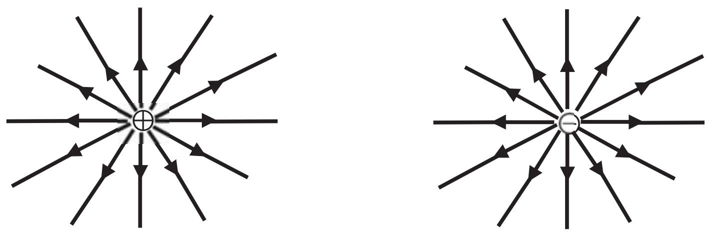
RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Properties of Electric field lines

The presence of an electric field is contingent upon the existence of an electric potential difference. In cases where charge distribution remains uniform across all points, regardless of the magnitude of the electric potential, there will be no electric field. Consequently, the interrelation between electric field and electric potential can be succinctly summarized as follows: "The electric field is the negative spatial derivative of the electric potential."



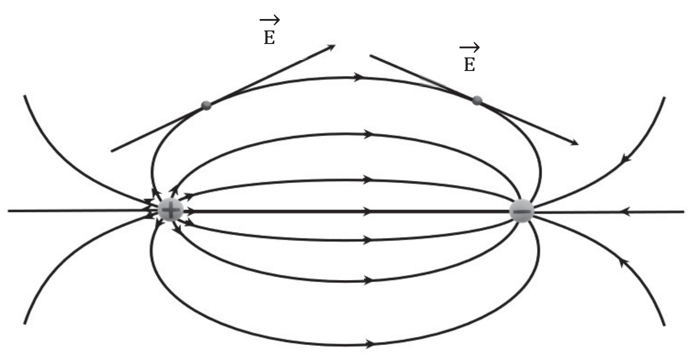
Radially Outwards   Radially Inwards

Mathematically, this relationship is represented by:

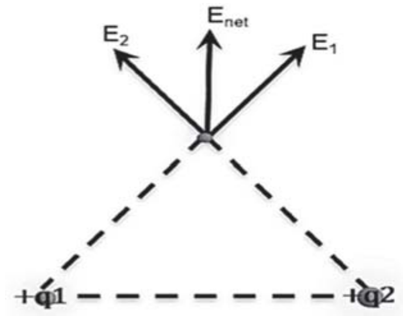
$$E = -\frac{dV}{dx}$$

Where:

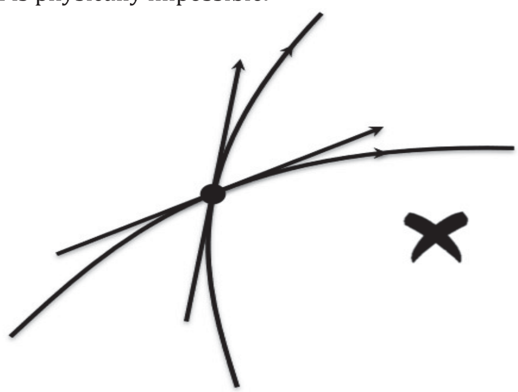
- E denotes the electric field.
- V represents the electric potential.
- dx signifies the path length.
- The negative sign in the equation indicates that the electric field always points from regions of higher electric potential to regions of lower electric potential.
- At any given point along an electric field line, the tangent to the line provides insight into the direction of both the electric field and the electric force acting at that precise location. This phenomenon is commonly referred to as electric lines of force. In essence, by examining the tangent to an electric field line, one can discern the orientation of the electric field and the force experienced by a charged particle placed at that particular point. This concept is pivotal in understanding and visualizing the behavior of electric fields and their effects on charged particles in electromagnetism.



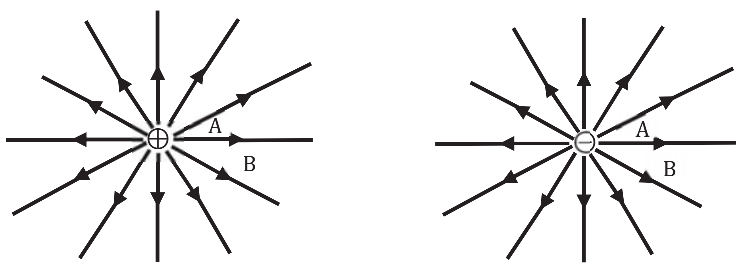
- The configuration of electric field lines consistently signifies the overall net electric field in a given region.



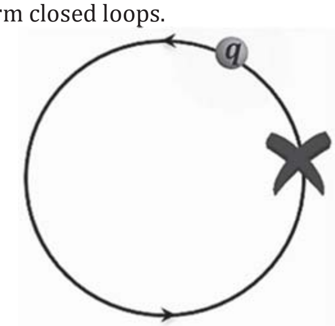
- Electric field lines never intersect one another. This is because at the point of intersection, there would be ambiguity in determining the direction of the electric field, as two conflicting directions would be indicated, which is physically impossible.



- In regions where the density of electric field lines is higher, the intensity or strength of the electric field will also be greater.

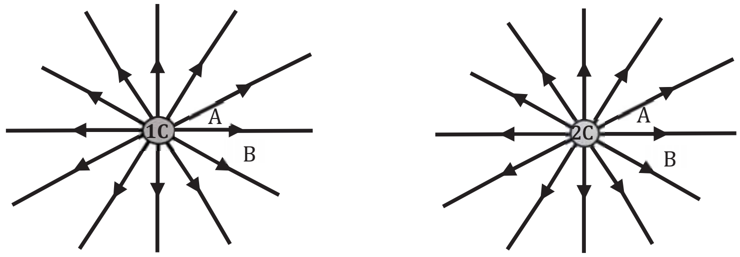


- The magnitude of the electric field intensity at point A $\left|\vec{E}_A\right|$ is greater than the magnitude of the electric field intensity at point B ($\left|\vec{E}_B\right|$).
- Electric field lines do not form closed loops.

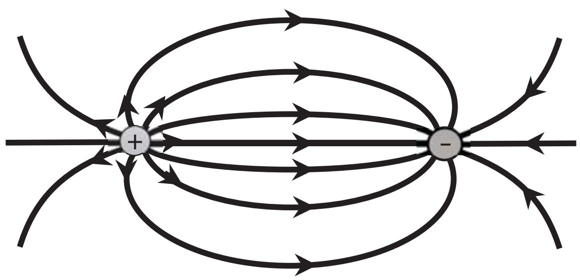


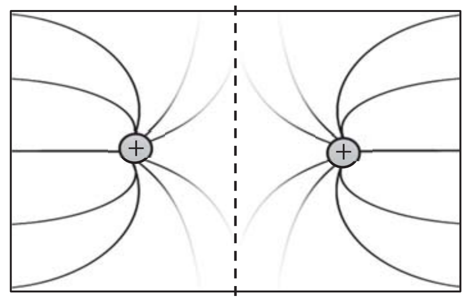
- If a closed loop is present, the work done along the loop is often believed to be W.n=qE$^2\pi$R. However, this contradicts the principle that the work done by a conservative force is zero for a closed loop, rendering this statement incorrect.
- If an electric field line originates from a positive charge, it cannot end on itself.

The quantity of electric field lines is directly proportional to the magnitude of the charge.

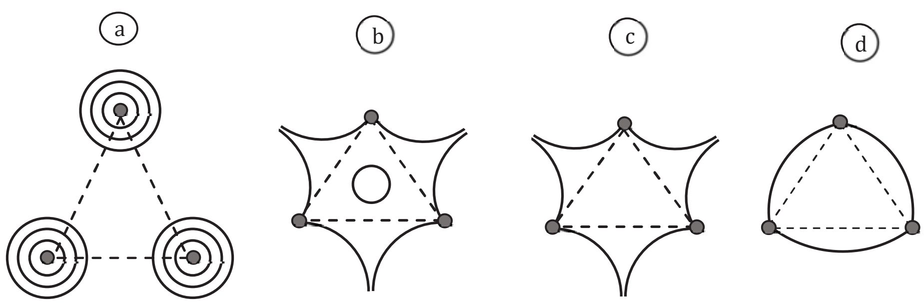


- The number of electric field lines originating from or terminating at a point is determined by the magnitude of the charge.
- Electric field lines invariably originate from a positive charge or from infinity and conclude at a negative charge or infinity.
- At any given point along an electric field line, the tangent indicates the direction of both the electric field and the electric force.
- These lines, known as electric lines of force, consistently represent the net electric field.
- They do not intersect because at points of intersection, there would be conflicting directions of the electric field, which is not feasible.
- When the density of electric field lines is high, the intensity of the electric field is correspondingly high, and vice versa.
- Electric field lines cannot form closed loops due to the conservative nature of the force field; consequently, the work done in a closed loop must be zero, which is not achievable.
- The number of electric field lines emanating from or entering a point is directly related to the magnitude of the charge.





Ex.: Three identical charges are positioned at the vertices of an equilateral triangle. Determine the accurate representation of the electric field lines for this configuration.

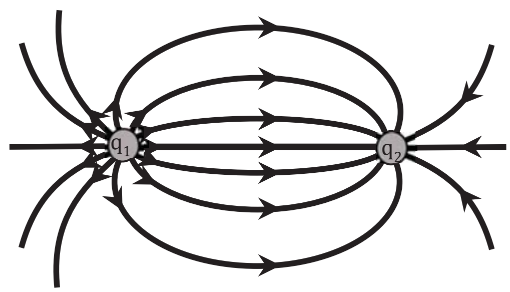


Solution:

- Since electric field lines do not form closed loops, options (a) and (b) are deemed incorrect.
- Additionally, electric field lines cannot originate and terminate at charges of the same polarity, rendering option (d) incorrect.
- Therefore, option (c) stands as the correct choice.

Ex: Select the appropriate choices.

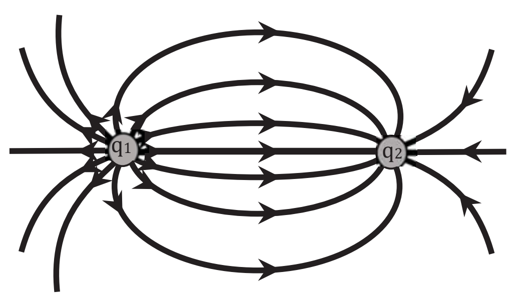
(a) $q_1 = +\text{ve}$, $q_2 = -\text{ve}$.
(b) $q_1 = -\text{ve}, q_2 = +\text{ve}$.
(c) $|q_1| > |q_2|$
(d) $|q_1| < |q_2|$.



Solution:

- Given that electric field lines originate from positive charges and terminate on negative charges, if q_1 is positive (+ve) and q_2 is negative (−ve), then option (a) is the correct choice.
- Since the quantity of electric field lines emanating from or entering a point is directly proportional to the magnitude of the charge, if the magnitude of q_1 is greater than the magnitude of q_2, denoted as $|q_1|>|q_2|$, then option (c) is also deemed correct.

Ex: Electric Field will be zero at_________



Solution: It is a known fact that the electric field strength becomes null along the line connecting two charges, particularly nearer to the charge with lesser magnitude when the charges exhibit opposite polarities.

Consequently, on the side to the right of the charge q_2, the electric field is rendered null.

When F(x,y,z) is given, $\Delta u = -\int F(x)dx + \int F(y)dy + \int F(z)dz$.
When U(x,y,z) is given, $\vec{F} = -\frac{\partial U}{\partial x}\hat{\imath} - \frac{\partial U}{\partial y}\hat{\jmath} - \frac{\partial U}{\partial z}\hat{k}$

} For conservative force

For electrostatics: $\vec{F} = q\vec{E}$ $U = qV$

The electric potential energy, U is calculated as qV, where q signifies charge and V represents electric potential.

$$F = -\frac{dU}{dr}$$

The force (F) is equal to the negative derivative of the electric potential energy (U) with respect to the distance (r), which can be expressed as $-\frac{dU}{dr}$.

in general,

$$\vec{E} = \frac{\partial U}{\partial x}\hat{\imath} - \frac{\partial U}{\partial y}\hat{\jmath} - \frac{\partial U}{\partial z}\hat{k}$$

Ex.: Given the potential variation expressed as U=x²+2y+z, determine the force at the point P (1,1,1).

Solution:

the expression is $U=x^2+2y+z$,

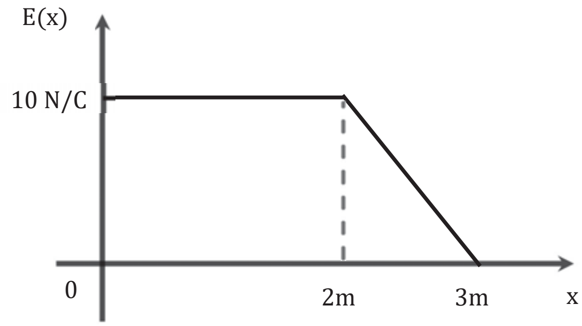
Partial differentiation:

$$\frac{\partial y}{\partial x} = 2x \ \frac{\partial u}{\partial y} = 2 \ \frac{\partial u}{\partial z} = 1$$
$$\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$
$$\vec{F} = -2\hat{i} - 2\hat{j} - \hat{k}$$

Ex.: If the electric potential at the origin is V (0,0) =10Volt, determine the electric potential at the point V (3,0).

Solution:

We have, $V_i - V_f = \int \vec{E} \cdot d\vec{x}$



Hence, the disparity in electric potential between the initial and final positions corresponds to the area beneath the E−x curve.

$$V_i - V_f = \frac{1}{2}(2+3) \times 10$$
$$V(0,0) - V(3,0) = 25$$
$$V(3,0) = 10 - 25 = -15V$$

The electric potential is recorded as -15 volts.

Relation between Electric field and Potential

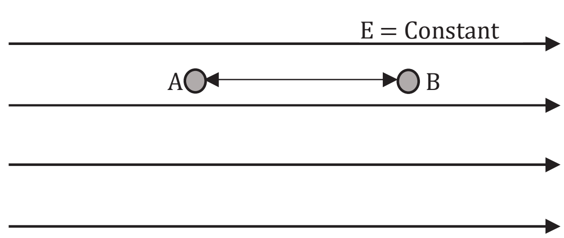
We have, $V_i - V_f = \int \vec{E} \cdot d\vec{r}$

In situations where the electric field remains uniform or steady,

$$V_i - V_f = \vec{E} \cdot \Delta\vec{r}$$
$$V_i - V_f = \vec{E} \cdot (\vec{r_f} - \vec{r_i})$$

Displacement vector: $\vec{r} = \vec{r_f} - \vec{r_i}$

$$V_i - V_f = \vec{E} \cdot \vec{r}$$
$$v_v - v_f = |\vec{E}||\vec{v}| \cos\theta$$

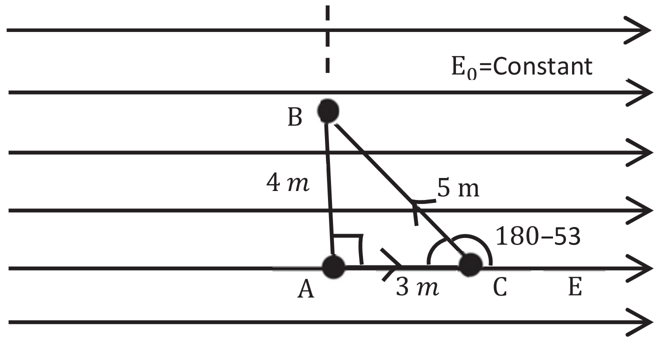


In figure, $\theta = 0°$, $|\vec{r}| = d$

$V_A - V_B = Ed \quad V_A > V_B$

Electric field lines progress from regions of higher electric potential to regions of lower electric potential.

Ex. Determine the difference in electric potential between the specified points.



Solution:

We have, $V_i - V_f = \int \vec{E} \cdot d\vec{r}$

As the electric field is constant, $V_1 - V_f = |\hat{E}||\bar{Y}|Cos\theta$

$V_A – V_C = E_0 \times 3 \times \cos 0^o = 3E_o$

$V_{A-}V_B = E_o \times 4 \times \cos 90^o = 0$

$V_A–V_C$	$3E_o$
$V_A–V_B$	0
$V_C–V_B$	$-3E_o$

$VC - VB = Eo \times 5 \times \cos (180 – 53)°$

$= 5Eo \times - 3\ 5 = -3Eo$

Because, $\text{Cos } (180\text{-}53)^o$

$= \text{-Cos } 53^o = \text{-}3/5$