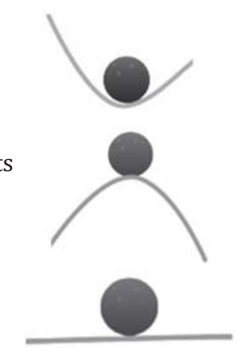
PROBLEMS ON EQUILIBRIUM AND LAMI'S THEOREM

**Equilibrium:**

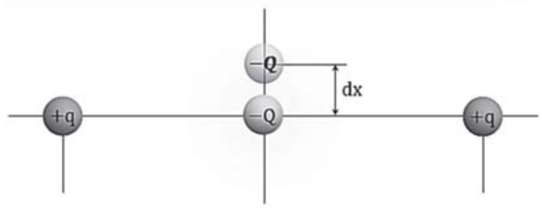
Equilibrium is a state for a mass/charge/particle when its state of motion remains unaffected, which implies that the net force acting on it is zero. Since the force is interconnected with the field, the previous statement also implies that the net field acting has to be zero.

**Types of equilibrium:**



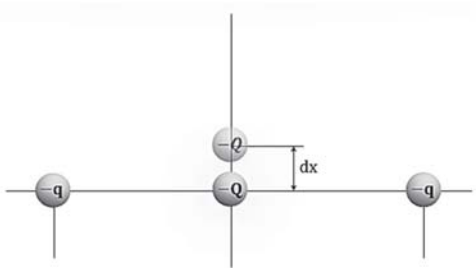
1. **Stable equilibrium:**
   In this case, if a particle is subjected to an impactive force, it tries to go back to its initial state of motion.
2. **Unstable equilibrium:**
   In this case, if a particle is subjected to an impactive force, it cannot go back to its initial state of motion.
3. **Neutral equilibrium:**
   In this case, if a particle is subjected to an impactive force, it gets displaced, but its current and initial states of motion have no difference.

## Stable And Unstable Equilibrium



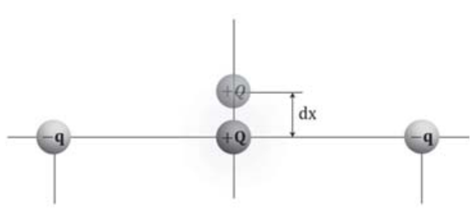

STABLE

When a charged particle experiences a slight displacement from its equilibrium position, and the resultant force restores it back to the original position, it is considered to be in stable electrostatic equilibrium. In this scenario, if we slightly displace the -Q charge perpendicular to the line connecting the two +q charges, it will tend to return to its initial position. The combined electrostatic force generated by the two positive charges will pull it back towards the initial position.



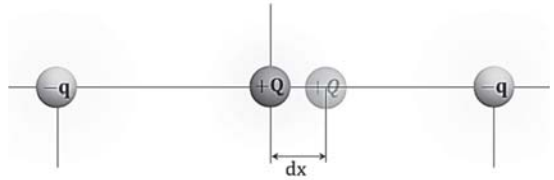

UNSTABLE

If a charged particle undergoes a slight displacement from its equilibrium position and the resulting force attempts to push the particle further from the equilibrium position, it is considered to be in an unstable electrostatic equilibrium. In this instance, since all three charges are negative, the collective force repels the charge -Q away from its initial position.




STABLE

In this scenario, if we make a slight displacement of the +Q charge perpendicular to the line connecting the two -q charges, it will tend to return to its original position. The combined electrostatic force generated by the two negative charges will pull it back towards the initial position. Since the charge being considered has the opposite sign compared to the other two charges, the resulting net force will be attractive.

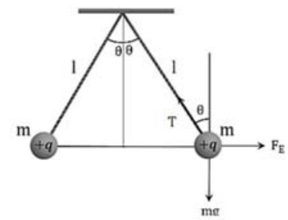



UNSTABLE

In this scenario, if we slightly displace the +Q charge along the line connecting the two -q charges, it will move in the direction of the displacement. As the distance between them diminishes, the electrostatic force will intensify. Consequently, the net electrostatic force generated by the two negative charges will act in the direction of the displacement. Thus, the charge will be in an unstable equilibrium. If all three charges have the same sign (i.e., all negative or all positive), and we slightly displace the center charge in either direction along the line connecting the other two charges, the center charge will undergo simple harmonic motion (SHM).

**Pendulum Problem**

**At equilibrium**



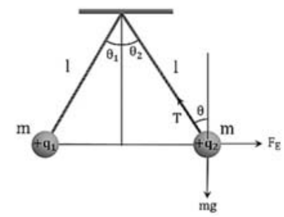

Balancing the forces in y-direction: $= T\cos\theta = mg$

Balancing the forces in x-direction: $= T\sin\theta = \frac{kq^2}{4l^2 - \sin^2\theta}$

**Ex.** Find the mathematical relation between $\theta_1$ and $\theta_2$ given that $q_1 > q_2$?

(a) $\theta_1 > \theta_2$ (b) $\theta_2 > \theta_1$

(c) $\theta_1 = \theta_2$ (d) $\theta_1 \geq \theta_2$



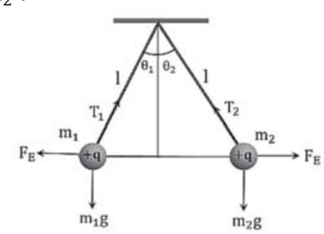

**Sol.** As we can see, the electrostatic force acting on these two charges is same. The length of the string and their masses are equal. So the angle by which they are rotated from the mean position will be equal.

The correct answer is (c) $\theta_1 = \theta_2$

**Ex.** Find the mathematical relation between $\theta_1$ and $\theta_2$ given that $m_1 > m_2$ ?

(a) $\theta_1 > \theta_2$ (b) $\theta_1 = \theta_2$

(c) $\theta_2 < \theta_1$ (d) $\theta_1 \geq \theta_2$

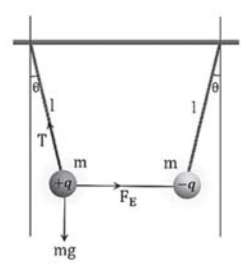



**Sol.** As we can see, the electrostatic force acting on these two charges is same. The length of the string is equal for both charges. But their mass is different. So the heavier charge will shift more towards the mean position. Hence it will have smaller angle subtended with the normal to the mean position.

The correct answer is (c) $\theta_2 < \theta_1$

**Pendulum Problems**

**At equilibrium**




$$f_\varepsilon = \frac{kq^2}{(d - 2l\sin\theta)^2}$$

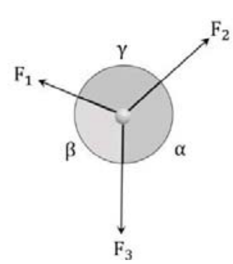
$$T\cos\theta = mg \quad \text{... (1)}$$

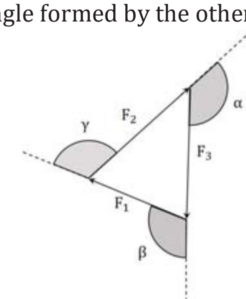
$$T\sin\theta = \frac{kq^2}{(d - 2ls\ \ \theta)^2} \quad \text{... (2)}$$

FBD of a charge at equilibrium,

**LAMI'S Theorem**

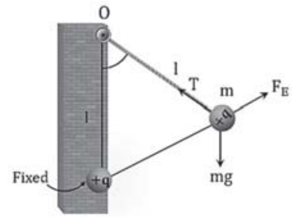
When three coplanar, concurrent forces act on a body to maintain it in static equilibrium, each force is directly proportional to the sine of the angle formed by the other two forces.




$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

**Ex.** A charge +q of mass $m$ is attached to a massless string of length l, the string is hinged at point O and is free to rotate about the hinge. Another charge +q is placed on the vertical wall at a depth of l from point O. Find the tension (T) in the string when the system reaches its equilibrium state.




**Sol.** From the triangle:

$$\theta + 2x = \pi$$

As the system is at equilibrium. So, by using Lami's theorem:

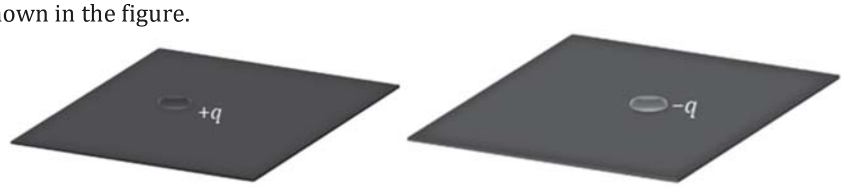
$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

$$\frac{T}{\mathrm{Sin}(\pi - x)} = \frac{mg}{\mathrm{Sin}(\pi - x)}$$

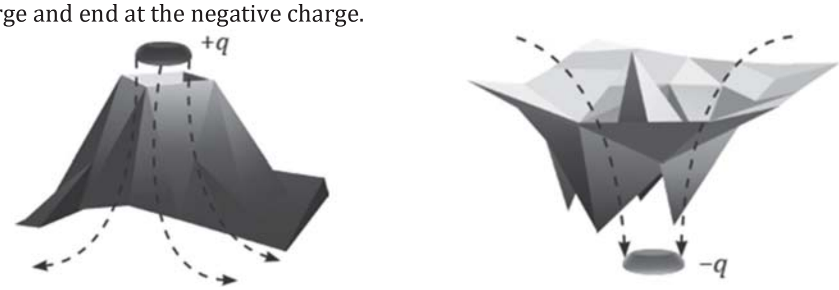
$$= T = mg$$

**Layman's View of Geography of Electric Charges:**

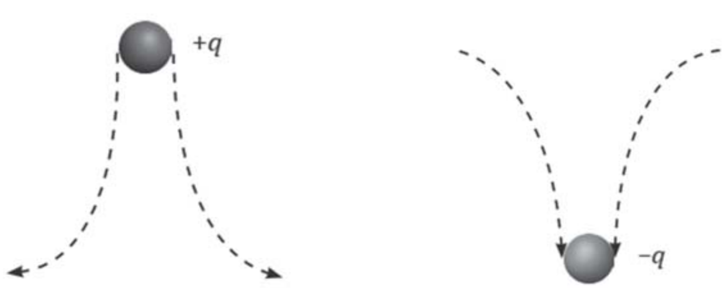
The electric charges also possess different states of equilibrium. To begin the discussion, we should remember that the electric field lines always begin on a positive charge and end on a negative charge. When we talk about positive and negative charges, we can consider them on the same level, as shown in the figure.




However, if the concept of field lines is incorporated with positive and negative charges, then the position of the positive charge can be thought of as the top of a hill from which a fountain originates. It is because field lines originate from the positive charge. The position of the negative charge can be thought of as a valley or the bottom of a funnel, because field lines originate from the positive charge and end at the negative charge.

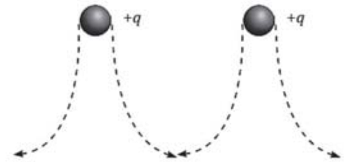



Hence, it can be said that the positive charge acts as the source and the negative charge acts as the sink. Therefore, it can be concluded that from the topological point of view, the positive charge and the negative charge are not at the same level. The side views of the geography of the charges are shown below.

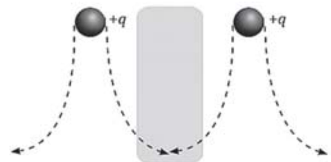


**Equilibrium of electric charges:**

When two positive charges of an equal magnitude are placed side by side, from layman's view of the geography of charges, they can be seen as shown in the figure.

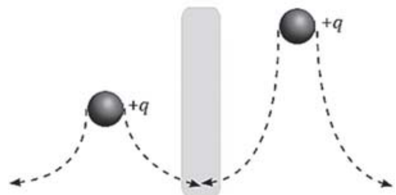



From the figure, the position of two positive charges of equal magnitude can be thought of as two different peaks of a hill of the same height. Now, the question arises that is there any possibility of equilibrium in this arrangement? Yes, the valley or region between the two peaks is the stable equilibrium position.

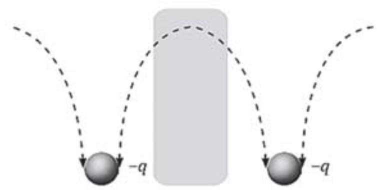



Zone of possibility for **stable equilibrium**

Suppose that two positive charges of unequal magnitude are placed side by side. For this case as well, the valley or region between the two peaks is at the stable equilibrium position, but the region becomes smaller here and it is closer to the charge with less magnitude.




When two negative charges of equal magnitude are placed side by side, from layman's view of the geography of charges, the position of the two negative charges of equal magnitude can be thought of as bases of funnels of the same depth. Now, the question arises that is there any possibility of equilibrium in this arrangement? Yes, the region (the peak) between the two charges is the unstable equilibrium position.



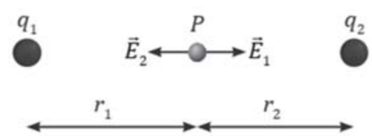

Zone of possibility for **unstable equilibrium**

**Null Point:**

The null point is a position where the net field turns out to be zero as a vector sum.

Consider two point charges $q_1$ and $q_2$ as shown in the figure.

For the net field to be zero at point P, the following should be true:



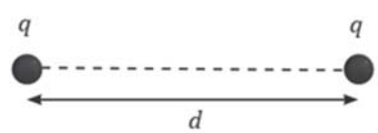

$$\vec{E}_{net} = \vec{0}$$

$$\vec{E}_1 + \vec{E}_2 = \vec{0}$$

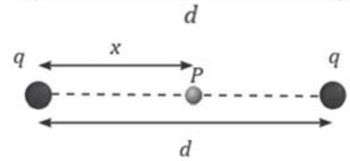
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1^2} - \frac{q_2}{r_2^2}\right] = 0 \left[\begin{array}{c}\text{The negative sign arises because the directions} \\ \text{of } \vec{E}_1 \text{ and } \vec{E}_2 \text{ are opposite.}\end{array}\right]$$

$$\frac{q_1}{r_1^2} = \frac{q_2}{r_2^2}$$

**Ex.** Find the position along the line joining two point charges where the net electric field is zero.




**Sol.** Let the electric field be zero at point P, which is x distance away from the charge on the left as shown in the figure.




Since both the charges are positive, the direction of the electric field on P due to these charges will be opposite.

Also, we assumed the net electric field at point P to be zero.

$$\vec{E}_{ngt} = \vec{0}$$

$$|\vec{E}_1| = |\vec{E}_2|$$

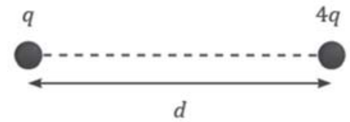
$$\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(d-x)^2}$$

$$x = \pm(d - x)$$

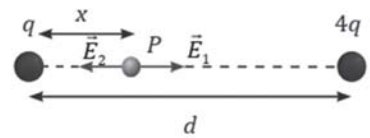
$x = \frac{d}{2}$ [By taking only the positive value]

Therefore, the equilibrium position or the null point will be at the middle of the line joining the two equal positive charges.

**Ex.** Find the position along the line joining two point charges where the net electric field is zero.




**Sol.** Let the electric field be zero at point P, which is x distance away from the charge at the left.




Since both the charges are positive, the direction of the electric field at P due to these charges will be opposite.

Also, we assumed the net electric field at point P to be zero.

$$\vec{E}_{net} = \vec{0}$$

$$|\vec{E}_1| = |\vec{E}_2|$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{4q}{(d-x)^2}$$

$$4x^2 = (d-x)^2$$

$$2x = \pm(d-x)$$

$$2x = (d-x)$$

[By taking only the positive value]

$$x = \frac{d}{3}$$

Therefore, the equilibrium position or the null point will be closer to the smaller positive charge along the line joining the two unequal positive charges.