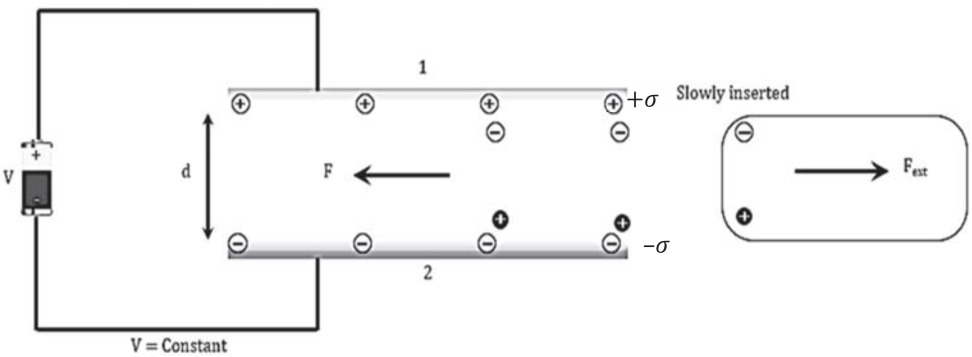
EFFECTS OF DIELECTRIC INSERTION IN CAPACITOR

When a dielectric material is inserted between the plates of a capacitor, it significantly impacts the capacitor's electrical properties and behavior.

Insertion of dielectric at constant potential

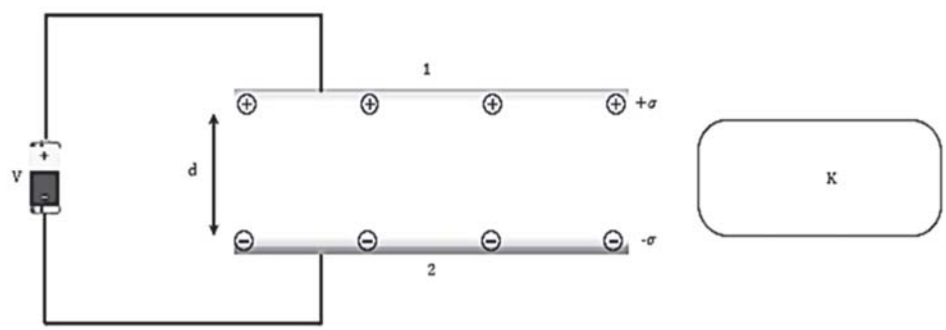
Let's examine the scenario where a dielectric material is gently introduced into a capacitor, as depicted. The presence of induced charges on the dielectric results in an attractive force, denoted as F, exerted upon it.



During this gradual insertion process, an external force, represented by F_{ext}, is applied in the opposite direction to ensure a controlled pace. Simultaneously, a battery is connected across the capacitor, maintaining a constant voltage due to the deliberate slow pace of insertion.

Following the insertion of the dielectric, the capacitance experiences a notable increase, precisely by a factor equal to the 'dielectric constant' multiplied by its original value. This augmentation underscores the significance of the dielectric material's impact on enhancing the capacitor's capacitance.

The alterations in parameters resulting from the presence of a dielectric are delineated in the following table:

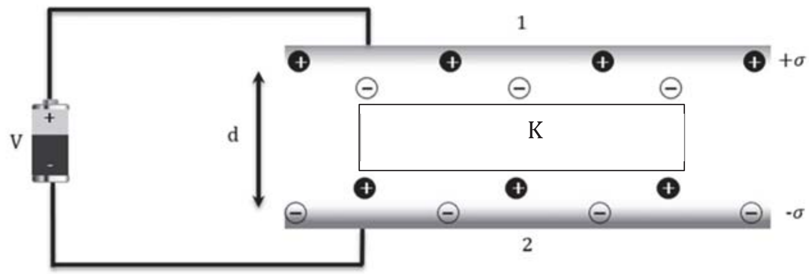


Parameter	Before insertion	After insertion
Charge	CV	KCV↑
Capacitance	C	KC↑
Potential	V	V
Electric field	V/d	V/d

Before the dielectric is inserted, the charge (Q) in the capacitor is determined by the product of capacitance (C) and potential (V), as denoted by CV. Upon insertion, the charge increases by a factor of the dielectric constant (K), leading to KCV in the capacitor.

The capacitance (C) of the capacitor prior to insertion remains unchanged, while after insertion, it increases by a factor of the dielectric constant (K), resulting in KC.

The potential (V) across the capacitor remains constant both before and after the insertion of the dielectric.



Similarly, the electric field (E), calculated as the ratio of potential (V) to distance (d), remains constant before and after the insertion.

The amount of charge that has passed through the battery $= Q_f - Q_i = K - 1\ C$v

$$U = \frac{1}{2}CV^2$$

The amount of stored energy within a capacitor is denoted by the symbol U.

Before insertion	After insertion
$\frac{1}{2}CV^2$	$\frac{1}{2}CV^2 \uparrow$

The work performed by the battery, denoted by the symbol W, refers to the energy expended or transferred by the battery to the electrical circuit or device it powers.

$$Q_i = CV \quad Q_f = KCV$$

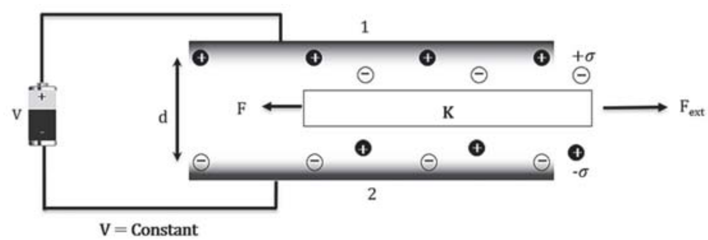
The work done in an electrical system is equal to the product of the charge flowing through the battery and the potential difference across its terminals.

$$W = (K-1)CV^2$$

The change in energy, denoted as ΔU, refers to the increase in energy stored within the capacitor.

$$U_i = \frac{1}{2}CV^2 \qquad U_f = \frac{1}{2}KCV^2$$

$$\Delta U = \frac{1}{2}(K-1)CV^2$$

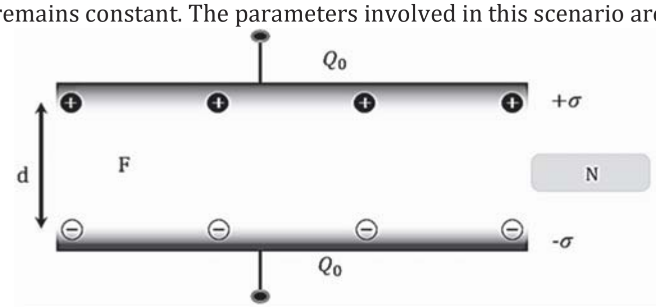


The rise in energy stored within the capacitor corresponds to half of the energy provided by the battery. The remaining half of the energy is utilized by an external entity for performing negative work. This work executed by the external agent constitutes the energy dissipated from the system. The negativity arises due to the opposing directions of both force and displacement vectors.

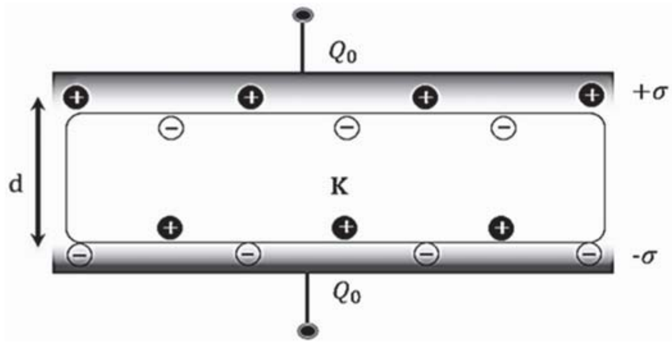
$$W_{\text{ext}}{}^{n} = -\frac{1}{2}(K-1)CV^2$$

Insertion of dielectric at constant charge

When a dielectric is inserted into a charged capacitor after removing the battery, the charge stored on the capacitor remains constant. The parameters involved in this scenario are outlined below:



Parameter	Before insertion	After insertion
Charge	$Q_\circ$	$Q_\circ$
Capacitance	$A\varepsilon_\circ/d$	$AK\varepsilon_\circ/d \uparrow$
Potential	Q/C	Q/KC ↓
Electric field	$Q/A\varepsilon_\circ$	$Q/AK\varepsilon_\circ \downarrow$
Energy	$Q^2/2C$	$Q^2/2KC \downarrow$



The exertion carried out, denoted as Wext is the work performed by an external entity or agent.

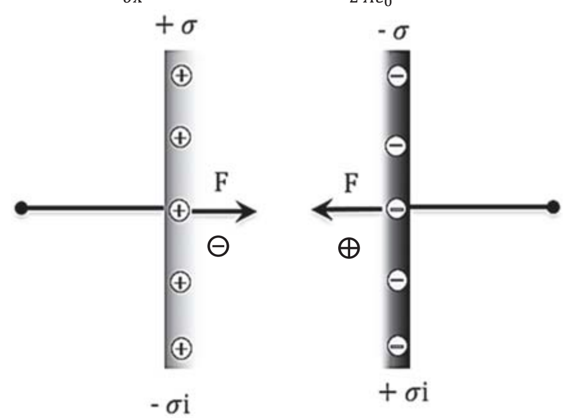
$$U_i = \frac{Q^2}{2C} \qquad U_f = \frac{Q^2}{2KC}$$

$$W_{ext} = \frac{Q^2}{2C}\left(\frac{1}{K} - 1\right)1$$

In this scenario, the energy stored within the capacitor diminishes as it is utilized to counteract an external force.

The term Force (F) denotes the force exerted between the plates.

$$F = -\frac{\partial U}{\partial x} \qquad F = -\frac{1}{2}\frac{Q^2}{A\varepsilon_0}$$



The negative sign signifies an attractive nature of the force between the plates. The magnitude of the force (F) between the plates remains constant.

Partially filled dielectric

the capacitance of a parallel plate capacitor is modified when a dielectric is present. As before, we have two large plates, each of area A, separated by a distance d. The charge on the plates is $\pm Q$, corresponding to the charge density $\pm\sigma$ (with $\sigma = Q/A$). When there is vacuum between the plates,

$$E_{air} = \frac{\sigma}{\varepsilon_0}$$

And the potential difference V_0 is

$$V_0 = E_0 d$$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d}$$

Consider next a dielectric inserted between the plates fully occupying the intervening region. The dielectric is polarized by the field and, as explained, the effect is equivalent to two charged sheets (at the surfaces of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.
That is,

$$E = \frac{\sigma - \sigma_p}{\varepsilon_0}$$

So that the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0} d$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

Where K is a constant characteristic of the dielectric. Clearly, $K > 1$.
We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K}$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d}$$

The product $\varepsilon_0 K$ is called the permittivity of the medium and is denoted by $\varepsilon = \varepsilon_0 K$
For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the permittivity of the vacuum.
The dimensionless ratio

$$K = \frac{\varepsilon}{\varepsilon_0}$$

Is called the dielectric constant of the substance. it is clear that K is greater than 1.

$$K = \frac{C}{C_0}$$

Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor. Though we arrived at the case of a parallel plate capacitor, it holds good for any type of capacitor and can, in fact, be viewed in general as a definition of the dielectric constant of a substance

Ex. A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4)d, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Sol. Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/K$. The potential difference will then be

$$V = E_0\left(\frac{1}{4}d\right) + \frac{E_0}{K}\left(\frac{3}{4}d\right)$$

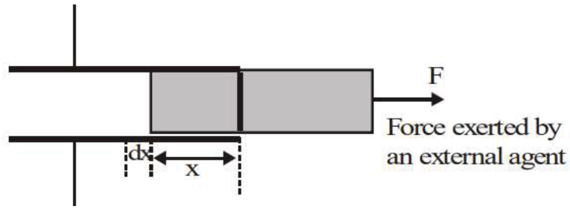
$$E_0 d\left(\frac{1}{4} + \frac{3}{4K}\right) = V_0 \frac{K+3}{4K}$$

The potential difference decreases by the factor $(K + 3)/4K$ while the free charge Q0 on the plates remains unchanged. The capacitance thus increases

$$C = \frac{Q_0}{V} = \frac{4K}{K+3}\frac{Q_0}{V_0} = \frac{4K}{K+3}C_0$$

Force on a Dielectric in a capacitor

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{Elketsonatic} + W_F = 0$, where $W_,$ denotes the work done by external agent in displacement dx



$$W_F = -W_{Electrostatic}\ W_F = \Delta U$$

$\Rightarrow$ $$-F \cdot dx = \frac{Q^2}{2} d\left[\frac{1}{C}\right]\left[W = \frac{Q^2}{2C}\right]$$

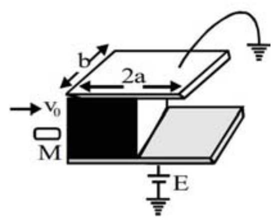
$\Rightarrow$ $$-F \cdot dx = \frac{-Q^2}{2C^2} dC$$

$\Rightarrow$ $$F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant

$$V = \frac{Q}{C} \Rightarrow F = \frac{1}{2}V^2\frac{dC}{dx}$$

Ex. A parallel plate capacitor is half filled with a dielectric (K) of mass M. Capacitor is attached with a cell of emf E. Plates are held fixed on smooth insulating horizontal surface. A bullet of mass M hits the dielectric elastically and it's found that dielectric just leaves out the capacitor. Find speed of bullet.



Sol. Since collision is elastic

$\therefore$ Velocity of dielectric after collision is v_0.

Dielectric will move and when it is coming out of capacitor a force is applied on

It by the capacitor $F = \frac{-d}{dx} = \frac{-E^2\varepsilon_0\, b(K-1)}{2d}$

Ex. The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted into the (i) before the introduction of the metal plate and (ii) after its introduction. What would be the potential difference if a plate of dielectric constant K = 2 is introduced in place of metal plate?

Sol. **1.** In case of a capacitor as $E = (V/d)$, the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5\text{kV}$$

2. Now as after charging battery is removed, capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate $q = CV = C'V'$ i.e., $V' = \frac{C}{C'}V$

And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d} \times V$

So in case of metal plate as $K = \infty$, $V_M = [\frac{d-t}{d}] \times V = [\frac{0.05-0.01}{0.05}] \times 1.5 = 1.2\text{kV}$

And if instead of metal plate dielectric with K = 2 is introduced

$$V_D = [\frac{(0.05-0.01)+(0001/2)}{0.05}] \times 1.5 = 1.35\text{kV}$$