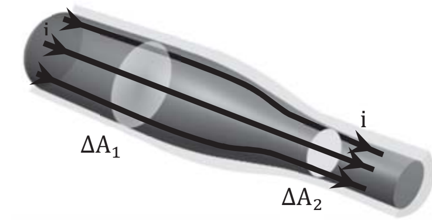
$$\rho = \frac{2m}{ne^2\tau} \text{ S.I Unit : } \Omega.\text{m}$$

Conductance, $G = \frac{1}{R}$ S.I Unit: ℧ or mho

Conductivity, $\sigma = \frac{1}{\rho}$ S.I Unit: ℧/m or mho/m

Electric Current Density

A vector quantity with a magnitude equivalent to the electric current per unit infinitesimal normal area at any point within a conductor.



$$j_{avg} = \frac{\Delta i}{\Delta A}$$

$\vec{J} \rightarrow$ direction same as i

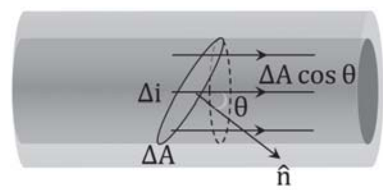
$\Delta A \rightarrow \perp$ to the current flow

The instantaneous current density is defined as,

$$j = \frac{di}{dA}$$

If a current Δi flows through an area ΔA, and the angle between the direction of the current flow and the direction of the area vector is θ, as depicted in the figure, then the current density is mathematically defined as follows:

$$|\vec{J}| = \frac{\Delta i}{\Delta A \cos\theta}$$



Hence, if the current density remains constant and the area is known, we can determine the current by applying the following equation:

$$i = \vec{j} \cdot \Delta\vec{A}$$

If the current density varies across the conductor, we can still determine the current using the following method:

$$i = \int \vec{j} \cdot d\vec{A}$$

Categorization of Materials Based on Conductivity

1. **Conductor**

   Certain materials exhibit weak binding of outer electrons in each atom or molecule. These electrons, commonly referred to as free electrons or conduction electrons, have a high degree of mobility within the material. When such a material is subjected to an electric field, these free electrons move in a direction opposite to the field materials with this characteristic are termed conductors.

2. **Insulator**

   Another category of materials is known as insulators, wherein all electrons are firmly bound to their respective atoms or molecules. In essence, there are no free electrons present. When these materials are exposed to an electric field, the electrons may undergo slight shifts in the direction opposite to the field, but they are unable to detach from their parent atoms or molecules. Consequently, they cannot traverse long distances. These materials are alternatively referred to as dielectrics.

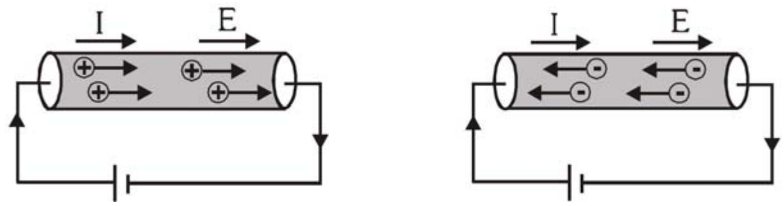
3. **Semiconductor**

   In semiconductors, at lower temperatures, their behavior resembles that of an insulator. However, at elevated temperatures, a limited number of electrons gain the freedom to respond to an applied electric field. Due to the considerably fewer free electrons in semiconductors compared to conductors, their behavior falls between that of a conductor and an insulator, giving rise to the term "semiconductor." When an electron is liberated in a semiconductor, it creates a vacancy in its usual bound position, and these vacancies also contribute to the conduction process.

Note

- Current is a fundamental quantity characterized by the dimensions $[M^0 L^0 T^0 A^1]$.
- Current is a scalar quantity with its SI unit ampere.
- Ampere: The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross–section of wire.

- The conventional direction of current is aligned with the flow of positive charge or the applied electric field. This direction is opposite to the flow of negatively charged electrons.



- The conductor remains electrically neutral when a current flows through it, as the rate of charge entering one end per second is equal to the rate of charge leaving the other end per second.
- For a given conductor current does not change with change in its cross–section because current is simply rate of flow of charge.
- If n particles each having a charge q pass per second per unit area then current associated with cross–sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross–sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$. The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi}$.

Behavior of conductor in absence of applied potential difference

In the absence of an applied potential difference, electrons exhibit random motion with zero average displacement and average velocity. Due to this thermal motion of free electrons in a conductor, there is no flow of current. The free electrons within a conductor acquire energy from the surrounding temperature, causing them to move randomly.

The speed gained by virtue of temperature is called as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron

At room temperature $T = 300$ K, $v_{rms} = 10^5$ m/s

Mean free path λ: $(\lambda \sim 10\text{Å})$, $\lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$