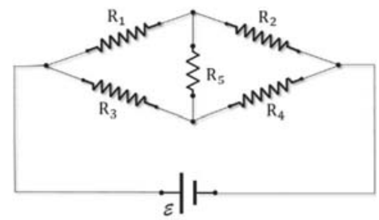
BALANCED AND UNBALANCED WHEATSTONE BRIDGE

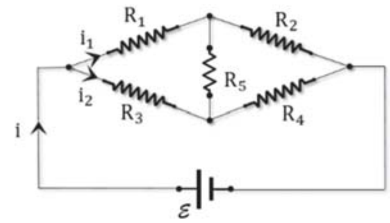
Wheatstone Bridge



A Wheatstone bridge connection comprises a specific arrangement of five resistors, where none of the resistors are directly in series or parallel with each other.

Typically, the majority of questions focus on balanced Wheatstone bridges.

Balanced Wheatstone Bridge



If there is no current through R_5 the bridge is said to be balanced.

The bridge is considered to be in a balanced state.

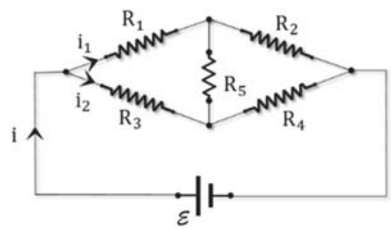
KVL between voltage V to x: $V - x = i_1R_1 = i_2R_3$ ... (1)

KVL between voltage x to 0: $x - 0 = l_1R_2 = l_2R_4$ ... (2)

Dividing equation (1) by (2)

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_1 \times R_4 = R_3 \times R_2$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ or } R_1R_4 = R_3R_2$$



If there is no current through R_5, the bridge is balanced

As no current flows through R_5 in balanced WB, circuit can be shown as given figure

Equivalent resistance of balanced WB:

$$R_{eq} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

Ex. Find the electric current through the segment BC in the circuit shown.

Sol. This is not a Wheatstone bridge. It would have been a Wheatstone bridge if battery had been connected between A and D.

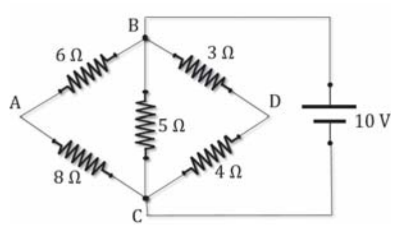
This is a simple circuit with three branches in parallel connection. Equivalent circuit is shown below.

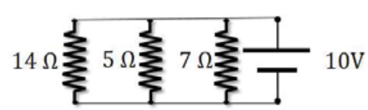
Thus, voltage at terminal B: $V_B = 10$ V

Voltage at terminal C: $V_C = 0$ V

Current through segment BC: $i_{BC} = \frac{10-0}{5} = 2A$

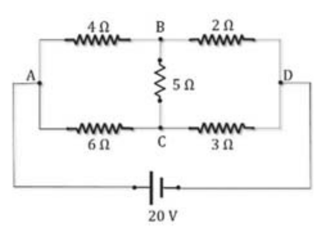
$$i = 2\text{ A}$$





Ex. Find the equivalent resistance between the points A and D in the circuit shown.

Sol. Here $\frac{R_1}{R_2} = \frac{R_3}{R_4}$



Thus, this is a balanced Wheatstone bridge.

Equivalent resistance of balanced WB

$$R_{eq} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

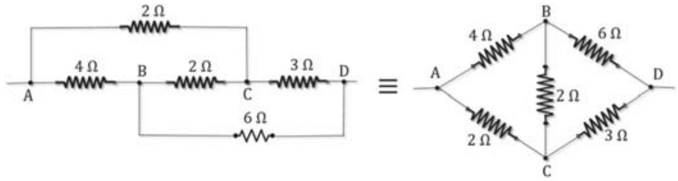
$$= \frac{(4+2)\times(6+3)}{4+6+2+3} = \frac{6\times 9}{15} = \frac{18}{5}\,\Omega$$

$$R_{eq} = \frac{18}{5}\,\Omega$$

Ex. Find equivalent resistance between the points A and D in the circuit shown.

Sol. This is a different representation of Wheatstone bridge. It can be reframed to usual form as shown in figure $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Thus, this is a balanced Wheatstone bridge.



Equivalent resistance of balanced WB:

$$R_{eq} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

$$R_{eq} = \frac{(4+6)\times(2+3)}{4+6+2+3}$$

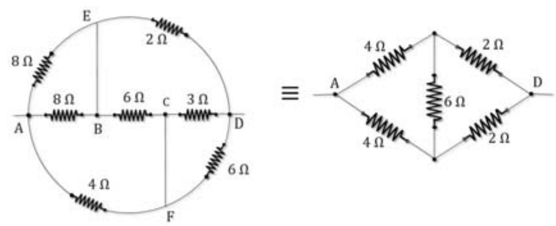
$$= \frac{10\times 5}{15}$$

$$= \frac{10}{3}\Omega$$

$$R_{eq} = \frac{10}{3}\Omega$$

Ex. Find equivalent resistance between the points A and D in the circuit shown.

Sol. Rearrange the given circuit as follows



Here, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Thus, this is a balanced Wheatstone bridge.

Equivalent resistance of balanced WB:

$$R_{eq} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_5+R_4}$$

$$= \frac{(4+2)\times(2+4)}{4+2+2+4}$$

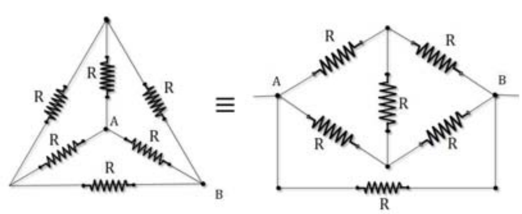
$$= \frac{6\times 6}{12} = 3\Omega$$

$$R_{eq} = 3\Omega$$

Ex. Find equivalent resistance between the points A and B in the circuit shown.

Sol. Rearrange the given circuit as follows:

Thus, circuit between point A and B is a Wheatstone bridge and one more resistance is connected parallel to it.



Equivalent resistance of balanced WB:

$$R_{AB} = \frac{(R_1+R_2)(R_3+R_4)}{R_1+R_2+R_3+R_4}$$

$$= \frac{(R+R)\times(R+R)}{R+R+R+R} = R$$

Equivalent resistance of the circuit:

$$R_{eq} = \frac{R \times R}{R+R} = \frac{R}{2}$$

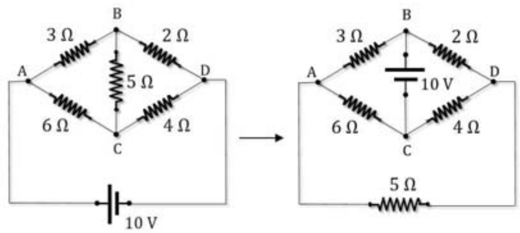
$$R_{eq} = \frac{R}{2}$$

Ex. Find current through 5 Ω resistance after interchanging position of 5 Ω resistance with the battery in the balanced Wheatstone bridge shown.

Sol. After interchanging the position of 5 Ω resistance, circuits becomes as shown in the figure
The resultant circuit is also a balanced Wheatstone bridge

Here $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Thus, this is a balanced Wheatstone bridge.



We know that the current through central resistance for balanced Wheatstone bridge is zero.

$$i = 0$$

Unbalanced Wheatstone Bridge

V − i method to determine the equivalent resistance of a circuit

Let's consider the potential at point B to be x and at point C to be y.

Current through branch BD: $i_1 = \frac{x}{2}$

Current through branch CD: $i_2 = \frac{y}{2}$

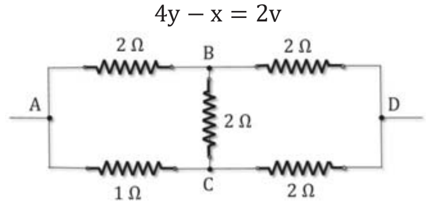
Applying KCL at point B, $\frac{x-V}{2} + \frac{x-y}{2} + \frac{x-0}{2} = 0$

$$3x - V = y \quad \text{... (1)}$$

Applying KCL at point C, $\frac{y-V}{1} + \frac{y-x}{2} + \frac{y-0}{2} = 0$

$$2y - 2v + y - x + y = 0$$

$$4y - x = 2v \quad \text{... (2)}$$



Putting value of y from equation (1) in equation (2),

$$4(3x - v) - x = 2v$$

$$12x - 4v - x = 2v$$

$$11x = 6v$$

$$x = \frac{6v}{11}$$

Putting value of x in equation (1),

$$\frac{18v}{11} - v = y$$

$$\frac{7v}{11} = y$$

Equivalent resistance of the circuit,

$$R_{eq} = \frac{V}{i} = \frac{V}{i_1 + i_2} = \frac{V}{\frac{x}{2} + yi_2}$$

$$= \frac{2v}{x+y} = \frac{2v}{\frac{6v}{11} + \frac{7v}{11}} = \frac{22\Omega}{13}$$

$$R_{eq} = \frac{22}{13}\Omega$$

Ex. Find the equivalent resistance between A and B for the given circuit diagram.

Sol. Assume the potential at point B and C be x and y respectively

Current through branch BD: $i_1 = \frac{x}{1}$

Current through branch CD: $i_2 = \frac{y}{2}$

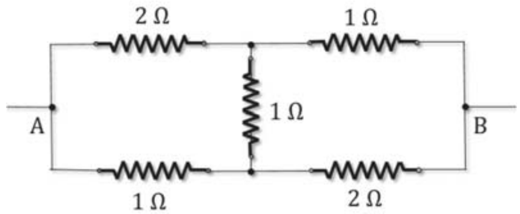
Applying KCL at point B,

$$\frac{x-v}{2} + \frac{x-y}{1} + \frac{x-0}{1} = 0$$

$$x - v + 2x - 2y + 2x = 0$$

$$5x - v = 2y \qquad \dots (1)$$

$$y = \frac{5x-v}{2}$$



Applying KCL at point C,

$$\frac{y-v}{1} + \frac{y-x}{1} + \frac{y-0}{2} = 0$$

$$2y - 2v + 2y - 2x + y = 0$$

$$5y - 2x = 2v \qquad \dots (2)$$

Solving equation (1) and (2),

$$y = \frac{5\cdot(\frac{3v}{7})-v}{2}$$

$$= \frac{8v}{14} = \frac{4v}{7}$$

Putting value of y in equation (2),

$$5(\frac{5x-v}{2}) - 2x = 2v$$

$$5(5x - v) - 4x = 4v$$

$$25x - 5v - 4x = 4v$$

$$21x = 9v$$

$$x = \frac{9v}{21} = \frac{3v}{7}$$

Equivalent resistance of the circuit,

$$R_{eq} = \frac{v}{i_1+i_2} = \frac{v}{x+yi_2}$$

$$= \frac{V}{\frac{3v}{7}+\frac{2v}{7}} = \frac{7}{5}\Omega$$

$$R_{eq} = \frac{7}{5}\Omega$$