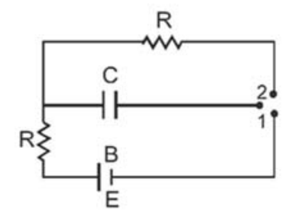
Charging of a Capacitor

In the given circuit, when switch 1 is closed, the capacitor undergoes a charging process. This charging process takes a finite amount of time to complete. At any given moment "t," the amount of charge accumulated in the capacitor is expressed by the equation:



$$g = g_0\left[1 - e^{-(t/RC)}\right]$$

Where q_0 = maximum final value of charge at t = ∞.

According to this equation the quantity of charge on the condenser increases exponentially with increase of time.

If t = RC = τ then

$$q = q_0\left[1 - e^{-(RC/RC)}\right] = q_0\left[1 - \frac{1}{e}\right]$$

or $$q = q_0(1 - 0.37) = 0.63q_0$$

$$= 63\% \text{ of } q_0$$

The time t = RC is referred to as the time constant in an RC circuit. It signifies the duration during which the charge accumulation on the capacitor plates reaches 63% of its maximum value.

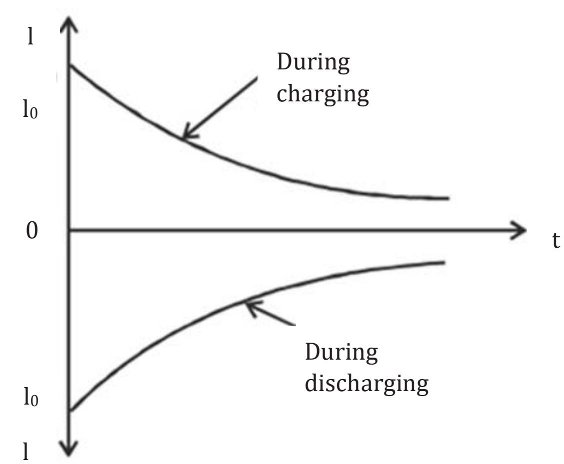
Mathematically, the potential difference across the capacitor plates at any given moment is expressed as:

$$V=V_o[1-e^{-(t/RC)}] \text{ volt}$$

This equation indicates that the potential curve resembles the charge curve. Throughout the charging process, a transient current flow in the circuit for a brief period. This transient current's magnitude at any time t is represented by:

$$1 = l_o e^{-(t/RC)}] \text{ amperes}$$

As per this equation, the current in the circuit decreases exponentially over time, as illustrated in the accompanying figure. When the time t equals the product of resistance and capacitance (RC), denoted as τ, it represents the time constant of the circuit.



$$I = I_0[e^{-(RC/RC)} = \frac{I_0}{e} = 0.37I_0 = 37\% \text{ of } I_0$$

Charging of R-C circuit:

Charging of an R-C circuit refers to the process of gradually accumulating electric charge on a capacitor when it is connected to a voltage source through a resistor. This process occurs in circuits containing a resistor (R) and a capacitor (C).

Here's a detailed explanation:

Initial State:

Initially, the capacitor is uncharged, and there is no potential difference across its plates. The circuit is in a steady state with no current flow.

Connection to Voltage Source:

When the circuit is closed, current begins to flow from the voltage source through the resistor and into the capacitor. This causes the capacitor to charge up gradually.

Charging Process:

As current flows into the capacitor, positive charge accumulates on one plate of the capacitor, while an equal amount of negative charge accumulates on the other plate. This process continues until the potential difference across the capacitor plates reaches the same value as the voltage of the source (V0).

Voltage Build-Up:

During the charging process, the voltage across the capacitor (Vc) gradually increases. It follows an exponential growth curve described by the equation:

$$V_c = V_0(1 - e^{-\frac{t}{RC}})$$

Where:

- V_c is the voltage across the capacitor at time t.
- V_0 is the voltage of the source.
- R_C is the time constant of the circuit.

Time Constant:

The time constant (RC) represents the time it takes for the capacitor to charge to approximately 63.2% of its maximum voltage (V_0). It is determined by the product of the resistance (R) in the circuit and the capacitance (C) of the capacitor (RC=R×C).

Completion of Charging:

The capacitor continues to charge until the potential difference across its plates reaches the same value as the voltage of the source (V_0). At this point, the charging process is complete, and the capacitor is fully charged.

Current at any time t in R-C circuit while charging the Capacitor:

To define the current at any given time "t" in an R-C (resistor-capacitor) circuit during the process of charging the capacitor, we must consider the behavior of both the resistor and the capacitor in the circuit.

Voltage across the Capacitor at any time t :

When a capacitor is charging in an R-C circuit, the current at any time "t" can be calculated using Ohm's Law and the exponential charging equation:

$$I(t) = \frac{V}{R} \times e^{-\frac{t}{RC}}.$$

This equation shows that the current decreases over time as the capacitor charges up. Initially, when the capacitor is empty and the voltage across it is zero, the current is at its maximum value determined solely by the voltage and resistance. As the capacitor charges and its voltage increases, the current gradually decreases until it eventually approaches zero as the capacitor becomes fully charged.

Heat dissipated during charging in R-C circuit

The heat dissipated during the charging process in an R-C (resistor-capacitor) circuit refers to the energy that is converted into heat as the capacitor charges up. This dissipation of heat occurs due to the flow of current through the resistor in the circuit, which generates thermal energy.

To define the heat dissipated during charging in an R-C circuit, we can express it as the integral of the power dissipated over time:

$$Q_{\text{disnipated}} = \int_0^t P(t)dt$$

This integral represents the accumulation of the power dissipated over the entire duration of the charging process, providing a measure of the total heat energy converted from electrical energy.

Numerical on charging of capacitor

Ex. A capacitor with a capacitance of C=10μF is connected to a 12-volt battery through a resistor with resistance R = 5kΩ. Calculate the voltage across the capacitor after t = 2seconds from the start of charging.

Sol. Given data:

Capacitance, C = 10μF

Voltage of the battery, V_0 = 12volts

Resistance, R = 5kΩ

Time, t = 2seconds

First, we calculate the time constant (RC):

$$RC = 5k\Omega \times 10\mu F = 50ms$$

Using the formula for the voltage across the capacitor at time t:

Vc=V0(1−e−RCt)

Substituting the given values: $V_c = 12volts(1-e-\frac{2\ seconds}{50\ ms})$

Using exponential function properties, we find:

$$e^{-\frac{2}{3n}} = e^{-40} \approx 0.000027$$

Now, we can calculate:

$$V_c = 12 \text{ volts } (1 - 0.000027) \approx 12 \text{ volts } \times 0.999973 \approx 11.999 \text{ volts}$$

Therefore, the voltage across the capacitor after t = 2seconds from the start of charging is approximately 11.999 volts.

General R-C circuit

A general R-C (resistor-capacitor) circuit is a basic electronic circuit configuration that consists of a resistor (R) and a capacitor (C) connected in series to a voltage source (such as a battery). This type of circuit is widely used in electronics for various applications, including timing circuits, signal processing, and filtering.

In a general R-C circuit, the resistor limits the flow of current in the circuit, while the capacitor stores electrical charge. When the circuit is connected to a voltage source, the capacitor charges up or discharges depending on the polarity of the voltage source.

The behavior of a general R-C circuit can be described using the following principles:

Charging:

When the circuit is connected to a voltage source, the capacitor charges up gradually. Initially, the capacitor acts as a short circuit, allowing a large current to flow through the circuit. As the capacitor charges up, the current decreases exponentially over time, while the voltage across the capacitor increases.

Discharging:

If the voltage source is removed or reversed, the capacitor discharges through the resistor. Initially, the capacitor acts as a voltage source, causing a current to flow in the opposite direction. As the capacitor discharges, the voltage across it decreases exponentially over time.

The behavior of a general R-C circuit is characterized by the time constant (τ), which is the product of the resistance (R) and the capacitance (C). The time constant represents the time it takes for the voltage across the capacitor to reach approximately 63.2% of its final value during charging or to decrease to approximately 36.8% of its initial value during discharging.

Mathematically, the time constant (τ) of an R-C circuit is given by the equation:

$$\tau = R \times C$$

In summary, a general R-C circuit provides a fundamental building block in electronics and is essential for understanding the behavior of more complex circuits and systems.