INTRODUCTION TO CAPACITOR, IT ' S CAPACITANCE AND TYPE

Introduction to Capacitor:

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

Definition of capacitance:

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

capacitance of an isolated conductor:

1. It is a scalar quantity.
2. Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^{2} T^{4}$
3. **1 Farad:** 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

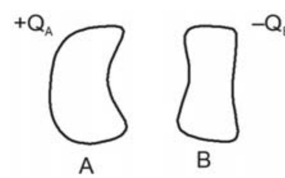
$$\text{Farad} = \frac{1\ \text{Coulomb}}{1\ \text{Volt}}$$

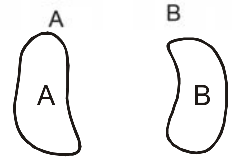
4. **Capacitance of an isolated conductor depends on following factors:**
   - (a) **Shape and size of the conductor:**
     On increasing the size, capacitance increases.
   - (b) **On surrounding medium:**
     With increase in dielectric constant K, capacitance increases
   - (c) **Presence of other conductors:**
     When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.
5. **Capacitance of a conductor do not depend on**
   - (a) Charge on the conductor
   - (b) Potential of the conductor
   - (c) Potential energy of the conductor.

Capacitor:

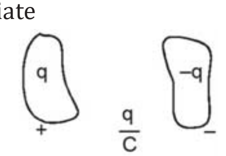
A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

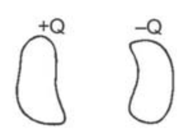
1. When uncharged conductor is brought near to a charged conductor, the charge on conductor's remains same but its potential decreases resulting in the increase of capacitance.
2. In capacitor two conductors have equal but opposite charges.
3. The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.
4. Formulae related with capacitors





   - (a) $\mathbf{Q = CV \Rightarrow C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}}$
     Q = Charge of positive plate of capacitor.
     V = Potential difference between positive and negative plates of capacitor
     C = Capacitance of capacitor.
   - (b) **Energy stored in the capacitor**
     Initially charge = 0
     Intermediate





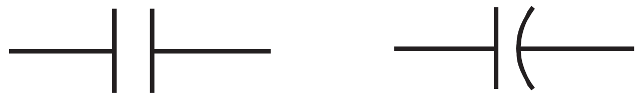
Finally, $$W = \int dW = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy stored in the capacitor $= U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

5. This energy is stored inside the capacitor in its electric field with energy density.

$$\frac{dU}{dV} = \frac{1}{2} \in E^2 \text{ or } \frac{1}{2}\epsilon_0\epsilon_r E^2$$

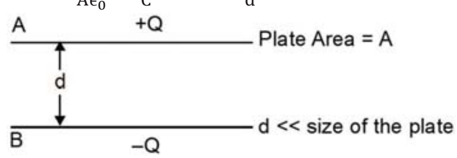
The capacitor is represented as following:



6. Based on shape and arrangement of capacitor plates there are various types of capacitors.
(a) Parallel plate capacitor.
(b) Spherical capacitor.
(c) Cylindrical capacitor
7. Capacitance of a capacitor depends on
(a) Area of plates.
(b) Distance between the plates.
(c) Dielectric medium between the plates
8. Electric field intensity between the plates of capacitors (air filled) $E = \sigma/\in_0 V/d$
9. Force experienced by any plate of capacitor $F = q^2/2A \in 0$

Ex. Find out the capacitance of parallel plate capacitor of plate area A and plate separation d.

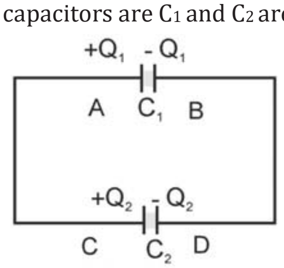
Sol. $mE = \frac{Q}{A\epsilon_0} \Rightarrow V_A - V_B = E.d. = \frac{Qd}{A\epsilon_0} = \frac{Q}{C} \Rightarrow C = \frac{\epsilon_0 A}{d}$



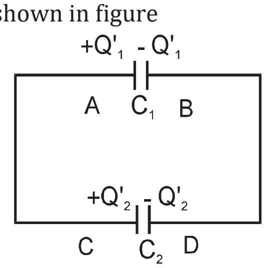
Where A = area of the plates.
d = distance between plates.

Distribution of charges on connecting two charged capacitors:

When two capacitors are C_1 and C_2 are connected as shown in figure



Initially



Finally

Before connecting the capacitors		
Parameter	Ist Capacitor	IInd Capacitor
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2
After connecting the capacitors		
Parameter	Ist Capacitor	IInd Capacitor
Parameter	C_1	C_2
Capacitance	Q_1	Q_2
Charge	V	V

1. Common potential:
By charge conservation of plates, A and C before and after connection.

$$Q_1 + Q_2 = C_1 V + C_2 V$$

$$V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1+C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

2. $Q' = C_1 V = \frac{C_1}{C_1+C_2}(Q_1 + Q_2)$

$$\Rightarrow Q_2{}' = C_2 V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$$

3. Heat loss during redistribution:

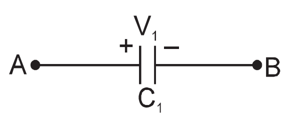
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1 C_2}{C_1+C_2}(V_1 - V_2)^2$$

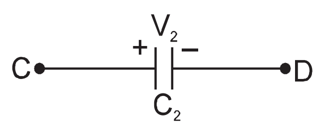
The loss of energy is in the form of Joule heating in the wire.

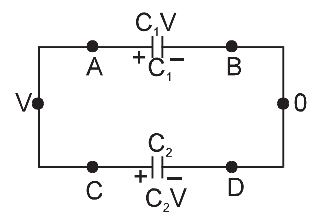
Note:

1. When plates of similar charges are connected with each other (+with +and-with-) then put all values (Q_1 Q_2 V_1 V_2) with positive sign
2. When plates of opposite polarity are connected with each other (+with-) then take charges and potential of one of the plates to be negative.

Derivation of above formulae:







Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$$C_1 V + C_2 V = C_1 V_1 + C_2 V_2$$

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \Rightarrow H = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

$$\frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2}\frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

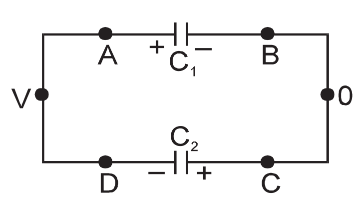
$$\frac{C_1^2 V_1^2 + C_1 C_2 V_1^2 + C_2 C_1 V_2^2 + C_2^2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2 - 2C_1 C_2 V_1 V_2}{C_1 + C_2}] = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$H = \frac{1}{2}\frac{C_1 C_2}{C_1+C_2}(V_1 - V_2)^2$$

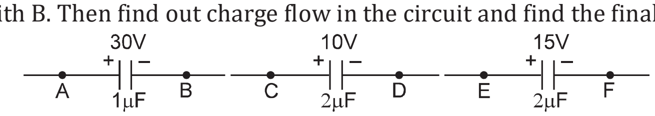
When oppositely charge terminals are connected then

$$C_1 V + C_2 V = C_1 V_1 - C_2 V_2$$

$$V = \frac{C_1 V_1 - C_2 V_2}{C_1 + C_2} \text{ and } H = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(V_1 + V_2)^2$$



Ex. Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged up to potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



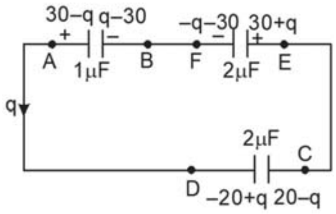
Sol. Let charge flow is q. Now applying Kirchhoff's voltage low

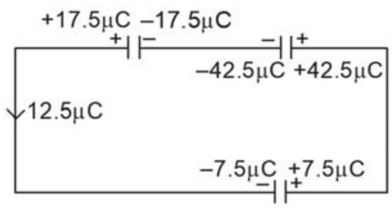
$$-\frac{(q-20)}{2}-\frac{(30+q)}{2}+\frac{30-q}{1}=0$$

$$-2q=-25$$

$$q=12.5\ \mu C$$

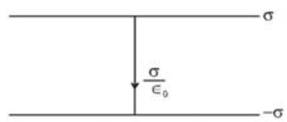
Final charges on plates



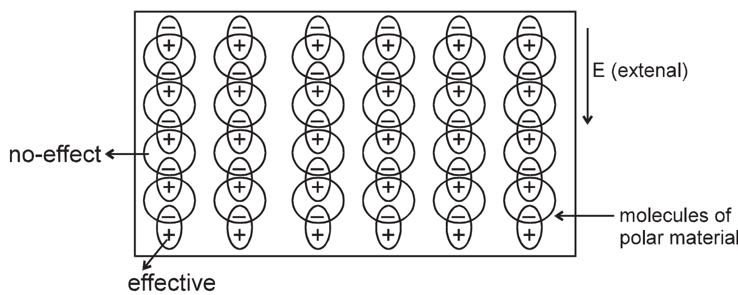


Capacitors With Dielectric

1. In absence of dielectric $E=\frac{\sigma}{\epsilon_0}$



2. When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.



σ_b = induced charge density (called bound charge because it is not due to free electrons).

For polar molecules dipole moment ≠ 0

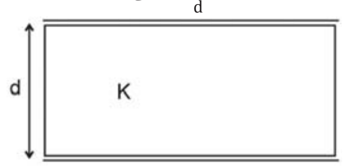
For non-polar molecules dipole moment = 0

3. Capacitance in the presence of dielectric

$$C=\frac{\sigma A}{V}=\frac{\sigma A}{\frac{\sigma}{K\epsilon_0}\cdot d}=\frac{AK\epsilon_0}{d}=\frac{AK\epsilon_0}{d}$$

Here capacitance is increased by a factor K.

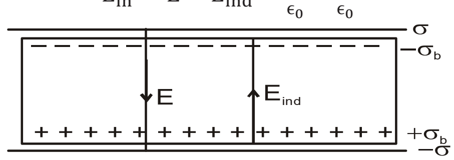
$$C=\frac{AK\epsilon_0}{d}$$



4. Polarization of material: When nonpolar substance is placed in electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarization of material. The induced charge also produces electric field.

σ_β = induced (bound) charge density.

$$E_{in}=E-E_{ind}=\frac{\sigma}{\epsilon_0}-\frac{\sigma_b}{\epsilon_0}$$



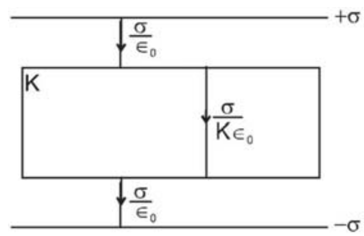
It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

$$E_{in} = \frac{\sigma}{K\epsilon_0} \Rightarrow \sigma_b = \sigma(1 - \frac{1}{K})$$

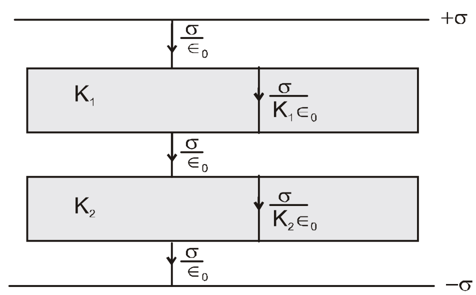
5. If the medium does not filled between the plates completely then electric field will be as shown in figure

Case: (1):

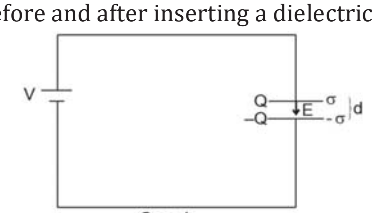
The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.



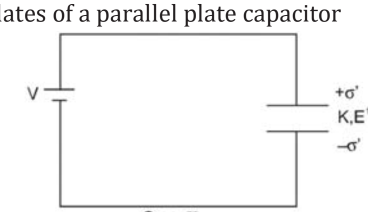
Case: (2)



6. Comparison of E (electric field), σ (surface char ges density), Q (charge), C (capacitance) and before and after inserting a dielectric slab between the plates of a parallel plate capacitor



Case I



Case II

$c = \frac{\epsilon_0 A}{d}$

$C = \frac{\epsilon_0 A}{d}$

$Q = CV$

$E = \frac{\sigma}{\epsilon_0} = \frac{CV}{A\epsilon_0}$

$= \frac{V}{d}$

Here potential difference between the plates,

$Ed = V$

$E = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma}{\epsilon_0}$

$C' = \frac{A\epsilon_0 K}{d}$

$C' = \frac{A\epsilon_0 K}{d}$

$Q' = C'V$

$E' = \frac{\sigma}{K\epsilon_0} = \frac{CV}{A\epsilon_0}$

$E' = \frac{V}{d}$

Here potential difference between the plates

$E'd = V$

$E' = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma'}{K\epsilon_0}$

Equating both

$$\frac{\sigma}{\epsilon_0} = \frac{\sigma'}{K\epsilon_0}$$

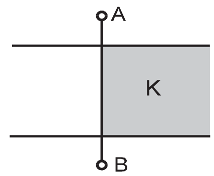
$$\sigma' = K\sigma$$

In the presence of dielectric, i.e. in case II capacitance of capacitor is more.

7. Energy density in a dielectric $= \frac{1}{2}\epsilon_0 \in E^2$

Ex. A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-

(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$ (C) $\frac{C}{2}[1+K]$ (D) $\frac{2[1+K]}{C}$



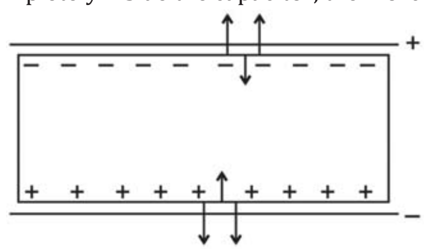
Sol. This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2 = \frac{\epsilon_0 A/2}{d} + \frac{\epsilon_0 (A/2)K}{d} = \frac{\epsilon_0 A}{2d}[1+K] = \frac{C}{2}[1+K]$$

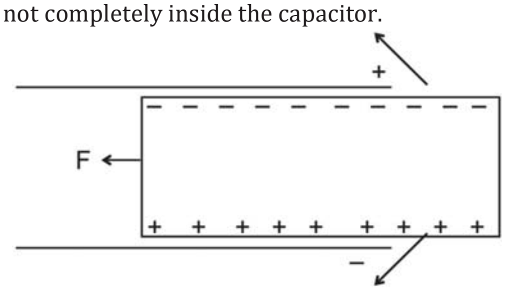
Hence the correct answer will be (C).

8. Force on a dielectric due to charged capacitor:

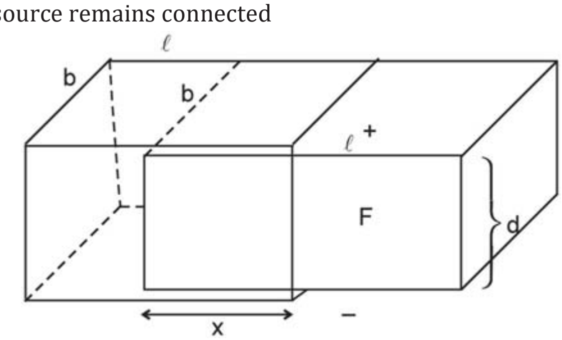
(a) If dielectric is completely inside the capacitor, then force is equal to zero.



(b) Dielectric is not completely inside the capacitor.



Case-I: Voltage source remains connected



V = constant.

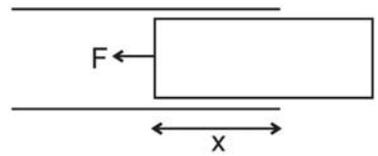
$$U = \frac{1}{2}C^2$$

$$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx} \text{ where } C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d} \Rightarrow C = \frac{\epsilon_0 b}{d}[Kx + \lambda - x]$$

$$\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)$$

$F = \frac{\epsilon_0 b(K-1)V^2}{2d} =$ constant (does not depend on x)

Case II: When charge on capacitor is constant

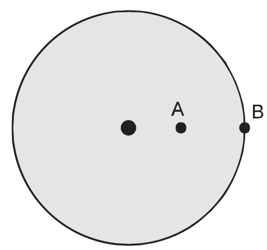


$$C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d}, U = \frac{Q^2}{2C}$$

$$F = (\frac{dU}{dx}) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \text{[where, } \frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)]$$

$$= \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \text{ (here force 'F' depends on x)}$$

Ex. What is potential at a distance r (<R) in a dielectric sphere of uniform charge density ⍰, radius R and dielectric constant ε_r.



Sol. Here

$$V_A = V_B + \frac{W_{B\to A}}{q}$$

$$V = \frac{Q}{4\pi\ _0R} + \int_R^r \frac{\rho r}{3\epsilon_0\epsilon_r}(-dr) = \frac{Q}{4\pi\epsilon_0 R} + \frac{\rho(R^2 - r^2)}{3\epsilon_0\epsilon_r}$$

$$V_{outiside} = \frac{KQ}{r}$$

Types of capacitor-Parallel plate capacitor, Spherical capacitor, cylindrical capacitor

Spherical capacitor:

This arrangement is known as spherical capacitor.

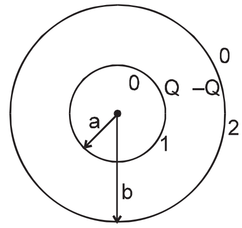
$$V_1 - V_2 = [\frac{KQ}{a} - \frac{KQ}{b}] - [\frac{KQ}{b} - \frac{KQ}{b}] = \frac{KQ}{a} - \frac{KQ}{b}$$

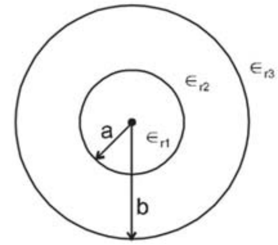
$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{ac}{K(b-a)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$C = \frac{4\pi\ _0ab}{b-a}$$

If b >> a then

$C = 4\pi \in_0 a$ (Like isolated spherical capacitor





If dielectric mediums are filled as shown then: $C = \frac{4\pi\epsilon_0\epsilon_{r_2}ab}{b-a}$

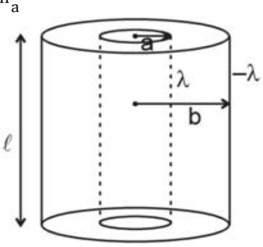
Cylindrical capacitor

There are two co-axial conducting cylindrical surfaces where λ >> a and λ >> b, where a and b is radius of cylinders.

Capacitance per unit length $C = \frac{\lambda}{V}$

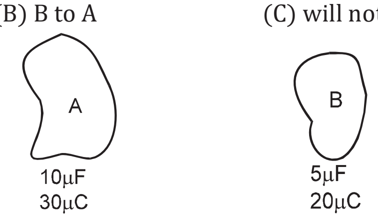
$$= \frac{\lambda}{2K \quad \frac{b}{a}} = \frac{4\pi\epsilon_0}{2\ln\frac{b}{a}} = \frac{2\pi \quad _0}{\ln\frac{b}{a}}$$

Capacitance per unit length $= \frac{2\pi\epsilon_0}{\ln\frac{b}{a}}$ F/m



Ex. When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.

(A) A to B (B) B to A (C) will not flow (D) cannot say.



Sol. Charge always flows from higher potential body to lower potential body

Hence, $V_A = \frac{30}{10} = 3V \Rightarrow V_B = \frac{20}{5} = 4V$ As $V_B > V_A$

(B) Is correct Answer.

Conductor

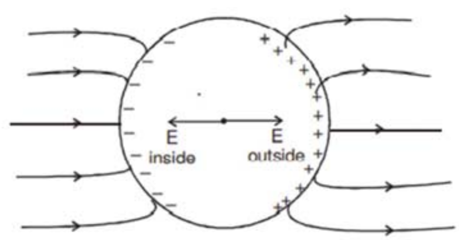
Type of materials

1. conductors (All electrons are free)
2. Semi-conductors (Some electrons are free)
3. Insulators (all electrons are bounded)

Conductors:

A conductor contains free electrons, which can move freely in the material, but cannot leave it.

On applying an external electric field on conductor charges of a conductor adjust themselves in such a fashion that the net electric field inside the conductor is zero under electrostatics conditions.

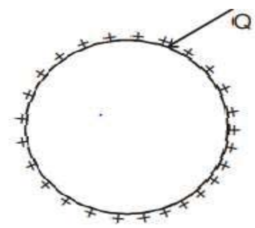


Net $\vec{E} = 0 \Rightarrow$ Potential is constant

Conductor behaves as an equipotential surface

Being an equipotential surface, electric field lines will terminate or originate perpendicularly Let us now consider the interior of a charged conducting object. Since it is a conductor, the electric field in the interior is everywhere zero. Let we analyses a Gaussian surface inside the conductor as close as possible to the surface of the conductor. Since the electric intensity E is zero everywhere inside the conductor, it must be zero for every point of the Gaussian surface. Hence the flux through the surface. $\oint \vec{E} \cdot d\vec{S}$ will be zero. Therefore, according to Gauss's law $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$ the net charge inside the Gaussian surface and hence inside the conductor must be zero. Since there can be no charge in the interior of the conductor charge given to the conductor will reside on the surface of the conductor.

All the charge given to the conductor reside on the surface of the conductor

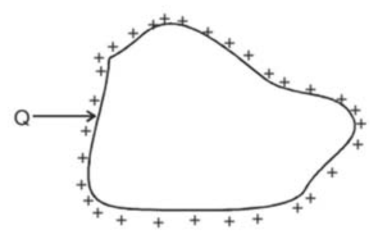


Till now we have only discussed the case of uniform shaped bodies on which the charge distribute itself uniformly.

But what about the charge distribution on irregular shaped bodies? Does in this case also uniform charge distribution take place? .... NO

In this case $\sigma \alpha \frac{1}{r_c}$

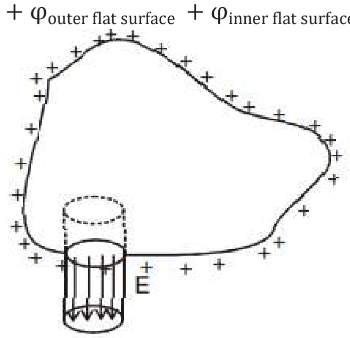
Charge per unit area   Radius of curvature



Let us consider a random shaped body and find Electric field due to a small portion of this body. However, the σ is not uniform everywhere but for a small area dA, we can assume that σ is constant. Considering a cylindrical Gaussian surface, we will calculate flux passing through the cross section dA.

$$\phi_{net} = \oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$\varphi_{net} = \varphi_{\text{curxed surface}} + \varphi_{\text{outer flat surface}} + \varphi_{\text{inner flat surface}} \ \varphi_{\text{curxed surface}} = 0$$



Because no flux is passing through lateral surface (electric field lines are perpendicular to area vector.)

$$\vec{E} \cdot d\vec{S} = 0$$

$$\varphi_{\text{inner flat surface}} = 0$$

Because $\vec{E}$ inside conductor = 0

$$\frac{q_{in}}{\varepsilon_0} = \varphi_{\text{outer flat surface}}$$

$$\frac{\sigma dA}{\varepsilon_0} = \vec{E} \cdot d\vec{S}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}$$