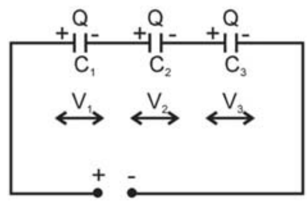
COMBINATION OF CHARGED CAPACITOR AND CIRCUIT THEORY

Combination of charged capacitor:

In electrical circuits, the combination of charged capacitors refers to the arrangement or configuration of multiple capacitors that have already been charged to specific voltages. When charged capacitors are combined, their voltages and charges interact according to the principles of circuit analysis.

Series Combination:

1. When initially uncharged capacitors are connected as shown then the combination is called series combination.



2. All capacitors will have same charge but different potential difference across them.
3. We can say that

$$V_1 = \frac{Q}{C_1}$$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$$V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}; \dots \dots$$

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

We can say that potential difference across capacitor is inversely proportional to its Capacitance in series combination.

$$V \propto \frac{1}{C}$$

Note: In series combination the smallest capacitor gets maximum potential.

5.
$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots\dots}V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots\dots}V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots\dots}V$$

Where $V = V_1 + V_2 + V_3$

6. Equivalent Capacitance: Equivalent capacitance of any combination is that **capacitance** which when connected in place of the combination, stores same charge and energy that of the combination. In series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

Note: In series combination equivalent capacitance is always less the smallest capacitor of combination.

7. Energy stored in the combination

$$U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$U_{combination} = \frac{Q^2}{2C_{eq}}$$

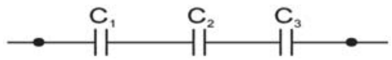
Energy supplied by the battery in charging the combination

$$U_{batterk} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$$

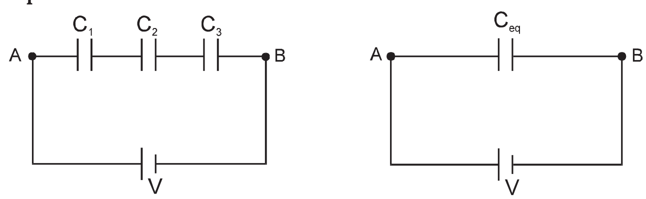
$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Note: Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Derivation of Formulae:



Meaning of equivalent capacitor



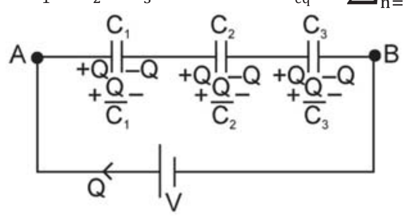
$$C_{eq} = \frac{Q}{V}$$

Now, initially, the capacitor has no charge. Applying Kirchhoff's voltage law

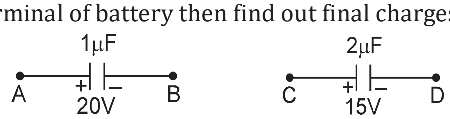
$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$$

$$V = Q[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}]; V = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ in general, } \frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$



Ex. Two capacitors of capacitance 1μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor



Sol. Now applying Kirchhoff voltage law

$$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$-40 - 2q - 30 - q = -60$$

$$3q = -10$$

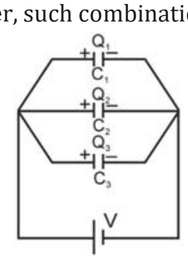
Charge flow $= -\frac{10}{3}$ μC.

Charge on capacitor of capacitance 1μF $= 20 + q = \frac{50}{3}$ μC

Charge on capacitor of capacitance 2μF $= 30 + q = \frac{80}{3}$ μC

Parallel Combination:

1. When one plate of each capacitor (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.



2. All capacitors have same potential difference but different charges.
3. We can say that:
$Q_1 = C_1V$
Q_1 = Charge on capacitor C_1
C_1 = Capacitance of capacitor C_1
V = Potential across capacitor C_1
5. $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$
The charge on the capacitor is proportional to its capacitance $Q \propto C$
6. $Q_1 = \frac{C_1}{C_1+C_2+C_3}Q \Rightarrow Q_2 = \frac{C_2}{C_1+C_2+C_3}Q$
$Q_3 = \frac{C_3}{C_1+C_2+C_3}Q$
Where $Q = Q_1 + Q_2 + Q_3 \ldots\ldots$

Note: Maximum charge will flow through the capacitor of largest value.
Equivalent capacitance of parallel combination $C_{eq} = C_1 + C_2 + C_3$

Note: Equivalent capacitance is always greater than the largest capacitor of combination.

7. Energy stored in the combination:

$$V_{\text{combination}} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \cdots . = \frac{1}{2}(C_1 + C_2 + C_3 \ldots\ldots)V^2 = \frac{1}{2}C_{eq}V^2$$

$$U_{\text{battery}} = QV = CV^2$$

$$\frac{U_{\text{combination}}}{U_{\text{battery}}} = \frac{1}{2}$$

Note: Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (If all capacitors are initially uncharged)

Capacitor- Circuit theory

In circuit theory, a capacitor is a passive electronic component that stores and releases electrical energy in the form of an electric field between two conductive plates separated by a dielectric material. Capacitors are widely used in electronic circuits for various purposes, including energy storage, signal filtering, timing, and coupling.

Series Concepts:

1. **Capacitance (C):** Capacitance is the ability of a capacitor to store electrical charge per unit voltage. It is measured in farads (F). The capacitance of a capacitor depends on its physical characteristics, such as the area of the plates, the distance between them, and the properties of the dielectric material between the plates.
2. **Voltage (V):** Voltage across a capacitor represents the potential difference between its plates. When a voltage is applied across a capacitor, it stores electrical energy in the form of an electric field between the plates. The voltage across a capacitor can change over time due to charging and discharging processes.
3. **Charge (Q):** Charge stored on a capacitor represents the amount of electrical charge accumulated on its plates. The charge stored on a capacitor is directly proportional to the voltage across it and its capacitance, as given by the formula Q = CV.
4. **Energy Storage:** Capacitors store electrical energy in the form of an electric field. The energy stored (U) in a capacitor can be calculated using the formula $U = \frac{1}{9}CV^2$. This energy can be released when needed, making capacitors useful for applications such as energy storage and power delivery.
5. **Transient Response:** Capacitors exhibit transient behavior during charging and discharging processes. The transient response of a capacitor describes how its voltage and charge change over time in response to changes in the circuit conditions. Analysis of transient responses is crucial for understanding capacitor behavior in time-varying circuits.

6. **Frequency Response:** Capacitors have frequency-dependent impedance, which affects their behavior in AC (alternating current) circuits. The impedance of a capacitor decreases with increasing frequency, making capacitors useful for filtering and frequency-selective applications in electronic circuits.

Combination of capacitors in series and parallel

- Charged capacitors can also be combined in a combination of series and parallel configurations. This involves grouping capacitors into series branches and then connecting these branches in parallel, or vice versa.
- The total voltage, charge, and capacitance of such a combination can be determined by analyzing each branch separately and then combining the results using the rules for series and parallel combinations.

Formulae Derivation for parallel combination:

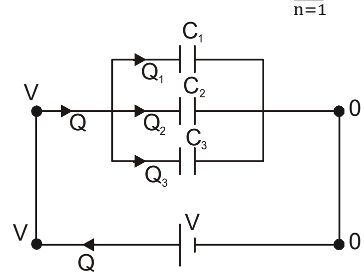
$$Q = Q_1 + Q_2 + Q_3$$

$$\frac{Q}{V} = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$$

$$= C_1 + C_2 + C_3$$

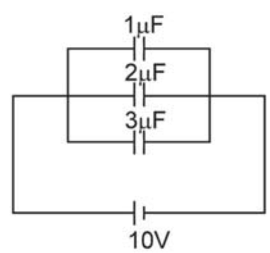
$$C_{eq} = C_1 + C_2 + C_3$$

$$\text{In general, } C_{eq} = \sum_{n=1}^{n} C_n$$



Ex. Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following

1. Charge flow from the battery
2. Total energy stored in the capacitors
3. Heat produced in the circuit
4. Potential energy in the 3⍰F capacitor

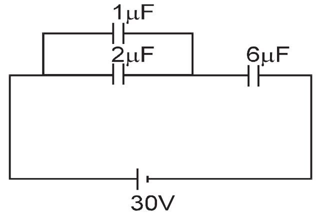


Sol.
1. $Q = (30 + 20 + 10)\mu C = 60\mu C$
2. $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\mu J$
3. Heat produced $= 60 \times 10 - 300 = 300\mu J$
4. $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\mu J$

Mixed Combination:

The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category. There are two types of mixed combinations Simple complex.

Ex. In the given circuit find out charge on 6μF and 1μF capacitor.



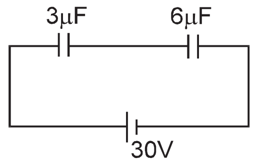
Sol. It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$ charge flow through the cell $= 30 \times 2\mu C$

$Q = 60\mu C$

Now charge on $3\mu F =$ Charge on $6\mu F = 60\mu C$

Potential difference across $3\mu F = \frac{60}{3} = 20$ V

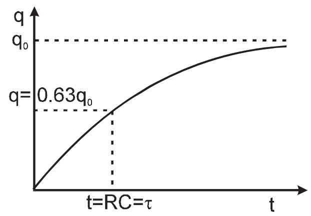
Charge on $1\mu F = 20\mu C$.

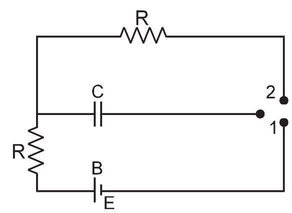


Charging And Discharging of a Capacitor:

Charging of a condenser:

1. In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0 [1 - e^{-(t/RC)}]$ Where q_0 = maximum final value of charge at $t = \infty$. According to these equations the quantity of charge on the condenser increases exponentially with increase of time.





2. If $t = RC = \tau$ then

$$q = q_0[1 - e^{-(RC/RC)}] = q_0[1 - \frac{1}{e}]$$
$$q = q_0(1 - 0.37) = 0.63q_0 = 63\% \text{ of } q_0$$

3. Time t = RC is known as time constant. I.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

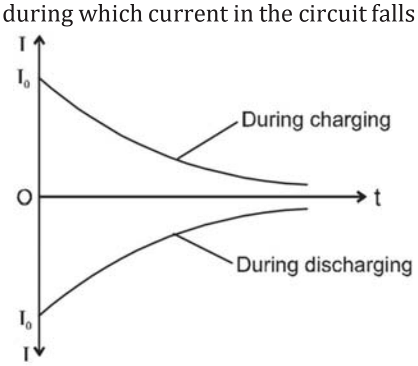
4. The potential difference across the condenser plates at any instant of time is given by $V = V_0 [1 - e^{-(t/RC)}]$ volt

5. The potential curve is also similar to that of charge. During charging process an electric current flow in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by $I = I_0[e^{-(t/RC)}]$ ampere According to this equation the current falls in the circuit exponentially.

6. If $t = RC = \tau =$ Time constant

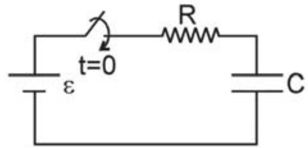
$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37I_0$$
$$= 37\% \text{ of } I_0$$

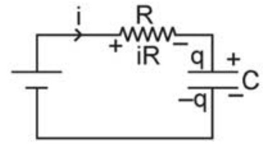
I.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.



Derivation of formulae for charging of capacitor

it is given that initially capacitor is uncharged. Let at any time charge on capacitor is q





Applying Kirchhoff voltage law.

$$\varepsilon - iR - \frac{q}{C} = 0 \Rightarrow iR = \frac{\varepsilon C - q}{C} i$$

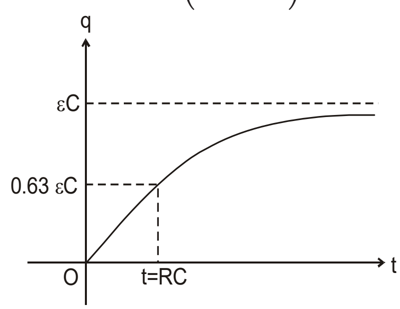
$$\frac{\varepsilon C - q}{CR} \Rightarrow \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \Rightarrow \frac{CR}{\varepsilon C - q} \cdot dq = dt.$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}; \varepsilon C - q = \varepsilon C \cdot e^{-t/RC}$$

$$q = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$

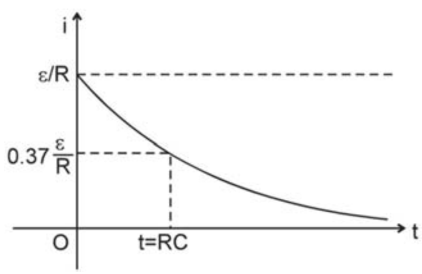


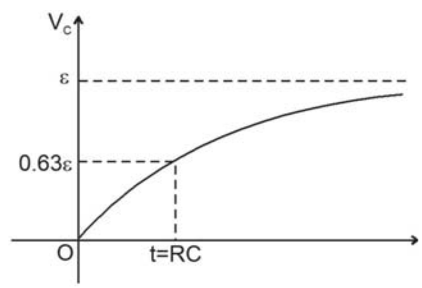
After one time constant $q = \varepsilon C(1 - \frac{1}{e}) = \varepsilon C(1 - 0.37) = 0.63\varepsilon C.$

Current at any time t

$$i = \frac{dq}{dt} = \varepsilon C(-e^{-t/Rc}(-\frac{1}{RC}))$$

$$= \frac{\varepsilon}{R} e^{-t/Rc}$$





Voltage across capacitor after one time constant $\mathbf{V = 0.63\varepsilon}$

$$Q = CV; V_C = \varepsilon(1 - e^{-t/RC})$$

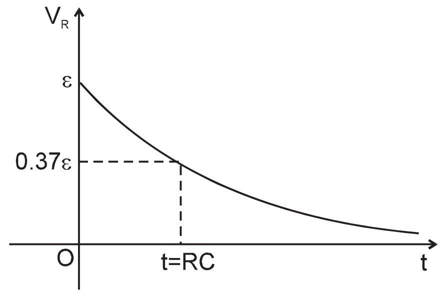
Voltage across the resistor

$$V_R = iR = \varepsilon e^{-t/RC}$$

By energy conservation,

Heat dissipated = work done by battery $-\Delta U_{capacitor}$

$$C\varepsilon(\varepsilon) - (\frac{1}{2}C\varepsilon^2 - 0) = \frac{1}{2}C\varepsilon^2$$



Ex. A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Sol. The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.
Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C(1 - e^{-t/RC})$.
Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is.

$$Q_0/C = 36V. \text{ So, } \Rightarrow 12V = 36\,V(1 - e^{-t/RC})$$

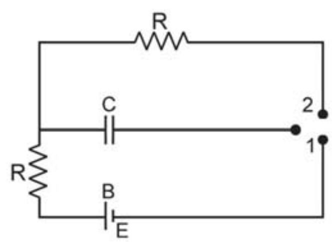
$$1 - e^{-t/RC} = \frac{1}{3} \text{ or, } e^{-t/RC} = \frac{2}{3}$$

$$\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0.405 \text{ or, } RC = \frac{t}{0.405} = \frac{2\mu}{0.45} = 4.936\mu s$$

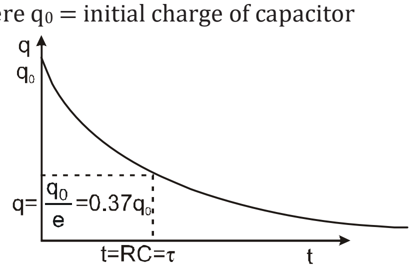
$$C = \frac{4.936\mu s}{20\Omega} = 0.25\mu F.$$

Discharging of a condenser:

1. In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.



2. The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$ I.e. the charge falls exponentially. Here q_0 = initial charge of capacitor



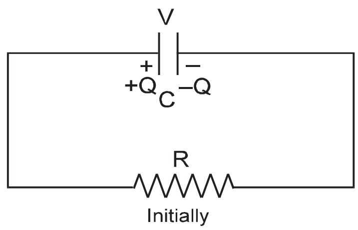
3. If $t = RC = \tau =$ time constant, then $q = \frac{q_0}{e} = 0.37q_0 = 37\%$ of q_0 i.e., the time constant is that time during which the charge on condense plates in discharge process, falls to 37%

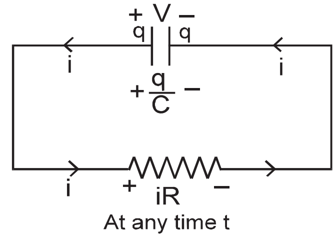
4. The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

5. The potential difference across the condenser plates at any instant of time t is given by $V = V_0 e^{-(t/RC)} V_{ol} t$.

6. The transient current at any instant of time is given by $I = -I_0 e^{-(t/RC)}$ ampere.i.e. The current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

Derivation of equation of discharging circuit:





Applying K.V.L. $+\frac{q}{C} - iR = 0$

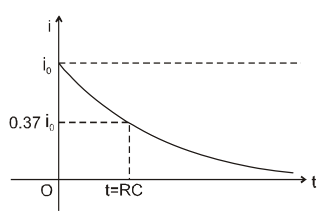
$$i = \frac{q}{CR}$$

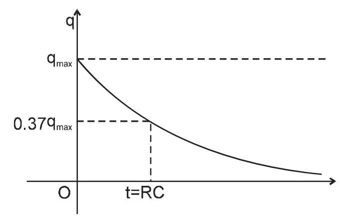
$$\int_{Q}^{q} \frac{-dq}{q} = \int_{0}^{t} \frac{dt}{CR}$$

$$-\ln \frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q_i e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC} e^{-t/RC} = i_0 e^{-t/RC}$$

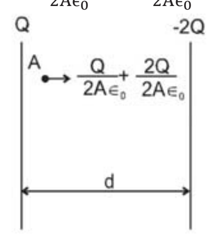




Ex. Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Sol. Capacitance = C

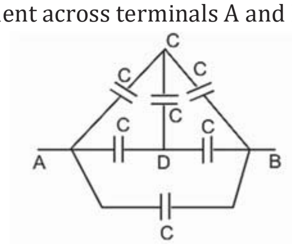
$$\text{Electric field } = \frac{3Q}{2A\epsilon_0}; V = \frac{3Qd}{2A\epsilon_0} \Rightarrow V = \frac{3Q}{2C}$$



Symmetrical circuits:

Symmetrical circuits refer to electrical circuits that exhibit symmetry in their structure or behavior. Symmetry in circuits can manifest in various forms, including geometric symmetry, electrical symmetry, or functional symmetry.

Ex. Determine the capacitance equivalent across terminals A and B.



Sol. Since line CD exhibits symmetry with respect to points A and B, it is eliminated from consideration.

$$= C_{e_q} = 2C$$

Geometric Symmetry:

Geometric symmetry in circuits refers to the physical arrangement of components or elements in a circuit. For example, a circuit may be symmetric if its components are arranged in a pattern that exhibits mirror symmetry or rotational symmetry.

Geometric symmetry often simplifies circuit analysis and design by allowing for the application of symmetry properties to reduce complexity and mathematical calculations.

Electrical Symmetry:

Electrical symmetry in circuits refers to the distribution of electrical properties, such as resistance, capacitance, or inductance, in a symmetric manner throughout the circuit.

In an electrically symmetrical circuit, the electrical properties are the same or exhibit a predictable pattern across different branches or elements of the circuit.

Electrical symmetry can lead to predictable circuit behavior and facilitate analysis and troubleshooting.

Functional Symmetry:

Functional symmetry in circuits refers to the balanced distribution of functions or operations within the circuit.

In a functionally symmetrical circuit, similar operations or functions are distributed symmetrically, leading to balanced performance and efficient operation.

Functional symmetry can enhance circuit reliability, stability, and performance by ensuring that the circuit operates uniformly under different conditions.

Applications and Benefits:

Symmetrical circuits are commonly used in various applications, including power distribution systems, signal processing circuits, and communication systems.

The benefits of symmetrical circuits include simplified analysis, improved reliability, reduced sensitivity to variations or disturbances, and enhanced performance under normal and fault conditions.